

Pacific Journal of Mathematics

**ON FINITE GROUPS WITH INDEPENDENT CYCLIC SYLOW
SUBGROUPS**

MARCEL HERZOG

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The purpose of this paper is to classify all perfect groups with cyclic Sylow p -subgroups which satisfy the condition

(TI) two different Sylow p -subgroups of G contain only the unit element in common and such that

$$o(G) < o(P)^3$$

where P is a Sylow p -subgroup of G .

The main result of this paper is the following

THEOREM 1. Let G be a perfect finite group with a cyclic Sylow p -subgroup P of order p^a and assume that the Sylow p -subgroups of G satisfy the (TI) condition. Assume, furthermore, that

$$o(G) < p^{3a}.$$

Then one of the following statements holds.

- (I) $a = 1$, $G \cong PSL(2, p)$, where $p > 3$ is a prime.
- (II) $a = 1$, $G \cong PSL(2, p - 1)$, where $p = 2^m + 1 > 5$ is a Fermat prime.
- (III) $a = 1$, $G \cong SL(2, p)$, where $p > 3$ is a prime.
- (IV) $a = 2$, $p = 3$, $G \cong PSL(2, 8)$.

Ten years ago E. Artin raised the following problem: what are the simple finite groups G of order g which are divisible by a prime $p > g^{1/3}$? This question was answered by R. Brauer and W. F. Reynolds in [1]. They found that the only groups satisfying the above conditions are $PSL(2, p)$, where $p > 3$ is a prime, and $PSL(2, p - 1)$ where $p > 3$ is a Fermat prime, $p = 2^m + 1$. In particular, the Sylow p -subgroups of these groups are of order p and therefore they are cyclic and satisfy the (TI) condition. Theorem 1 thus generalizes these results.

As a matter of fact we will prove a more general statement than Theorem 1.

THEOREM 1*. Let G be a finite group with a cyclic Sylow p -subgroup P of order p^a and assume that the Sylow p -subgroups of G satisfy the (TI) condition. Assume, furthermore, that

$$o(G) < p^{3a}$$

and no homomorphic image of G is isomorphic to $N_G(P)/W$, where

W is the normal complement of P in $C_G(P)$. Then one of the following statements holds.

- (I)* $a = 1$, $G \cong PSL(2, p)$, where $p > 3$ is a prime.
- (II)* $a = 1$, $G \cong PSL(2, p - 1)$, where $p = 2^m + 1 > 5$ is a Fermat prime.
- (III)* $a = 1$, $G \cong SL(2, p)$, where $p > 3$ is a prime.
- (IV)* $a = 2$, $p = 3$, $G \cong PSL(2, 8)$.
- (V)* $a = 1$, $G \cong PGL(2, p)$, where $p > 3$ is a prime.
- (VI)* $a = 1$, $G \cong PSL(2, p) \times M$, where $p > 3$ is a prime and $o(M) = 2$.

Since $G = G'$ implies the last condition of Theorem 1*, Theorem 1 follows immediately from Theorem 1*. In this paper the group $N_G(P)/W$ will be referred to as the p -metacyclic group of order qp^a .

Theorem 1* follows from the following more general result:

THEOREM 2. *Let G be a finite group with a cyclic Sylow p -subgroup P of order $p^a > 1$ and assume that the Sylow p -subgroups of G satisfy the (TI) condition. Suppose that no homomorphic image of G is isomorphic to the p -metacyclic group of order $p^a q$. Then*

$$o(G) = qwp^a(1 + np^a)$$

where $wp^a = o(C_G(P))$, $q = [N_G(P) : C_G(P)] > 1$ and n is a positive integer.

Furthermore, let G_0 be the minimal normal subgroup of G for which G/G_0 is solvable, and let M be the maximal normal subgroup of G_0 of order prime to p . Denote G_0/M by G^* . Then one of the following statements holds.

- (A) $n = (hvp^a + h + v^2 + v)/(v + 1)$
where h and v are positive integers and $v + 1 \mid h(p^a - 1)$.
- (B) $a = 1$, $n = 1$, $G^* \cong PSL(2, p)$ where $p > 3$ is a prime.
- (C) $a = 1$, $n = (p - 3)/2$, $G^* \cong PSL(2, p - 1)$ where $p = 2^m + 1 > 5$ is a Fermat prime.
- (D) $a = 2$, $p = 3$, $n = (p^2 - 3)/2$, $G^* \cong PSL(2, 8)$.

Theorem 2 immediately yields

COROLLARY. *Let G satisfy the assumptions of Theorem 2 and suppose that $n < (p^a + 3)/2$. Then G^* is of type (B), (C) or (D).*

In §2 some basic properties of groups with a Sylow subgroup satisfying the TI-property are derived. Section 3 contains the proof of Theorem 2, from which Theorem 1* is deduced in §4.

We use the standard notation $C_G(T)$, $N_G(T)$, $o(T)$, T^* , and $\langle T \rangle$,

where T is a subset of the group G , to denote respectively: the centralizer, normalizer, number of elements, the nonunit elements and the group generated by T . We will say that $N_G(T)/C_G(T)$ acts frobeniusly on T if $\theta^\eta = \theta$ for $\theta \in T^*$ and $\eta \in N_G(T)$ implies that $\eta \in C_G(T)$. An element of G is called a p' -element, where p is a prime number, if p does not divide its order. The principal character and the commutator subgroup of G will be denoted by 1_G and G' respectively. Finally, if a and b are integers, then (a, b) denotes their greatest common divisor and $a | b$ means: a divides b .

2. *TIP*-groups. A finite group will be called a *TIP*-group if its Sylow p -subgroups are nontrivial and satisfy the *TI*-property. This section deals with properties of *TIP*-groups in general, followed by a study of *TIP*-groups with a cyclic Sylow p -subgroup.

PROPOSITION 2.1. *Let G be a *TIP*-group with a Sylow p -subgroup P of order p^a . Then the following statements hold.*

(a) $C_G(P) = W \times P$

where $o(W) = w$ and $(w, p) = 1$.

(b) $o(G) = qwp^a(1 + np^a)$

where $q = [N_G(P) : C_G(P)]$ and n is a nonnegative integer.

(c) *Any normal subgroup L of G of order divisible exactly by $p^b > 1$ is a *TIP*-group of order $q_L w_L p^b(1 + np^a)$.*

(d) *If H is a normal subgroups of G of order prime to p , then G/H is a *TIP*-group.*

Proof. Let $C = C_G(P)$, $N = N_G(P)$.

(a) Since P is a normal Hall-subgroup of C , it has a complement W and $(w, p) = 1$. Since elements of W commute with elements of P , $C = W \times P$.

(b) Consider the conjugates $\{P_i\}$ of P , other than P . If $\sigma \in P$ and $P_i^\sigma = P_i$, where $P_i = P^\tau$, $\tau \in G$, then $P^{\tau\sigma\tau^{-1}} = P$, $\tau\sigma\tau^{-1} \in N_G(P)$ and $\sigma \in N_G(P^\tau)$, $\sigma \in P \cap P^\tau = \{1\}$. Thus P acts by conjugation fixed point free on $\{P_i\}$ and therefore $o\{P_i\} = np^a$ for some nonnegative integer n . Hence $[G : N] = 1 + np^a$ and $o(G) = qwp^a(1 + np^a)$.

(c)—(d) The proof of Lemma 1 in [6] obviously holds also for general *TIP*-groups, with $p \neq 2$. Thus any subgroup of G of order divisible by p is a *TIP*-group and (d) holds. Let $o(L) = q_L x_L p^b(1 + n_L p^b)$. Since L and G have the same number of Sylow p -subgroups $1 + n_L p^b = 1 + np^a$ proving (c).

PROPOSITION 2.2. *Let G be a *TIP*-group with a cyclic Sylow p -subgroup P of order p^a . Then in addition to properties (a)—(d) of Proposition 2.1, and using the same notation, the following state-*

ments hold.

(e) $C_G(P) = C_G(\sigma)$ and $N_G(P) = N_G(\langle \sigma \rangle)$
for all $\sigma \in P^*$.

(f) q divides $p - 1$.

(g) $o(G/H) = q\bar{w}p^a(1 + \bar{n}p^a)$

and there exists a nonnegative integer z such that

$$n = z + \bar{n} + z\bar{n}p^a .$$

If $z = 0$ then $H \subset W$.

(h) If K is a normal subgroup of G and K does not contain P then

$$N_K(P) = C_K(P) .$$

(i) If also $o(K \cap P) > 1$, then G can be mapped homomorphically on the p -metacyclic group of order p^aq .

Proof. (e) Let $\sigma \in P^*$; then by Lemma 2.1.b in [3] $C_G(\sigma) \cap N_G(P) = C_G(P)$. It follows from the TI-property that $C_G(\sigma) \subset N_G(P)$ and $N_G(\langle \sigma \rangle) \subset N_G(P)$. Thus $C_G(\sigma) = C_G(P)$ and since P is cyclic $N_G(\langle \sigma \rangle) = N_G(P)$.

(f) By Lemma 2.1.d of [3] N/C acts frobeniusly on P and P is cyclic. Therefore $q = [N : C]$ divides $p - 1$.

(g) The proof of Proposition 2 in [1] holds, with the obvious changes, also in the present case. It is clear from the proof in [1] that if $z = 0$ then $H \subset C_G(P)$.

(h) Suppose that $K \cap N \not\subset C$ and let $\sigma \in K \cap N - C$. Since N/C acts frobeniusly on P , it follows that the elements $\sigma\rho^{-1}\sigma^{-1}\rho$, $\rho \in P$, are distinct and belong to $P \cap K$. Thus P is contained in K , a contradiction. Consequently $K \cap N \subset C$ and $K \cap N = K \cap C$, as required.

(i) Let p^b be the exact power of p dividing $o(K)$. Then $1 < p^b < p^a$ and by Proposition 2.1.c and (h) $o(K) = w_K p^b(1 + np^a)$, where $w_K p^b = o(C_K(P \cap K)) = o(N_K(P \cap K))$. By the Burnside Theorem K has a characteristic subgroup T of order $w_K(1 + np^a)$. T is normal in G and $G = NT$. Consequently WT is a normal subgroup of G and G/WT is isomorphic to the p -metacyclic group of order p^aq .

3. Proof of Theorem 2. If either $p = 2$ or $q = 1$, then $C_G(P) = N_G(P)$ and by the Burnside Theorem P has a normal complement in G , in contradiction to our assumption. Thus $p > 2$ and $q > 1$.

If P is normal in G and $C_G(P) = W \times P$, then W is normal in G , again a contradiction. Thus P is not normal in G and the first statement of Theorem 2 follows from Proposition 2.1.b.

It follows from Proposition 3 in [1] and Proposition 2.2.i that

$P \subset G_0$. Indeed, if $P \not\subset G_0$ then either $a = 1$ or $a > 1$ and G contains a normal subgroup U such that $1 < o(U \cap P) < p^a$. In both cases the above mentioned propositions yield a contradiction to our assumptions.

The definition of G_0 forces it to be its own commutator subgroup and the same is true for G^* . Moreover, G^* does not have nontrivial normal subgroups of order prime to p .

From now on we will assume that (A) is not satisfied and will show that then one of the statements (B), (C), or (D) holds.

Let $o(G_0) = q_0 w_0 p^a (1 + n p^a)$, $o(G^*) = q_0 w^* p^a (1 + n^* p^a)$. Since $G^* = (G^*)'$, $n^* \neq 0$. By Proposition 2.2.g there exists a nonnegative integer z such that

$$n = z + n^* + z n^* p^a .$$

If $z \neq 0$, let $h = (z + 1)n^*$, $v = z$. Then:

$$n = v + h/(v + 1) + v h p^a / (v + 1)$$

in contradiction to our assumptions. Thus $z = 0$ and $n^* = n$.

Consequently, it suffices to show that if G satisfies the assumptions of Theorem 2 and in addition, $G = G'$, G has no nontrivial normal subgroup of order prime to p and n does not satisfy (A), then G is isomorphic to one of the simple groups described in (B), (C), and (D).

We will use the following notation: $N = N_G(P)$, $C = C_G(P) = W \times P$ where $o(W) = w$ and $(w, p) = 1$.

Let B be the principal p -block of G . Then by Proposition 2.1 of [3] B contains $t = (p^a - 1)/q$ exceptional characters X_λ of degree x_λ , $\lambda = 1, \dots, t$ and q nonexceptional characters X_i of degree x_i , $i = 1, \dots, q$. If $\sigma \in P^\#$ and π is a p' -element of $C_G(\sigma) = C$ then:

$$(1) \quad \begin{aligned} X_\lambda(\sigma\pi) &= -\varepsilon_0 \sum_{\rho \in R} \zeta_\lambda^\rho(\sigma) && \text{for } \lambda = 1, \dots, t \\ X_j(\sigma\pi) &= \varepsilon_j && \text{for } j = 1, \dots, q \end{aligned}$$

where R is a set of coset representatives of C in N , $\{\zeta_\lambda \mid \lambda = 1, \dots, t\}$ is a set of representatives of the t transitivity classes of characters of P under conjugation by N (see [3], Lemma 2.2), and $\varepsilon_j = \pm 1$ for $j = 0, 1, \dots, q$. It follows also by Corollary 2.1 of [3] that the following relations hold:

$$(2) \quad \begin{aligned} x_i &\equiv \varepsilon_i \pmod{p^a} && \text{for } i = 1, \dots, q \\ t x_0 &\equiv \varepsilon_0 \pmod{p^a} \end{aligned}$$

and

$$(3) \quad \sum_{i=0}^q \varepsilon_i x_i = 0 .$$

We are now ready to state

LEMMA 3.1.

$$(4) \quad tx_j \mid (p^a - 1)(1 + np^a) \quad \text{for } j = 0, \dots, q.$$

Proof. If $\sigma \in P^\ddagger$, then $C = C_\sigma(\sigma)$ and it is well-known that the expression

$$\frac{o(G) \cdot X_j(\sigma)}{o(C) \cdot x_j}$$

is an algebraic integer for all j . It follows, from Proposition 2.1 and (1), that for $j = 1, \dots, q$

$$qwp^a(1 + np^a)/wp^ax_j$$

is an algebraic integer and consequently

$$tx_j \mid tq(1 + np^a) = (p^a - 1)(1 + np^a).$$

For $j = 0$, it follows from (1), Proposition 2.1 and Lemma 2.2 of [3] that

$$\sum_{\lambda=1}^t \frac{o(G)X_\lambda(\sigma)}{o(C)x_0} = \frac{qwp^a(1 + np^a)\varepsilon_0}{wp^ax_0}$$

is an algebraic integer and therefore $tx_0 \mid (p^a - 1)(1 + np^a)$.

Since the block B contains 1_G as a nonexceptional character, we may assume that $X_1 = 1_G$. We have then

LEMMA 3.2. For $j = 0, 2, 3, \dots, q$

$$\bar{x}_j = \begin{cases} 1 + np^a & \text{if } \varepsilon_j = 1 \\ p^a - 1 & \text{if } \varepsilon_j = -1 \end{cases}$$

where $\bar{x}_j = x_j$ for $j = 2, \dots, q$ and $\bar{x}_0 = tx_0$.

Proof. We will show first that if

$$up^a + \varepsilon \mid (p^a - 1)(1 + np^a), \quad \varepsilon = \pm 1$$

then either n satisfies statement (A) or one of the following relations holds:

$$\begin{array}{lll} up^a + \varepsilon = 1 & \text{or } np^a + 1 & \text{if } \varepsilon = 1 \\ up^a + \varepsilon = p^a - 1 & \text{or } (p^a - 1)(np^a + 1) & \text{if } \varepsilon = -1. \end{array}$$

To do so, it suffices to show that if n does not satisfy (A) then the only solutions of

$$(5) \quad (vp^a + 1)(wp^a - 1) = (p^a - 1)(1 + np^a)$$

in nonnegative integers v and w are: $v = 0, wp^a - 1 = (p^a - 1)(1 + np^a)$ and $v = n, w = 1$.

Suppose that $v \neq 0$ and $w > 1$. Then $vp^a + 1 < 1 + np^a, v < n$. By multiplying out equation (5), adding 1 to both sides and dividing by p^a we get

$$(6) \quad wvp^a + w - v = np^a - n + 1.$$

Now by (6):

$$\begin{aligned} (vp^a + 1)(n - wv) &= vp^an - v(wvp^a) + n - wv \\ &= vp^an + vw - v^2 - vnp^a + vn - v + n - wv \\ &= (n - v)(v + 1). \end{aligned}$$

Since $n > v$, the left hand side of the equation is positive and so we may put $h = n - wv$, where h is a positive integer. Solving for n we get a contradiction to the assumption that n does not satisfy (A). Thus either $v = 0$ or $w = 1$ and the above assertion follows.

Now we have seen that for $j = 0, 2, 3, \dots, q$

$$\bar{x}_j \equiv \varepsilon_j \pmod{p^a} \text{ and } \bar{x}_j \mid (p^a - 1)(1 + np^a).$$

Since X_1 is the only character of G of degree 1, it follows that for $j = 0, 2, 3, \dots, q$

$$\bar{x}_j = \begin{cases} 1 + np^a & \text{if } \varepsilon_j = 1 \\ p^a - 1 \text{ or } (p^a - 1)(1 + np^a) & \text{if } \varepsilon_j = -1. \end{cases}$$

Thus it suffices to show that for $j = 0, 2, 3, \dots, q$

$$\bar{x}_j \neq (p^a - 1)(1 + np^a).$$

Indeed, if the equality holds, then by (3):

$$0 = \sum_{i=0}^q \varepsilon_i x_i \leq 1 + (q - 1)(1 + np^a) - (p^a - 1)(1 + np^a)/t = -np^a$$

a contradiction. The proof of Lemma 3.2 is complete.

We will proceed with the proof of Theorem 2. It follows from (3) that at least one of the ε_j 's, $j = 0, 1, \dots, q$, is negative. If $\varepsilon_0 = -1$, let $X = \sum_{\lambda=1}^t X_\lambda$ and if $\varepsilon_i = -1$ for some $i \geq 2$ let $X = X_i$. In either case, by Lemma 3.2 X is a character of G of degree $p^a - 1$ and by (1) and Lemma 2.2 of [3]

$$X(\sigma\pi) = -1$$

for $\sigma \in P^*, \pi \in W$, where $C = P \times W$. Denote the restriction of X to

C also by X ; then X is a character of $P \times W$ and therefore for $\rho \in P$ and $\pi \in W$ we have:

$$(7) \quad X(\rho\pi) = \sum_{i=1}^r \psi_i(\pi)\varphi_i(\rho)$$

where ψ_i , $i = 1, \dots, r$ are distinct irreducible characters of W and φ_i , $i = 1, \dots, r$ are characters of P . Let $\sigma \in P^\#$, $\pi \in W$; as $X(\sigma\pi) = -1$, it follows from (7) and from the linear independence of the irreducible characters of W , that the principal character appears among the ψ_i , say $\psi_1 = 1_W$, and

$$\varphi_1(\sigma) = -1, \varphi_2(\sigma) = \dots = \varphi_r(\sigma) = 0.$$

Suppose that $r > 1$. Then φ_2 vanishes on $P^\#$ and therefore p^a divides $\varphi_2(1)$, in contradiction to (7) and the fact that $X(1) = p^a - 1$. Thus $r = 1$ and

$$X(\rho\pi) = \varphi_1(\rho) \quad \text{for all } \rho \in P, \pi \in W.$$

In particular $X(\pi) = \varphi_1(1) = X(1)$ for all $\pi \in W$. Let V denote the kernel of X ; then V is a normal subgroup G and $W \subset V$. Suppose that $W \neq \{1\}$. Then it follows from the assumption that G has no nontrivial subgroups of order prime to p and from Proposition 2.2.i that $P \subset V$, in contradiction to the fact that $X(\sigma) = -1$ for $\sigma \in P^\#$. Consequently $W = \{1\}$, P contains the centralizer of each of its nonunit elements and by Theorem 2 of [2] G is either of type (B), or of type C, or $G \cong PSL(2, p^a - 1)$, where $a > 1$ and $p^a - 1 = 2^b$. In view of Lemma 3.1 of [3], the only solution of the above equation with $a > 1$ is: $p = 3$, $a = 2$ and $b = 3$. Thus if $a > 1$, $G \cong PSL(2, 8)$. Since the groups of types (B), (C) and (D) satisfy the conditions of Theorem 2, the proof is complete.

4. **Proof of Theorem 1***. It follows from Theorem 2 that one of the statements (B), (C) and (D) holds. Statement (A) could not occur, since then

$$n \geq (p^a + 3)/2, \quad o(G) \geq 2p^a(p^a + 3)p^a/2 > p^{3a}$$

a contradiction.

In cases (C) and (D) $o(G^*) > p^{3a}/2$ and therefore $G \cong G^*$, yielding statements (II)* or (IV)*. In case (B), $o(G^*) = (p^{3a} - p^a)/2$ and therefore either $[G : G_0] = 2$, $G_0 \cong G^*$, or $G = G_0$, $o(M) = 1$ or 2. If $[G : G_0] = 2$, then $o(M) = 1$, G is isomorphic to a subgroup of the automorphism group of $PSL(2, p)$ and by [5, Lemma 2] $G \cong PGL(2, p)$, yielding (V)*. If $G = G_0$ and $o(M) = 1$, then $G \cong PSL(2, p)$, $p > 3$, and (I)* holds. Finally, if $G = G_0$ and $o(M) = 2$, then it follows from a theorem of

Schur [4, p. 120] that G is either isomorphic to $SL(2, p)$ and (III)* holds, or it is isomorphic to $PSL(2, p) \times M$ and (VI)* holds. Since the groups mentioned in Theorem 1* satisfy the conditions of that theorem, the proof is complete.

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