ON NORMALOID OPERATORS

I. H. Sheth
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The purpose of the present paper is to extend an earlier theorem of the author's on hyponormal operators to the following, on normaloid operators.

THEOREM. Let $N$ be an operator such that $N - zI$ is normaloid for all complex values of $z$. If $AN = N^*A$, for an arbitrary operator $A$, for which $0 \in \text{Cl}(W(A))$, then $N = N^*$.

2. Notations. We consider bounded linear operators defined on a Hilbert space $H$. As usual, the symbols $s(T)$, $\Sigma(T)$, $W(T)$ and $\text{Cl}(W(T))$ stand for the spectrum of an operator $T$, the closed convex hull of $s(T)$, the numerical range of $T$ and the closure of $W(T)$ respectively.

An operator $T$ is said to be normaloid if $||T|| = \sup \{|z|; z \in s(T)\}$ and hyponormal, if $T^*T - TT^* \geq 0$. It is known that if $T$ is hyponormal, then $T$ is normaloid and $T - zI$ is also hyponormal for all complex numbers $z$.

When the original version of this paper was submitted, the referee told me of [3] then existing as a preprint and this makes possible the following shorter proof.

Proof of Theorem. Since $AN = N^*A$ and $0 \in \text{Cl}(W(A))$, $s(N)$ is real [3]. Also $\Sigma(N) = \text{Cl}(W(N))$ for such a normaloid operator $N$ [1]. Hence $\text{Cl}(W(N))$ is real. This completes the proof of theorem.

The corresponding result for hyponormal operators now follows as corollary from this theorem and the remark made above.

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REFERENCES


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