

Pacific Journal of Mathematics

THE SCHWARZIAN DERIVATIVE AND MULTIVALENCE

W. J. KIM

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A generalization of the Schwarzian derivative and a sufficient condition for disconjugacy of the n th-order differential equation with analytic coefficients are obtained. These results are then used to establish a multivalence criterion for a certain family of analytic functions.

Let y_1 and y_2 be linearly independent solutions of the differential equation

$$(1.1) \quad y'' + p(z)y = 0$$

and let

$$(1.2) \quad w = \frac{y_2}{y_1}.$$

Then, by a classical formula,

$$(1.3) \quad p = \frac{1}{2}\{w, z\}$$

where $\{w, z\}$ is the Schwarzian derivative of w , i.e.,

$$\{w, z\} = \left(\frac{w''}{w'}\right)' - \frac{1}{2}\left(\frac{w''}{w'}\right)^2.$$

Conversely, the general solution w of (1.3) is of the form (1.2).

Utilizing the above relations, Nehari [5] proved that for an analytic function f to be univalent in the unit disk $D = \{z: |z| < 1\}$ it is necessary that

$$|\{f, z\}| \leq \frac{6}{(1 - |z|^2)^2}, \quad z \in D,$$

and sufficient that

$$|\{f, z\}| \leq \frac{2}{(1 - |z|^2)^2}, \quad z \in D.$$

Generalizations of formula (1.3) for higher-order differential equations have recently been obtained. Vodička [9] considered the n th-order equation of the type

$$(1.4) \quad y^{(n)} + p(z)y = 0$$

and derived a relation between the coefficient p and the function $w =$

y_2/y_1 , where y_1 and y_2 are any two linearly independent solutions of (1.4). In a recent paper, Lavie [4] established relations between the coefficients of the differential equation

$$(1.5) \quad y^{(n)} + p_{n-1}(z)y^{(n-1)} + \cdots + p_0(z)y = 0$$

and the function $w = y_2/y_1$, where y_1 and y_2 are certain linearly independent solutions of (1.5).

In §2 we shall consider the n th-order differential equation (1.5) and derive relations in which each coefficient p_i is expressed as a function of the ratios y_i/y_n , $i = 1, 2, \dots, n-1$, where y_1, y_2, \dots, y_n are linearly independent solutions of (1.5).

In §3, using the relations derived in §2, we establish a sufficient condition for p -valence of a p -parameter family of analytic functions.

2. In this section we shall obtain some invariants which play a role in the study of differential equation

$$(2.1) \quad y^{(n)} + p_{n-2}(z)y^{(n-2)} + \cdots + p_0(z)y = 0$$

which is analogous to that played by (1.3) in the study of (1.1). We remark that there is no loss of generality in considering (2.1) because any homogeneous n th-order linear differential equation can be put into the form (2.1) by a standard transformation.

Let y_i , $i = 1, 2, \dots, n$, be linearly independent solutions of (2.1) and set

$$f_1 = \frac{y_1}{y_n}, \dots, f_{n-1} = \frac{y_{n-1}}{y_n}.$$

We seek relations of the type

$$(2.2) \quad p_i = \Phi_i(f_1, f_2, \dots, f_{n-1}), \quad i = 0, 1, \dots, n-2.$$

Since the left-hand side in (2.2) is independent of the particular choice of n linearly independent solutions, the right-hand side must remain invariant under the transformation

$$f_i \longrightarrow \frac{a_{i0} + a_{i1}f_1 + \cdots + a_{in-1}f_{n-1}}{b_0 + b_1f_1 + \cdots + b_{n-1}f_{n-1}}, \quad i = 1, 2, \dots, n-1,$$

where the a 's and b 's are constants.

THEOREM 2.1. *Let y_i , $i = 1, 2, \dots, n$, be linearly independent solutions of (2.1), let*

$$(2.3) \quad f_1 = \frac{y_1}{y_n}, \dots, f_{n-1} = \frac{y_{n-1}}{y_n}$$

and let W_i be the determinant defined by

$$W_i = \begin{vmatrix} f'_1 & f'_2 & \cdots & f'_{n-1} \\ & \dots & & \\ f_1^{(i-1)} & f_2^{(i-1)} & \cdots & f_{n-1}^{(i-1)} \\ f_1^{(i+1)} & f_2^{(i+1)} & \cdots & f_{n-1}^{(i+1)} \\ & \dots & & \\ f_1^{(n)} & f_2^{(n)} & \cdots & f_{n-1}^{(n)} \end{vmatrix},$$

$i = 1, 2, \dots, n$. Then we have

$$(2.4) \quad p_i = \frac{1}{W_n \sqrt[n]{W_n}} \left[\sum_{j=0}^{n-i} (-1)^{2n-j} (1 - \delta_{nj}) \binom{n-j}{n-j-i} W_{n-j} (\sqrt[n]{W_n})^{(n-j-i)} \right],$$

$i = 0, 1, \dots, n - 2$, where $\delta_{nn} = 1$ and $\delta_{nj} = 0$ otherwise.

Conversely, the general solution $(f_1, f_2, \dots, f_{n-1})$ of the system (2.4) of differential equations is of the form (2.3).

Proof. It is easily confirmed that $1, f_1, \dots, f_{n-1}$ are linearly independent solutions of the differential equation

$$y^{(n)} - \frac{W_{n-1}}{W_n} y^{(n-1)} + \dots + (-1)^{n+1} \frac{W_1}{W_n} y' = 0$$

and that $W_{n-1} = W'_n$. Put

$$y = Y \cdot \exp \left(\frac{1}{n} \int \frac{W_{n-1}}{W_n} dz \right) = Y \cdot \sqrt[n]{W_n}.$$

Then the function Y satisfies the differential equation

$$(2.5) \quad Y^{(n)} + q_{n-2}(z) Y^{(n-2)} + \dots + q_0(z) Y = 0$$

where

$$q_i = \frac{1}{W_n \sqrt[n]{W_n}} \left[\sum_{j=0}^{n-i} (-1)^{2n-j} (1 - \delta_{nj}) \binom{n-j}{n-j-i} W_{n-j} (\sqrt[n]{W_n})^{(n-j-i)} \right],$$

$i = 0, 1, \dots, n - 2$. Furthermore, it is evident that

$$\frac{f_1}{\sqrt[n]{W_n}}, \dots, \frac{f_{n-1}}{\sqrt[n]{W_n}}, \frac{1}{\sqrt[n]{W_n}}$$

are linearly independent solutions of (2.5).

We now assert that

$$(2.6) \quad \frac{f_1}{\sqrt[n]{W_n}} = Ky_1, \dots, \frac{f_{n-1}}{\sqrt[n]{W_n}} = Ky_{n-1}, \frac{1}{\sqrt[n]{W_n}} = Ky_n$$

for some constant K . But, if this assertion is true, it would imply that the differential equations (2.1) and (2.5) have the same set of linearly independent solutions y_1, \dots, y_n . In other words, (2.1) and (2.5) are identical, i.e., $p_i = q_i, i = 0, 1, \dots, n - 2$, which proves the theorem. To prove the equalities in (2.6), it suffices to prove only the last equality. It is easily confirmed that

$$(-1)^{n-1} W_n = \frac{W}{y_n^n},$$

where W is the Wronskian of y_1, \dots, y_n (see, e.g., [7]). Since the Wronskian W is constant, we may set $K = -1/\sqrt[n]{W}$ to obtain the last equality in (2.6).

The converse is easy to prove; it follows from the fact that

$$\frac{f_1}{\sqrt[n]{W_n}}, \dots, \frac{f_{n-1}}{\sqrt[n]{W_n}}, \frac{1}{\sqrt[n]{W_n}}$$

are linearly independent solutions of (2.1).

For the second-order equation (1.1), the formulas in (2.4) yield the familiar relation (1.3); and for the third-order equation $y''' + p_1(z)y' + p_0(z)y = 0$,

$$p_0 = \frac{-1}{3} \left[\frac{2}{9} \left(\frac{f_1' f_2'''}{f_1' f_2''} - \frac{f_1''' f_2'}{f_1'' f_2'} \right)^3 - \left(\frac{f_1' f_2'''}{f_1' f_2''} - \frac{f_1''' f_2'}{f_1'' f_2'} \right)'' - \left(\frac{f_1' f_2'''}{f_1' f_2''} - \frac{f_1''' f_2'}{f_1'' f_2'} \right) \left(\frac{f_1' f_2'''}{f_2' f_2''} - \frac{f_1''' f_2'}{f_1' f_2''} \right) \right],$$

$$p_1 = \frac{f_1'' f_2'''}{f_1' f_2''} - \frac{f_1''' f_2'}{f_1'' f_2'} + \left(\frac{f_1' f_2'''}{f_1' f_2''} - \frac{f_1''' f_2'}{f_1'' f_2'} \right)' - \frac{1}{3} \left(\frac{f_1' f_2'''}{f_1' f_2''} - \frac{f_1''' f_2'}{f_1'' f_2'} \right)^2.$$

3. Let p_0, \dots, p_{n-2} in (2.1) be analytic functions which are regular in a domain D of the complex plane. The differential equation (2.1) is said to be disconjugate in D if no nontrivial solution of (2.1) has more than $n - 1$ zeros (where the zeros are counted with their multiplicities) in D . We now state an elementary principle which relates disconjugacy with a certain function-theoretic aspect of (2.1), as a theorem for convenient reference.

THEOREM 3.1. *Let y_1, y_2, \dots, y_n be linearly independent solutions of (2.1), and let $f_i = y_i/y_n, i = 1, 2, \dots, n - 1$. Then the differential equation (2.1) is disconjugate in D if and only if every nontrivial linear combination of f_1, f_2, \dots, f_{n-1} is $(n - 1)$ -valent in D , i.e., it does not take on any one value more than $n - 1$ times in D .*

Proof. If (2.1) is not disconjugate in D , then there exists a

nontrivial solution $y = \sum_{i=1}^n a_i y_i$, for some constants $a_i \neq 0, i = 1, 2, \dots, n$, which has more than $n - 1$ zeros in D . Without loss of generality, we may assume that none of the zeros of y_n coincide with the zeros of y . Thus, we find that $a_n + \sum_{i=1}^{n-1} a_i f_i$ has more than $n - 1$ zeros in D , i.e., the linear combination $\sum_{i=1}^{n-1} a_i f_i$ assumes the value $-a_n$ more than $n - 1$ times in D . Conversely, if some nontrivial linear combination $\sum_{i=1}^{n-1} a_i f_i$ takes on the value $-a_n$ more than $n - 1$ times in D , the nontrivial solution $y = \sum_{i=1}^n a_i y_i$ has more than $n - 1$ zeros in D .

Next we shall establish a sufficient condition for disconjugacy of (2.1). We first require the following lemma.

LEMMA 3.1. *Let y be analytic in a region R . If $y(a_i) = 0, a_i \in R, i = 1, 2, \dots, n$, then*

$$(3.1) \quad y^{(k)}(z) = \sum_{j=1}^{k+1} \binom{k}{j-1} P_{n-j}^{(k+1-j)}(z) I_j(z) (a_j - z)^{-j+1},$$

$k = 0, 1, \dots, n - 1$, where

$$I_n(z) = \int_{a_n}^z (a_n - \zeta)^{n-1} y^{(n)}(\zeta) d\zeta,$$

$$I_j(z) = \int_{a_j}^z \frac{(a_j - \zeta)^{j-1}}{(a_{j+1} - \zeta)^{j+1}} I_{j+1}(\zeta) d\zeta, j = 1, 2, \dots, n - 1,$$

and

$$P_{n-j}(z) = \prod_{i=j+1}^n (a_i - z).$$

Proof. It is easily confirmed that $y = P_{n-1} I_1$, which proves (3.1) for $k = 0[1, 3]$. The rest follows from induction on k .

We remark that the a_i 's in the above lemma are not necessarily distinct; we may put $a_k = a_{k+1} = \dots = a_{k+m-1}$ if the y has a zero of order m at a_k .

THEOREM 3.2. *Let p_0, \dots, p_{n-1} be analytic in the unit disk $D = \{z: |z| < 1\}$. If*

$$(3.2) \quad \sum_{k=1}^{n-1} \frac{(1 + |z|)^{n-k}}{(n - k)!} |p_k(z)| + \frac{(1 - |z|)(1 + |z|)^{n-1}}{n!} |p_0(z)| \leq 1,$$

then the differential equation

$$(3.3) \quad y^{(n)} + p_{n-1}(z)y^{(n-1)} + \dots + p_0(z)y = 0$$

is disconjugate in D .

Proof. Suppose that (3.3) has a nontrivial solution y with n zeros, i.e., $y(a_i) = 0, a_i \in D, i = 1, 2, \dots, n$. Then from Lemma 3.1 we have

$$(3.4) \quad \begin{aligned} y(z) &= (a_n - z) \cdots (a_2 - z) \int_{a_1}^z \frac{1}{(a_2 - \zeta_1)^2} \int_{a_2}^{\zeta_1} \frac{a_2 - \zeta_2}{(a_3 - \zeta_2)^3} \\ &\cdots \int_{a_{n-1}}^{\zeta_{n-2}} \frac{(a_{n-1} - \zeta_{n-1})^{n-2}}{(a_n - \zeta_{n-1})^n} \int_{a_n}^{\zeta_{n-1}} (a_n - \zeta_n)^{n-1} y^{(n)}(\zeta_n) d\zeta_n \cdots d\zeta_1. \end{aligned}$$

Let H be the convex hull of a_1, \dots, a_n . Since $|y^{(n)}(z)|$ is continuous in H , it attains its maximum in H at some point $z = z_0 \in H$. Taking the absolute values in (3.4) and performing the n -fold integration along the straight line segments connecting a_k and ζ_{k-1} , we arrive at

$$(3.5) \quad \begin{aligned} |y(z)| &\leq \frac{1}{n!} |y^{(n)}(z_0)| \prod_{i=1}^n |a_i - z| \\ &< \frac{1}{n!} |y^{(n)}(z_0)| (1 + |z|)^n, z \in H. \end{aligned}$$

Similarly,

$$(3.6) \quad |y^{(k)}(z)| < \frac{(1 + |z|)^{n-k}}{(n - k)!} |y^{(n)}(z_0)|, z \in H,$$

$k = 1, 2, \dots, n - 1$. It is easily confirmed that

$$|I_j| \leq \frac{(j - 1)!}{n!} |y^{(n)}(z_0)| |a_j - z|^j,$$

and that $P_{n-j}^{(k+1-j)}(z)$ is the sum of $(n - j)!/(n - k - 1)!$ terms of the form $\prod_{l=1}^{n-k-1} (a_{il} - z)$. Therefore, we obtain from (3.1)

$$\begin{aligned} |y^{(k)}(z)| &< |y^{(n)}(z_0)| \sum_{j=1}^{k+1} \binom{k}{j-1} \frac{(n - j)!}{(n - k - 1)!} \frac{(j - 1)!}{n!} (1 + |z|)^{n-k} \\ &= \frac{(1 + |z|)^{n-k}}{(n - k)!} |y^{(n)}(z_0)|, z \in H, \end{aligned}$$

which proves (3.6).

We remark that the second inequality in (3.5) may be improved; by a result of Schwarz [8],

$$\prod_{i=1}^n |a_i - z| < (1 - |z|)(1 + |z|)^{n-1}, z \in H,$$

and therefore

$$(3.7) \quad |y(z)| < \frac{1}{n!} (1 - |z|)(1 + |z|)^{n-1} |y^{(n)}(z_0)|, z \in H.$$

Finally, we deduce from (3.3), (3.6), and (3.7) that

$$|y^{(n)}(z)| < |y^{(n)}(z_0)| \left[\sum_{k=1}^{n-1} \frac{(1 + |z|)^{n-k}}{(n - k)!} |p_k(z)| + \frac{1}{n!} (1 - |z|)(1 + |z|)^{n-1} |p_0(z)| \right], z \in H,$$

which, for $z = z_0 \in H$, yields

$$1 < \sum_{k=1}^{n-1} \frac{(1 + |z_0|)^{n-k}}{(n - k)!} |p_k(z_0)| + \frac{1}{n!} (1 - |z_0|)(1 + |z_0|)^{n-1} |p_0(z_0)|,$$

contrary to (3.2). This contradiction proves the theorem.

We add two remarks. A slight modification of the above proof will establish the following statements: Let R be a convex region with diameter δ . If

$$\sum_{k=0}^{n-1} \frac{\delta^{n-k}}{(n - k)!} |p_k(z)| \leq 1, z \in R,$$

then (3.3) is disconjugate in R . Theorem 3.2 generalizes a result recently obtained by Hadass [2, Th. 2].

There are known to the author a few other disconjugacy criteria for higher-order equations with analytic coefficients [4, 6].

We are now ready to state the disconjugacy condition (Theorem 3.2) as a multivalence criterion. From Theorems 2.1 and 3.1 we see that every nontrivial linear combination of f_1, f_2, \dots, f_{n-1} is $(n - 1)$ -valent if the equation

$$y^{(n)} + p_{n-2}(z)y^{(n-2)} + \dots + p_0(z)y = 0,$$

where p_0, \dots, p_{n-2} are defined as in (2.4), is disconjugate. In view of this relation and Theorem 3.2, we have the following theorem.

THEOREM 3.3. *Let f_1, f_2, \dots, f_{n-1} be analytic in the unit disk $D = \{z: |z| < 1\}$. Define p_0, p_1, \dots, p_{n-2} as in (2.4). If $\det(f_j^{(i)})_{i,j=1}^{n-1}$ does not vanish in D , and if*

$$\sum_{k=1}^{n-2} \frac{(1 + |z|)^{n-k}}{(n - k)!} |p_k(z)| + \frac{1}{n!} (1 - |z|)(1 + |z|)^{n-1} |p_0(z)| \leq 1, z \in D,$$

then every nontrivial linear combination of f_1, f_2, \dots, f_{n-1} is $(n - 1)$ -valent in D .

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Received May 28, 1969.

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Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley, California, 94708.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), 7-17, Fujimi 2-chome, Chiyoda-ku, Tokyo, Japan.

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