

Pacific Journal of Mathematics

**ANALYTIC SHEAF COHOMOLOGY GROUPS OF DIMENSION
 n OF n -DIMENSIONAL NONCOMPACT COMPLEX
MANIFOLDS**

YUM-TONG SIU

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In this paper the following question is considered: if X is a σ -compact noncompact complex manifold of dimension n and \mathcal{F} is a coherent analytic sheaf on X , does $H^n(X, \mathcal{F})$ always vanish? The answer is in the affirmative.

This question was first proposed by Malgrange in [6] and in the same paper he gave the affirmative answer for the special case when \mathcal{F} is locally free.

THEOREM. *If X is an n -dimensional σ -compact noncompact complex manifold and \mathcal{F} is a coherent analytic sheaf on X , then $H^n(X, \mathcal{F}) = 0$.*

Proof. I. For $0 \leq p \leq n$ let $\mathcal{A}^{(0,p)}$ denote the sheaf of germs of $C^\infty(0, p)$ -forms on X and \mathcal{O} denote the structure-sheaf of X . Since at a point in a complex number space the ring of C^∞ function-germs as a module over the ring of holomorphic function-germs is flat ([7], Ths. 1 and 2 bis), the sequence

$$0 \longrightarrow \mathcal{F} \longrightarrow \mathcal{A}^{(0,0)} \otimes_{\mathcal{O}} \mathcal{F} \xrightarrow{\bar{\partial}'} \dots \\ \xrightarrow{\bar{\partial}'} \mathcal{A}^{(0,n-1)} \otimes_{\mathcal{O}} \mathcal{F} \xrightarrow{\bar{\partial}'} \mathcal{A}^{(0,n)} \otimes_{\mathcal{O}} \mathcal{F} \longrightarrow 0$$

obtained by tensoring

$$0 \longrightarrow \mathcal{O} \longrightarrow \mathcal{A}^{(0,0)} \xrightarrow{\bar{\partial}} \dots \xrightarrow{\bar{\partial}} \mathcal{A}^{(0,n-1)} \xrightarrow{\bar{\partial}} \mathcal{A}^{(0,n)} \longrightarrow 0$$

with \mathcal{F} over \mathcal{O} is exact (cf. [8], Th. 3).

The theorem follows if we can prove that

$$\beta_X: \Gamma(X, \mathcal{A}^{(0,n-1)} \otimes_{\mathcal{O}} \mathcal{F}) \longrightarrow \Gamma(X, \mathcal{A}^{(0,n)} \otimes_{\mathcal{O}} \mathcal{F})$$

induced from

$$\bar{\partial}': \mathcal{A}^{(0,n-1)} \otimes_{\mathcal{O}} \mathcal{F} \longrightarrow \mathcal{A}^{(0,n)} \otimes_{\mathcal{O}} \mathcal{F}$$

is surjective.

II. Suppose $0 \leq p \leq n$ and

$$\mathcal{O}^r \xrightarrow{\phi} \mathcal{O}^s \xrightarrow{\psi} \mathcal{F} \longrightarrow 0$$

is an exact sequence of sheaf-homomorphisms on an open subset U of

X which is biholomorphic to an open subset of \mathbb{C}^n . Tensoring the sequence with $\mathcal{A}^{(0,p)}$ over \mathcal{O} , we obtain an exact sequence

$$(\mathcal{A}^{(0,p)})^r \xrightarrow{\phi'} (\mathcal{A}^{(0,p)})^s \xrightarrow{\psi'} \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F} \longrightarrow 0.$$

Since $\text{Im } \phi'$ and $\text{Ker } \phi'$ are fine sheaves,

$$\Gamma(U, (\mathcal{A}^{(0,p)})^r) \xrightarrow{\tilde{\phi}} \Gamma(U, (\mathcal{A}^{(0,p)})^s) \xrightarrow{\tilde{\psi}} \Gamma(U, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F}) \longrightarrow 0$$

is exact. $\Gamma(U, (\mathcal{A}^{(0,p)})^s)$ is a Fréchet space if it is given the topology of uniform convergence of derivatives of coefficients on compact subsets. Since $\tilde{\phi}$ is defined by a matrix of holomorphic functions, by paragraph 1 of [7], $\text{Im } \tilde{\phi}$ is a closed subspace of $\Gamma(U, (\mathcal{A}^{(0,p)})^s)$ (cf. [8], Th. 5). We give $\Gamma(U, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$ the quotient topology and it becomes a Fréchet space.

Suppose G is an open subset of X . We can find a countable Stein open cover $\{U_k\}_{k=1}^\infty$ of G such that U_k is biholomorphic to an open subset of \mathbb{C}^n and on U_k we have an exact sequence of sheaf-homomorphisms

$$\mathcal{O}^{r_k} \xrightarrow{\phi_k} \mathcal{O}^{s_k} \xrightarrow{\psi_k} \mathcal{F} \longrightarrow 0.$$

We give $\Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$ the smallest topology that makes every restriction map $\Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F}) \rightarrow \Gamma(U_k, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$ continuous. This topology of $\Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$ is independent of the choices of $\{U_k\}$, $\{\phi_k\}$, and $\{\psi_k\}$. $\Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$ is a Fréchet space.

$$\beta_G: \Gamma(G, \mathcal{A}^{(0,n-1)} \otimes_{\mathcal{O}} \mathcal{F}) \longrightarrow \Gamma(G, \mathcal{A}^{(0,n)} \otimes_{\mathcal{O}} \mathcal{F})$$

induced from

$$\bar{\partial}': \mathcal{A}^{(0,n-1)} \otimes_{\mathcal{O}} \mathcal{F} \longrightarrow \mathcal{A}^{(0,n)} \otimes_{\mathcal{O}} \mathcal{F}$$

is continuous (cf. [8], pp. 21–24).

III. Suppose G is an open subset of X . Denote the strong dual of $\Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$ by $(\Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F}))^*$, $0 \leq p \leq n$. Suppose $T \in (\Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F}))^*$. The support of T , denoted by $\text{Supp } T$, is defined as the complement in G of the largest open subset H such that, if $a \in \Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$ and $\text{Supp } a \subset H$, then $T(a) = 0$. $\text{Supp } T$ is well-defined, because H exists by partition of unity. Observe that, if $a_k \in \Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$ and for every compact subset K of G $(\bigcup_{k=m}^\infty \text{Supp } a_k) \cap K = \emptyset$ for some m depending on K , then $a_k \rightarrow 0$ in $\Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$. We have:

- (1) If V is a bounded subset of $(\Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F}))^*$, then there is a compact subset K of G such that $\text{Supp } T \subset K$ for $T \in V$.

IV. Suppose G is an open subset of X . Fix

$$T \in (\Gamma(G, \mathcal{A}^{(0,n)} \otimes \mathcal{F}))^*$$

and let $\text{Supp } T\beta_G = K$. Let \hat{K} denote the union of K together with all the components of $G - K$ relatively compact in G . We are going to prove that $\text{Supp } T \subset \hat{K}$. Let L be a component of $G - K$ not relatively compact in G . We need only prove that $L \cap \text{Supp } T = \emptyset$. Suppose the contrary. Since L is not relatively compact in G , $L \not\subset \text{Supp } T$ ($\text{Supp } T$ is compact by (1)). $\text{Supp } T$ has a boundary point x_0 in L . We would have a contradiction if we can prove:

- (2) Every boundary point x of $\text{Supp } T$ is a boundary point of $\text{Supp } T\beta_G$.

To prove (2) we suppose that x is a boundary point of $\text{Supp } T$ and x is not a boundary point of $\text{Supp } T\beta_G$. Since $\text{Supp } T\beta_G \subset \text{Supp } T$, $x \in X - \text{Supp } T\beta_G$. On some connected open neighborhood D of x in $X - \text{Supp } T\beta_G$ we have a sheaf-epimorphism $\theta: \mathcal{O}^s \rightarrow \mathcal{F}$. Tensoring it with $\mathcal{A}^{(0,p)}$ over \mathcal{O} , we obtain a sheaf-epimorphism $\theta'_p: (\mathcal{A}^{(0,p)})^s \rightarrow \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F}$. $\tilde{\theta}_p: \Gamma(D, (\mathcal{A}^{(0,p)})^s) \rightarrow \Gamma(D, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$ induced by θ'_p is surjective.

Let $\{N_k\}_{k=1}^\infty$ be a sequence of compact subsets of D such that $N_k \subset \text{Int } N_{k+1}$ and $\bigcup_{k=1}^\infty N_k = D$. Let $\Gamma_{N_k}(D, (\mathcal{A}^{(0,p)})^s)$ be the set of all elements of $\Gamma(D, (\mathcal{A}^{(0,p)})^s)$ having supports contained in N_k . Give $\Gamma_{N_k}(D, (\mathcal{A}^{(0,p)})^s)$ the topology induced from $\Gamma(D, (\mathcal{A}^{(0,p)})^s)$. Give $\Gamma_*(D, (\mathcal{A}^{(0,p)})^s) = \bigcup_{k=1}^\infty \Gamma_{N_k}(D, (\mathcal{A}^{(0,p)})^s)$ the topology as the strict inductive limit of $\{\Gamma_{N_k}(D, (\mathcal{A}^{(0,p)})^s)\}$. $\Gamma_*(D, (\mathcal{A}^{(0,p)})^s)$ and its topology are independent of the choice of $\{N_k\}$.

For $a \in \Gamma_*(D, (\mathcal{A}^{(0,p)})^s)$, since $\text{Supp } \tilde{\theta}_p(a) \subset D$ is compact, $\tilde{\theta}_p(a)$ can be trivially extended to an element $(\tilde{\theta}_p(a))' \in \Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$. The map $\xi_p: \Gamma_*(D, (\mathcal{A}^{(0,p)})^s) \rightarrow \Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$ defined by $\xi_p(a) = (\tilde{\theta}_p(a))'$ is a continuous linear map.

- (3) If $b \in \Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$ and $\text{Supp } b$ is a compact subset of D , then $b \in \text{Im } \xi_p$.

The following diagram is commutative:

$$\begin{array}{ccc} \Gamma_*(D, (\mathcal{A}^{(0,n-1)})^s) & \xrightarrow{\tilde{\delta}} & \Gamma_*(D, (\mathcal{A}^{(0,n)})^s) \\ \xi_{n-1} \downarrow & & \xi_n \downarrow \\ \Gamma(G, \mathcal{A}^{(0,n-1)} \otimes_{\mathcal{O}} \mathcal{F}) & \xrightarrow{\beta_G} & \Gamma(G, \mathcal{A}^{(0,n)} \otimes_{\mathcal{O}} \mathcal{F}) \end{array}$$

Since $\text{Supp } (T\beta_G) \cap D = \emptyset$, $T\xi_n\tilde{\delta} = T\beta_G\xi_{n-1} = 0$. $T\xi_n$ can be represented by an s -tuple of distribution- $(n, 0)$ -forms on D (cf. the argument on p. 42, [2]). $T\xi_n\tilde{\delta} = 0$ implies that $T\xi_n$ can be represented by an s -tuple of holomorphic $(n, 0)$ -forms on D . Since $\text{Supp } T\xi_n \subset \text{Supp } T$ and $D \not\subset \text{Supp } T$, the s -tuple of holomorphic forms representing $T\xi_n$ must be identically zero. Hence $T\xi_n = 0$. By (3) $\text{Supp } T$ is disjoint from all compact subsets of D . x is not a boundary point of $\text{Supp } T$.

Hence (2) is proved. We have:

(4) $\text{Supp } T \subset (\widehat{\text{Supp } T\beta_G})$ for $T \in (\Gamma(G, \mathcal{A}^{(0,n)} \otimes_{\mathcal{O}} \mathcal{F}))^*$.

Denote the transpose of β_G by $(\beta_G)^*$. (4) implies that

(5) $(\beta_G)^*$ is injective,

because every component of G is noncompact.

V. By Lemma 3, [6], we have:

(6) For every point x of X there is an open neighborhood U of x in X such that $H^n(W, \mathcal{F}) = 0$ for every open subset W of U .

Suppose K is a compact subset of X . By (6) we can find two finite collections $\mathfrak{U}, \mathfrak{B} = \{B_k\}_{k=1}^m$ of relatively compact open Stein subsets of X such that (i) both \mathfrak{U} and \mathfrak{B} cover K ; (ii) intersections of subcollections of \mathfrak{U} and intersections of subcollections of \mathfrak{B} are Stein; (iii) the closure of any member of \mathfrak{U} is contained in some member of \mathfrak{B} ; and (iv) for any open subset W of any $B_k, 1 \leq k \leq m, H^n(W, \mathcal{F}) = 0$.

Let G and H be respectively the union of all the members of \mathfrak{U} and \mathfrak{B} . Define inductively $G_0 = G$ and $G_k = G_{k-1} \cup B_k, 1 \leq k \leq m$. $H^n(G_k, \mathcal{F}) \rightarrow H^n(G_{k-1}, \mathcal{F}) \oplus H^n(B_k, \mathcal{F}) \rightarrow H^n(G_{k-1} \cap B_k, \mathcal{F})$ is exact (Part a of § 17, [1]). $H^n(G_{k-1} \cap B_k, \mathcal{F}) = 0$ implies that the restriction map $H^n(G_k, \mathcal{F}) \rightarrow H^n(G_{k-1}, \mathcal{F})$ is surjective for $1 \leq k \leq m$. Since $H = G_m$, the restriction map $H^n(H, \mathcal{F}) \rightarrow H^n(G, \mathcal{F})$ is surjective. $H^n(G, \mathcal{F})$ is finite-dimensional (cf. Proof of Th. 11, § 17, [1]). Since $H^n(G, \mathcal{F}) \approx \text{Coker } \beta_G$, $\text{Im } \beta_G$ is closed. $\text{Im } (\beta_G)^*$ is weakly closed ([5], Préliminaires, § 3, Th. 2). Therefore we have:

(7) Every compact subset K of X has an open neighborhood G in X such that $\text{Im } (\beta_G)^*$ is weakly closed.

VI. By (5) and Th. 2, § 3, Préliminaires, [5], the theorem follows if we can prove that the intersection of $\text{Im } (\beta_X)^*$ with every weakly compact subset of $(\Gamma(X, \mathcal{A}^{(0,n-1)} \otimes_{\mathcal{O}} \mathcal{F}))^*$ is weakly compact. Suppose V is a weakly compact subset of $(\Gamma(X, \mathcal{A}^{(0,n-1)} \otimes_{\mathcal{O}} \mathcal{F}))^*$. V is strongly bounded ([3], Th. 3). By (1) there exists a compact subset K of X such that

(8) $\text{Supp } S \subset K$ for $S \in V$.

\hat{K} is compact ([5], Chap. IV, § 3, Lemma 3). By (7) there exists an open neighbourhood G of \hat{K} in X such that $\text{Im } (\beta_G)^*$ is weakly closed. By (4) and (8) we have:

(9) $\text{Supp } T \subset \hat{K}$ if $T \in (\Gamma(X, \mathcal{A}^{(0,n)} \otimes_{\mathcal{O}} \mathcal{F}))^*$ and $T\beta_X \in V$.

Let g be a C^∞ function on G having compact support and being identically one on some neighborhood of \hat{K} . Suppose $0 \leq p \leq n$. Let $\sigma_p: \Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F}) \rightarrow \Gamma(X, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$ be defined by trivial extension after multiplication by g . σ_p is continuous. Let $\rho_p: \Gamma(X, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F}) \rightarrow \Gamma(G, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F})$ be the restriction map.

(10) If $R \in (\Gamma(X, \mathcal{A}^{(0,p)} \otimes_{\mathcal{O}} \mathcal{F}))^*$ and $\text{Supp } R \subset \hat{K}$, then $R\sigma_p\rho_p = R$.

To prove that $\text{Im } (\beta_X)^* \cap V$ is weakly compact, it suffices to prove that it is weakly closed. Suppose $\{S_i\}_{i \in I}$ is a net in $\text{Im } (\beta_X)^* \cap V$ con-

verging weakly to $S \in V$. By (8) $\text{Supp } S \subset K$. $S_i = T_i \beta_X$ for some $T_i \in (\Gamma(X, \mathcal{A}^{(0,n)} \otimes_{\mathcal{O}} \mathcal{F}))^*$. By (9) $\text{Supp } T_i \subset \hat{K}$. $\text{Supp } T_i \sigma_n \subset \hat{K}$ and $\text{Supp } S \sigma_{n-1} \subset \hat{K}$. The following diagram is commutative:

$$\begin{CD} \Gamma(X, \mathcal{A}^{(0,n-1)} \otimes_{\mathcal{O}} \mathcal{F}) @>\beta_X>> \Gamma(X, \mathcal{A}^{(0,n)} \otimes_{\mathcal{O}} \mathcal{F}) \\ @V\rho_{n-1}VV @VV\rho_nV \\ \Gamma(G, \mathcal{A}^{(0,n-1)} \otimes_{\mathcal{O}} \mathcal{F}) @>\beta_G>> \Gamma(G, \mathcal{A}^{(0,n)} \otimes_{\mathcal{O}} \mathcal{F}) . \end{CD}$$

Take $a \in \Gamma(G, \mathcal{A}^{(0,n-1)} \otimes_{\mathcal{O}} \mathcal{F})$. Let $b = \sigma_{n-1}(a) \in \Gamma(X, \mathcal{A}^{(0,n-1)} \otimes_{\mathcal{O}} \mathcal{F})$. Then $\rho_{n-1}(b) = ga$. Since $\hat{K} \cap \text{Supp } \beta_G(a - ga) = \emptyset$,

$$T_i \sigma_n \beta_G(a) = T_i \sigma_n \beta_G(ga) = T_i \sigma_n \beta_G \rho_{n-1}(b) = T_i \sigma_n \rho_n \beta_X(b) = T_i \beta_X(b)$$

by (10). Since $\hat{K} \cap \text{Supp } (a - ga) = \emptyset$,

$$S \sigma_{n-1}(a) = S \sigma_{n-1}(ga) = S \sigma_{n-1} \rho_{n-1}(b) = S(b) .$$

Since $T_i \beta_X(b) \rightarrow S(b)$, $T_i \sigma_n \beta_G(a) \rightarrow S \sigma_{n-1}(a)$. Hence $T_i \sigma_n \beta_G \rightarrow S \sigma_{n-1}$ in the weak topology of $(\Gamma(G, \mathcal{A}^{(0,n-1)} \otimes_{\mathcal{O}} \mathcal{F}))^*$. Since $\text{Im } (\beta_G)^*$ is weakly closed, there exists $T' \in (\Gamma(G, \mathcal{A}^{(0,n)} \otimes_{\mathcal{O}} \mathcal{F}))^*$ such that $T' \beta_G = S \sigma_{n-1}$. Let $T = T' \rho_n$. Then

$$T \beta_X = T' \rho_n \beta_X = T' \beta_G \rho_{n-1} = S \sigma_{n-1} \rho_{n-1} = S .$$

$S \in \text{Im } (\beta_X)^* \cap V$. $\text{Im } (\beta_X)^* \cap V$ is weakly closed.

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