

Pacific Journal of Mathematics

***F'*-SPACES AND z -EMBEDDED SUBSPACES**

MARK MANDELKER

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A completely regular Hausdorff space is an F' -space if disjoint cozero-sets have disjoint closures. Here the theory of prime z -filters is applied to the study of F' -spaces. A z -embedded subspace is one in which the zero-sets are all intersections of the subspace with zero-sets in the larger space. It is shown that every z -embedded subspace of an F' -space is also an F' -space. Also, a new characterization of F' -spaces is obtained: Every z -embedded subspace is C^* -embedded in its closure.

F' - and F'' -spaces were introduced in [4] in connection with the study of finitely generated ideals in rings of continuous functions; further results on F' -spaces are found in [1] and [2].

Throughout this paper we shall use the terminology and notation of the Gillman-Jerison treatise [5]. Only completely regular Hausdorff spaces will be considered.

As noted above, a subspace Y of a space X is z -embedded in X if for every zero-set Z in Y there is a zero-set W in X such that $Z = W \cap Y$. For example, a C^* -embedded subspace is clearly z -embedded; also, a Lindelöf subspace is always z -embedded (Jerison, [9, 5.3]). Relations between z -, C^* -, and C -embedding have been given by Hager [7]. We shall find that z -embedded subspaces are of interest in problems concerning z -filters, and thus in problems concerning F' -spaces.

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1. Traces and induced z -filters. If $Y \subseteq X$, we define the *trace* $\mathcal{F}|Y = \{Z \cap Y : Z \in \mathcal{F}\}$ of any z -filter \mathcal{F} on X , and the *induced z -filter* $\mathcal{F}^* = \{Z \in \mathcal{Z}(X) : Z \cap Y \in \mathcal{F}\}$ for any z -filter \mathcal{F} on Y .

We now consider six basic lemmas in the calculus of traces and induced z -filters; the first two are easy to verify and the third is proved in [10].

LEMMA 1. *If \mathcal{P} is a prime z -filter on Y , then \mathcal{P}^* is a prime z -filter on X . [5, 4.12].*

LEMMA 2. *If Y is z -embedded in X and \mathcal{F} is a z -filter on Y , then $\mathcal{F} = \mathcal{F}^*|Y$.*

LEMMA 3. *Let Y be a z -embedded subspace of X . If \mathcal{F} is a*

z-filter on X every member of which meets Y , then $\mathcal{F}|Y$ is a *z*-filter on Y . If \mathcal{F} is prime, then $\mathcal{F}|Y$ is also prime. [10, Th. 5.2].

We shall use \mathcal{N}^p and \mathcal{O}^p to denote the *z*-filters $Z[M^p]$ and $Z[\mathcal{O}^p]$, respectively. For example, if $p \in X$, then \mathcal{O}_X^p is the *z*-filter of all zero-set-neighborhoods of p in X . In the next two lemmas we use induced *z*-filters and traces to relate \mathcal{O}_X^p with the corresponding *z*-filter on a subspace of X that contains p . The first lemma is immediate.

LEMMA 4. *If V is a neighborhood of p in X , then $\mathcal{O}_X^p = (\mathcal{O}_V^p)^{\#}$.*

LEMMA 5. *If Y is *z*-embedded in X , and $p \in Y$, then $\mathcal{O}_Y^p = \mathcal{O}_X^p|Y$.*

Proof. Clearly $\mathcal{O}_X^p|Y \subseteq \mathcal{O}_Y^p$. On the other hand, if $Z \in \mathcal{O}_Y^p$, there is $W \in \mathcal{O}_X^p$ such that $W \cap Y \subseteq Z$. Since Y is *z*-embedded, by Lemma 3 $\mathcal{O}_X^p|Y$ is a *z*-filter on Y , and since $W \cap Y$ is in $\mathcal{O}_X^p|Y$, so is Z .

LEMMA 6. *For any X , and any $Y \subseteq X$, if \mathcal{P} and \mathcal{Q} are prime *z*-filters on Y contained in the same *z*-ultrafilter on Y , then $\mathcal{P}^{\#}$ and $\mathcal{Q}^{\#}$ are contained in the same *z*-ultrafilter on X .*

Proof. If not, then $\mathcal{P}^{\#}$ and $\mathcal{Q}^{\#}$ contain distinct *z*-filters \mathcal{O}^p ; hence they, and thus also \mathcal{P} and \mathcal{Q} , have disjoint members, so that \mathcal{P} and \mathcal{Q} could not be contained in the same *z*-ultrafilter.

2. Subspaces of F' -spaces. We are now ready to use traces of *z*-filters to obtain our first result.

THEOREM 1. *Every *z*-embedded subspace of an F' -space is also an F' -space.*

Proof. According to [4, 8.13] (see also Theorem 3 below), a space T is an F' -space if and only if \mathcal{O}_T^p is prime for every $p \in T$.

Let Y be *z*-embedded in an F' -space X . For any $p \in Y$, we have $\mathcal{O}_Y^p = \mathcal{O}_X^p|Y$, by Lemma 5. Since X is an F' -space, \mathcal{O}_X^p is prime, and hence by Lemma 3, \mathcal{O}_Y^p is also prime. Thus Y is an F' -space.

This result generalizes Corollary 1.6 and Theorem 1.11 of [1] which give the result in the case of a Lindelöf subspace or a C^* -embedded subspace. An example of a *z*-embedded subspace of an F' -space that is neither Lindelöf nor C^* -embedded is the subspace $X - Y$ of the space X constructed in [4, 8.14].

It is easily verified (see for example [6, 3.1]) that every cozero-set is *z*-embedded. Hence as an application of Theorem 1 we find that

every cozero-set in an F' -space is also an F' -space. Thus we also obtain an immediate proof of a result in [1, § 4]: in any space, a point with an F' -neighborhood admits a fundamental system of F' -neighborhoods.

A zero-set in X need not be z -embedded in X ; for example, it is easily seen that the zero-set D of the space Γ of [5, 3K] is not z -embedded.

The z -filters \mathcal{O}^p may also be used to obtain other properties of F' -spaces. For example, by Lemmas 1 and 4 we see that, as noted in [1, § 4], F' is a local property, i.e., if every point of X has an F' -neighborhood, then X is an F' -space. Since it is clear that *any local property that is inherited by cozero-sets is also inherited by all open subspaces*, Theorem 1 also yields the following result of [1].

COROLLARY 1. [1, § 4]. *Every open subspace of an F' -space is also an F' -space.*

A space is an F -space if any two disjoint cozero-sets are completely separated [5, 14N.4]. Since cozero-sets are z -embedded, it is easily seen that “cozero-set” is transitive, i.e., if S is a cozero-set in X and T is a cozero-set in S , then T is also a cozero-set in X . Thus it is clear that a cozero-set in an F -space is also an F -space, as noted in [5, 14.26]. Hence the analog for F -spaces of the statement above on fundamental systems is also true, as noted in [1, § 4]. We note that “zero-set” is *not* transitive; for example the zero-set D above has many zero-sets that are not zero-sets of Γ . But in a *normal* space, “zero-set” is transitive.

It is well-known that if X is any locally-compact, σ -compact space, then $\beta X - X$ is an F -space ([5, 14.27]; see also [12, 3.3] or [11, Corollary 1]), and thus for any X , any zero-set (i.e., compact G_δ) in βX that does not meet X is an F -space [5, 14O.1]. Here is an analog for F' -spaces. *For any X , any locally compact G_δ in βX that does not meet X is an F' -space.* To see this, let Y be such a set and let $p \in Y$. Then p has a compact zero-set neighborhood Z in Y . Since Z is a G_δ in Y , it is a compact G_δ in βX , and hence an F -space. Since F' is a local property, Y is an F' -space.

In particular, *if X is σ -compact, and locally compact at infinity (i.e., $\beta X - X$ is locally compact, see [8, p. 94]), then $\beta X - X$ is an F' -space.*

For example, the space Σ of [5, 4M] is σ -compact but not locally compact. According to [8, 3.1], a space X is locally compact at infinity if and only if the set $R(X)$, of all points of X at which X is

not locally compact, is compact. Since $R(\Sigma) = \{\sigma\}$, Σ is locally compact at infinity; hence $\beta\Sigma - \Sigma$ is an F' -space. However, since $\beta\Sigma - \Sigma$ is an open subspace of $\beta\mathbf{N} - \mathbf{N}$, this is a special case of Corollary 1.

For an application not covered by Corollary 1, we consider the following.

EXAMPLE. Let $A_0 = \beta\mathbf{R} - \mathbf{N}$. A moment's reflection shows that A_0 is σ -compact and that $R(A_0) = \beta\mathbf{N} - \mathbf{N}$; hence $\beta A_0 - A_0$ is an F' -space. This example also shows the usefulness of [8, 3.1] in a situation in which it is not convenient to examine $\beta X - X$ directly.

The analog of Corollary 1 for F -spaces is not settled. However, under the continuum hypothesis it is shown in [3, 4.2] that all open subsets of the particular F -spaces $\beta\mathbf{R} - \mathbf{R}$ and $\beta\mathbf{N} - \mathbf{N}$ are also F -spaces.

As to *closed* subspaces, it is trivial that a closed subspace of a *compact* F -space is also an F -space, since it is C^* -embedded [5, 14.26]. For *locally compact* F -spaces we have the following.

COROLLARY 2. *Every closed subspace of a locally compact F -space is an F' -space.*

Proof. Let X be a locally compact F -space and G a closed subspace. It is shown in [5, 14.25] that X is an F -space if and only if βX is an F -space (this also follows immediately from Lemmas 1 and 3 using the relations $\mathcal{O}_{\beta X}^p = (\mathcal{O}_X^p)^{\#}$ and $\mathcal{O}_X^p = \mathcal{O}_{\beta X}^p|X$ which follow from [5, 7.12(a)]). Hence βX is a compact F -space and thus $\text{cl}_{\beta X} G$ is an F -space. Also, X is open in βX and hence $G = X \cap \text{cl}_{\beta X} G$ is an open subspace of $\text{cl}_{\beta X} G$. Hence G is an F' -space by Corollary 1.

3. Continuous images. Our z -filters also yield a simple proof of the following result, which is essentially the content of the lemma in [2].

THEOREM (Comfort-Ross). *An open continuous image of an F' -space is also an F' -space.*

Proof. Let $\tau : X \rightarrow Y$ be an open continuous mapping of an F' -space X onto a space Y . For any $p \in X$, since \mathcal{O}_X^p is prime, so is its sharp-image $\tau^{\#}\mathcal{O}_X^p$ [5, 4.12], and hence any z -filter containing $\tau^{\#}\mathcal{O}_X^p$ is also prime [5, 2.9]. If $Z \in \tau^{\#}\mathcal{O}_X^p$, then $\tau^{-}[Z]$ is a neighborhood of p , so that Z is a neighborhood of τp ; hence $\tau^{\#}\mathcal{O}_X^p \subseteq \mathcal{O}_Y^{\tau p}$, and thus $\mathcal{O}_Y^{\tau p}$ is prime. Hence Y is an F' -space.

We note that a closed continuous image of an F' -space need not be an F' -space. For example, if X is the open unit disk in the plane, and the compactification BX is the closed disk, then the unit circle $BX - X$ is a closed continuous image of the F' -space $\beta X - X$, but is not an F' -space, since a metrizable F -space must be discrete [5, 14N.3].

4. **Induced mappings.** In attempting to extend Theorem 1 to the case that X is an F' -space and $\tau : Y \rightarrow X$ is a continuous mapping of Y into X , a reasonable condition which generalizes z -embedding is that for every zero-set Z in Y there is a zero-set W in X such that $Z = \tau^{-1}[W]$. In this case Y is also an F' -space; however, the following result, an analog of [5, Th. 10.3(b)], shows that this situation is essentially the same as that of Theorem 1.

THEOREM 2. *Let $\tau : Y \rightarrow X$ be a continuous mapping of Y into X , and τ' the induced mapping $W \rightarrow \tau^{-1}[W]$ of $Z(X)$ into $Z(Y)$. Then τ' is onto $Z(Y)$ if and only if τ is a homeomorphism whose image is z -embedded in X .*

Proof. For any zero-set W in X we have $\tau^{-1}[W] = \tau^{-1}[W \cap \tau[Y]]$, where $W \cap \tau[Y]$ is a zero-set in $\tau[Y]$. Thus in proving the necessity we may assume that τ is onto X . Any two distinct points p_1 and p_2 of Y have disjoint zero-set-neighborhoods of the form $\tau^{-1}[W_1]$ and $\tau^{-1}[W_2]$, where W_1 and W_2 are zero-sets in X ; it follows that W_1 and W_2 are disjoint and hence $\tau p_1 \neq \tau p_2$. Thus τ is one-to-one. In both Y and X the closure of a set is the intersection of the zero-sets containing it. It follows that for any subset E of Y , we have $\text{cl}_Y E = \tau^{-1}[\text{cl}_X \tau[E]]$. Thus $\tau[\text{cl}_Y E] = \text{cl}_X \tau[E]$, and τ is a homeomorphism. The sufficiency is clear.

5. **Characterization of F' -spaces.** We now give a characterization of F' -spaces in terms of z -embedded subspaces (see condition (4) below), and include for convenience several other known characterizations. Characterization (5) is due to Comfort, Hindman, and Njeregontis [2, Th. 1.1], while the others are from [4] and [5].

THEOREM 3. *For any X , the following are equivalent.*

- (1) *For every $p \in X$, the ideal O^p [resp. z -filter \mathcal{O}^p] is prime.*
- (2) *The prime ideals [resp. prime z -filters] contained in any given fixed maximal ideal [resp. fixed z -ultrafilter] form a chain.*
- (3) *Given $p \in X$ and $f \in C(X)$, there is a neighborhood of p on which f does not change sign.*

- (4) *Every z -embedded subspace is C^* -embedded in its closure.*
 (5) *Every cozero-set is C^* -embedded in its closure.*
 (6) *For each $f \in C(X)$, $\text{pos } f$ and $\text{neg } f$ have disjoint closures.*
 (7) *Disjoint cozero-sets have disjoint closures (i.e., X is an F' -space).*

Proof. As in [5, 14.25], the equivalence of (1), (2), and (3) follows directly from [5; 7.15, 14.8(a), 14.2(a), 2.8, 2.9].

(2) implies (4). Let Y be z -embedded in X . According to [5, 6.4], Y is C^* -embedded in $\text{cl } Y$ if every point of $\text{cl } Y$ is the limit of a *unique* z -ultrafilter on Y . Let \mathcal{M}_1 and \mathcal{M}_2 be z -ultrafilters on Y converging to the same point p in $\text{cl } Y$. By Lemma 1 the induced z -filters \mathcal{M}_1^\ddagger and \mathcal{M}_2^\ddagger are prime. Let $Z \in \mathcal{M}_1^\ddagger$; thus $Z \cap Y \in \mathcal{M}_1$. If V is any neighborhood of p in X , then $V \cap \text{cl } Y$ contains some member of \mathcal{M}_1 [5, 6.2]; hence $V \cap \text{cl } Y$ meets $Z \cap Y$ and thus $V \cap Z \neq \emptyset$. It follows that $p \in Z$. Thus \mathcal{M}_1^\ddagger is contained in the z -ultrafilter \mathcal{M}_X^\ddagger , and similarly \mathcal{M}_2^\ddagger . By hypothesis, \mathcal{M}_1^\ddagger and \mathcal{M}_2^\ddagger are comparable. If, say, $\mathcal{M}_1^\ddagger \subseteq \mathcal{M}_2^\ddagger$, then since Y is z -embedded, we have by Lemma 2, $\mathcal{M}_1 = \mathcal{M}_1^\ddagger|Y \subseteq \mathcal{M}_2^\ddagger|Y = \mathcal{M}_2$, so that $\mathcal{M}_1 = \mathcal{M}_2$. Hence Y is C^* -embedded in $\text{cl } Y$.

(4) implies (5). As noted in §2, every cozero-set is z -embedded.

(5) implies (6). Put $T = \text{cl}_X(\text{pos } f \cup \text{neg } f)$. Put $g = 1$ on $\text{pos } f$ and $g = -1$ on $\text{neg } f$, and extend g to $h \in C^*(T)$. Since $h = 1$ on $\text{cl}_X(\text{pos } f)$ and $h = -1$ on $\text{cl}_X(\text{neg } f)$, these closures are disjoint.

(6) implies (7). If $X - Z(f)$ and $X - Z(g)$ are disjoint, then $X - Z(f) \subseteq \text{pos}(f^2 - g^2)$ and $X - Z(g) \subseteq \text{neg}(f^2 - g^2)$.

(7) implies (1). If Z and W are zero-sets with $Z \cup W = X$, then $X - Z$ and $X - W$ are disjoint cozero-sets and thus have disjoint closures. Hence $\text{int } Z \cup \text{int } W = X$, and thus $Z \in \mathcal{O}^p$ or $W \in \mathcal{O}^p$. By [5, 2E], \mathcal{O}^p is prime.

We may use Theorem 3 to obtain an alternative proof of Theorem 1 as follows. Let Y be z -embedded in an F' -space X . Let T be a z -embedded subspace of Y . Then T is z -embedded in X , and thus C^* -embedded in $\text{cl}_X T$, hence in $\text{cl}_Y T$. Thus Y is an F' -space. Still another instructive proof may be based on condition (2) and Lemmas 6 and 2.

Theorem 3 also yields the following extension of [1, Th. 1.8]. *Any F' -space with a dense normal z -embedded subspace is an F -space.* The proof given in [1] serves here as well.

The above characterization of F' -spaces in terms of z -embedded subspaces has an analog for F -spaces, [7]; it may also be obtained from our characterization of F' -spaces as follows.

COROLLARY (Hager). *A space X is an F -space if and only if every z -embedded subspace is C^* -embedded.*

Proof. According to [5, 14.25], X is an F -space if and only if every cozero-set is C^* -embedded in X . Since a cozero-set is z -embedded, the sufficiency is clear. Now let X be an F -space and Y a z -embedded subspace. Since X is z -embedded in βX , so is Y . Since βX is an F -space, it follows from Theorem 3 that Y is C^* -embedded in $\text{cl}_{\beta X} Y$. The latter space is compact, hence C^* -embedded in βX . Thus Y is C^* -embedded in βX , hence in X .

REFERENCES

1. W. W. Comfort, N. Hindman, and S. Negrepointis, *F'-spaces and their product with P-spaces* (to appear in Pacific J. Math.)
2. W. W. Comfort and K. A. Ross, *On the infinite product of topological spaces*, Arch. Math. (Basel) **14** (1963), 62-64.
3. N. J. Fine and L. Gillman, *Extension of continuous functions in $\beta\mathbb{N}$* , Bull. Amer. Math. Soc. **66** (1960), 376-381.
4. L. Gillman and M. Henriksen, *Rings of continuous functions in which every finitely generated ideal is principal*, Trans. Amer. Math. Soc. **82** (1956), 366-391.
5. L. Gillman and M. Jerison, *Rings of continuous functions*, Van Nostrand, Princeton, 1960.
6. L. Gillman and M. Jerison, *Quotient fields of residue class rings of function rings*, Illinois J. Math. **4** (1960), 425-436.
7. A. W. Hager, *C-, C^* -, and z -embedding* (to appear).
8. M. Henriksen and J. R. Isbell, *Some properties of compactifications*, Duke Math. J. **25** (1958), 83-106.
9. M. Henriksen and D. G. Johnson, *On the structure of a class of archimedean lattice-ordered algebras*, Fund. Math. **50** (1961), 73-94.
10. M. Mandelker, *Prime z -ideal structure of $C(\mathbb{R})$* , Fund. Math. **63** (1968), 145-166.
11. ———, *Prime ideal structure of rings of bounded continuous functions*, Proc. Amer. Math. Soc. **19** (1968), 1432-1438.
12. S. Negrepointis, *Absolute Baire sets*, Proc. Amer. Math. Soc. **18** (1967), 691-694.

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