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ON $(m - n)$ PRODUCTS OF BOOLEAN ALGEBRAS

ROBERT HAMOR LA GRANGE, JR.

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R. H. LA GRANGE

This discussion begins with the problem of whether or not all $(m - n)$ products of an indexed set $\{\mathfrak{A}_t\}_{t \in T}$ of Boolean algebras can be obtained as m -extensions of a particular algebra \mathcal{F}_n^* . The construction of \mathcal{F}_n^* is similar to the construction of the Boolean product of $\{\mathfrak{A}_t\}_{t \in T}$; however the \mathfrak{A}_t are embedded in \mathcal{F}_n^* in such a way that their images are n -independent. If there is a cardinal number n' , satisfying $n < n' \leq m$, then $(m - n')$ products are not obtainable in this manner. For the case $n = m$ an example shows the answer to be negative. It is explained how the class of m -extensions of \mathcal{F}_n^* is situated in the class of all $(m - n)$ products of $\{\mathfrak{A}_t\}_{t \in T}$. A set of m -representable Boolean algebras is given for which the minimal $(m - n)$ product is not m -representable and for which there is no smallest $(m - n)$ product.

These problems have been proposed by R. Sikorski (see [2]). Concerning $\{\mathfrak{A}_t\}_{t \in T}$, it is assumed throughout that each of these algebras has at least four elements. m and n will always denote infinite cardinals with $n \leq m$. All definitions are taken from [2]. An m -homomorphism is a homomorphism that is conditionally m -complete. We denote the class of $(m - n)$ products of $\{\mathfrak{A}_t\}_{t \in T}$ by P_n and the class of $(m - 0)$ products by P . Let $\{\{i_t\}_{t \in T}, \mathcal{B}\}$ and $\{\{j_t\}_{t \in T}, \mathcal{C}\}$ be elements of P . We say that

$$\{\{i_t\}_{t \in T}, \mathcal{B}\} \leq \{\{j_t\}_{t \in T}, \mathcal{C}\}$$

provided there is an m -homomorphism h from \mathcal{C} onto \mathcal{B} such that $h \circ j_t = i_t$ for $t \in T$. The relation " \leq " is a quasi-ordering of P . Two $(m - 0)$ products are isomorphic if each is \leq to the other.

The particular product, $\{\{g_t^*\}_{t \in T}, \mathcal{F}_n^*\}$ of $\{\mathfrak{A}_t\}_{t \in T}$ mentioned above is defined as follows. For each $t \in T$ let X_t be the Stone space of \mathfrak{A}_t and let g_t be an isomorphism from \mathfrak{A}_t onto the field \mathcal{F}_t of all open and closed subsets of X_t . Let X be the Cartesian product of the sets X_t , and for each $t \in T$ and each $b \in \mathfrak{A}_t$, set

$$(1) \quad g_t^*(b) = [x \in X: x(t) \in g_t(b)] .$$

Let G_n be the set of all subsets a of X which satisfy the following condition:

$$a = \bigcap_{t \in S} g_t^*(b_t) \text{ where } b_t \in \mathfrak{A}_t, S \subseteq T \text{ and } \bar{S} \leq n .$$

Finally, let \mathcal{F}_n^* be the field of subsets of X which is generated by G_n .

\mathcal{F}_n^* is a base for the n -topology on X . g_i^* is a complete isomorphism from \mathfrak{A}_i into \mathcal{F}_n^* . The set $\{g_i^*(\mathfrak{A}_i)\}$, of subalgebras, is n -independent.

A Boolean $(m - n)$ product $\{\{i_t\}_{t \in T}, \mathcal{B}\}$ is said to *belong to* E_n if and only if there is an m -isomorphism h (from \mathcal{F}_n^* into \mathcal{B}) such that $\{h, \mathcal{B}\}$ is an m -extension of \mathcal{F}_n^* and for each $t \in T$ $h \circ g_t^* = i_t$.

For every m -extension $\{h, \mathcal{B}\}$ of \mathcal{F}_n^* , $\{h \circ g_t^*\}_{t \in T}, \mathcal{B} \in E_n$. Clearly $E_n \subseteq P_n$ and E_n is not empty. m -extensions of \mathcal{F}_n^* seem to provide the most natural examples of Boolean $(m - n)$ products.

1. LEMMA 1.1. *Let $\{\mathcal{B}_i\}_{i \in T}$ be an n -independent set of subalgebras of a Boolean algebra \mathfrak{A} and let S and S' be subsets of T with $\bar{S} \leq n$ and $\bar{S}' \leq n$. For each t let a_t and b_t be nonzero elements of \mathcal{B}_t . Then*

- (i) $\prod_{t \in S} a_t \leq \prod_{t \in S} b_t$ if and only if $a_t \leq b_t$ for each $t \in S$;
- (ii) $\prod_{t \in S} a_t = \prod_{t \in S'} b_t$ implies that $a_t = b_t$ for $t \in S \cap S'$, $a_t = 1$ for $t \in S - S'$, and $b_t = 1$ for $t \in S' - S$.

Proof. (i) Assume that for some $t_0 \in S$, $a_{t_0} \not\leq b_{t_0}$. Define

$$C_t = \begin{cases} a_t & \text{if } t \in S \text{ and } t \neq t_0, \\ a_{t_0} \cdot (-b_{t_0}) & \text{if } t = t_0. \end{cases}$$

Set $c = \prod_{t \in S} c_t$, and note that $c \neq 0$, $c \leq \prod_{t \in S} a_t$, and $c \cdot \prod_{t \in S} b_t = 0$. The converse is clear.

To prove (ii) we define

$$x_t = \begin{cases} a_t & \text{if } t \in S, \\ 1 & \text{if } t \in S' - S; \end{cases} \quad \text{and} \quad y_t = \begin{cases} b_t & \text{if } t \in S', \\ 1 & \text{if } t \in S - S'. \end{cases}$$

Now

$$\prod_{t \in S \cup S'} x_t = \prod_{t \in S} a_t = \prod_{t \in S'} b_t = \prod_{t \in S \cup S'} y_t$$

and (ii) follows from (i).

LEMMA 1.2. *Let $\{\mathcal{B}_i\}_{i \in T}$ be an n -independent set of subalgebras of a Boolean algebra \mathfrak{A} . Let G be the set of all meets $\prod_{t \in S} a_t$ such that $S \subseteq T$, $\bar{S} \leq n$, and for each $t \in S$ a_t is a nonzero element of \mathcal{B}_t . Assume further that G generates \mathfrak{A} . Then G is dense in \mathfrak{A} .*

Proof. First note that for $g, g' \in G$ either $g \cdot g' = 0$ or else $g \cdot g' \in G$. Thus every nonzero element of \mathfrak{A} is a finite join of elements of the form $g \cdot \prod_{i < k} (-g_i)$ with $g, g_i \in G$ and k finite. (This notation is intended

to include the special cases g and $-g$.) Now suppose $g \cdot \prod_{i < k}^{\mathfrak{A}} (-g_i) \neq 0$, so that $g \not\cong \sum_{i < k} g_i$. We write a common form $g = \prod_{t \in S} a_t$, and for each $i < k$ $g_i = \prod_{t \in S} a_{i,t}$ where $S \subseteq T$, $\bar{S} \leq n$, and for each $t \in S$ a_t and $a_{i,t}$ are nonzero elements of \mathcal{B}_i . Since k is finite every Boolean algebra is $(k - n)$ -distributive (see [2], p. 62). We have

$$\prod_{t \in S} a_t \not\cong \sum_{i < k} \prod_{t \in S} a_{i,t} = \prod_{\psi \in S^k} \sum_{i < k} a_{i,\psi(i)} .$$

(Here S^k denotes the set of all functions from $k = \{0, 1, \dots, k - 1\}$ into S .) Choose $\phi \in S^k$ such that $\prod_{t \in S} a_t \not\cong \sum_{i < k} a_{i,\phi(i)}$. We have, for each $s \in \{\phi(i) : i < k\}$, $a_s \not\cong \sum_{\phi(i)=s} a_{i,\phi(i)}$. Define

$$b_t = \begin{cases} a_t & \text{if } t \in S - \{\phi(i) : i < k\} \\ a_t \cdot \sum_{\phi(i)=t} a_{i,\phi(i)} & \text{if } t \in \{\phi(i) : i < k\} . \end{cases}$$

Finally let $b = \prod_{t \in S} b_t$. Clearly $b \neq 0$, $b \in G$ and $b \leq g$. For each $t \in \{\phi(i) : i < k\}$, $b_t \cdot \sum_{\phi(i)=t} a_{i,\phi(i)} = 0$, so that $b \cdot \sum_{i < k} a_{i,\phi(i)} = 0$. It follows that $b \cdot \sum_{i < k} g_i = 0$, hence $b \leq g \cdot \prod_{i < k} (-g_i)$.

COROLLARY 1.3. *If $\bar{S} > n$, and for each $t \in S$, $a_t \neq 1$, then $\prod_{t \in S}^{\infty} a_t = 0$.*

THEOREM 1.4. *Let $\{\{i_t\}_{t \in T}, \mathcal{B}\} \in \mathbf{P}_n$. There is one and only one isomorphism h_n from \mathcal{F}_n^* into \mathcal{B} which satisfies the following completeness condition:*

$$(c) \quad h_n\left(\prod_{t \in S}^{\mathcal{F}_n^*} g_t^*(a_t)\right) = \prod_{t \in S}^{\mathcal{B}} i_t(a_t) \text{ whenever } S \subseteq T, \bar{S} \leq n, \\ a_t \in \mathfrak{A}_t \text{ and } a_t \neq 0 .$$

Proof. Let G be the set of all meets $\prod_{t \in S}^{\mathcal{B}} i_t(a_t)$ such that $S \subseteq T$, $\bar{S} \leq n$, each $a_t \in \mathfrak{A}_t$ and $a_t \neq 0$. Let \mathfrak{A} be the subalgebra of \mathcal{B} which is generated by G . For $\prod_{t \in S}^{\mathcal{B}} i_t(a_t) \in G$ it is clear that $\prod_{t \in S}^{\mathcal{B}} i_t(a_t) = \prod_{t \in S}^{\mathfrak{A}} i_t(a_t)$. By Lemma 1.2 G is dense in \mathfrak{A} . Also G_n is dense in \mathcal{F}_n^* . For $a \in G_n$ write $a = \bigcap_{t \in S} g_t^*(a_t) = \prod_{t \in S}^{\mathcal{F}_n^*} g_t^*(a_t)$. Define $h(a) = \prod_{t \in S}^{\mathfrak{A}} i_t(a_t)$. It is easily seen, using Lemma 1.1, that

- (i) h is a one to one function from G_n onto G ;
- (ii) for $a, b \in G_n$, $a \leq b$ if and only if $h(a) \leq h(b)$.

It follows (see [2], p. 37) that h can be extended to an isomorphism h_n from \mathcal{F}_n^* onto \mathfrak{A} . h_n is uniquely determined by condition (c) because G_n generates \mathcal{F}_n^* .

COROLLARY 1.5. *The product $\{\{i_t\}_{t \in T}, \mathcal{B}\} \in \mathbf{E}_n$ if and only if h_n is m-complete.*

Proof. Let $\{\{i_t\}_{t \in T}, \mathcal{B}\} \in E_n$. There is an m -isomorphism f from \mathcal{F}_n^* into \mathcal{B} such that for each $t \in T, f \circ g_t^* = i_t$. f satisfies condition (c) so $f = h_n$.

COROLLARY 1.6. *Assume $\bar{T} > n$ and that $m \geq n' > n$. Then $P_{n'} \cap E_n$ is empty.*

Proof. Let $\{\{i_t\}_{t \in T}, \mathcal{B}\} \in P_n$. Consider the isomorphism h_n from \mathcal{F}_n^* into \mathcal{B} . Choose $S \subseteq T, \bar{S} = n^+$, and for each $t \in S$ choose $a_t \in \mathfrak{A}_t$ with $a_t \neq 0, a_t \neq 1$. By Corollary 1.3

$$\prod_{t \in S}^{*} g_t^*(a_t) = 0.$$

However $0 \neq \prod_{t \in S} i_t(a_t) = \prod^{*} h_n \circ g_t^*(a_t)$ so that h_n is not m -complete.

There is an interesting contrast between E_n and $P_{n'}$, (under the hypotheses of Corollary 1.6). Let $\{\{i_t\}_{t \in T}, \mathcal{B}\}$ and $\{\{j_t\}_{t \in T}, \mathbb{C}\}$ be elements of P_n with $\{\{i_t\}_{t \in T}, \mathcal{B}\} \leq \{\{j_t\}_{t \in T}, \mathbb{C}\}$. It is known (see [2], p. 179) that if $\{\{i_t\}_{t \in T}, \mathcal{B}\} \in P_{n'}$, then $\{\{j_t\}_{t \in T}, \mathbb{C}\} \in P_{n'}$. On the other hand if $\{\{j_t\}_{t \in T}, \mathbb{C}\} \in E_n$ then we have $\{\{i_t\}_{t \in T}, \mathcal{B}\} \in E_n$.

COROLLARY 1.7. *Assume $\bar{T} > n$ and $m > n$. Then $E_n \cup P_{n^+} \neq P_n$.*

Proof. Let $S \subseteq T$ with $\bar{S} = n^+$. Choose, for each $t \in S, d_t \in \mathfrak{A}_t$ with $d_t \neq 0, d_t \neq 1$. Let $d = \bigcap_{t \in S} g_t^*(d_t)$. Let \mathcal{F} be the field of subsets of X which is generated by $\mathcal{F}_n^* \cup \{d\}$. Note that g_t^* is a complete isomorphism from \mathfrak{A}_t into \mathcal{F} . Let $\{f, \mathbb{C}\}$ be any m -extension of \mathcal{F} . It is easily seen that $\{\{f \circ g_t^*\}_{t \in T}, \mathbb{C}\} \in P_n$.

Consider the isomorphism h_n from \mathcal{F}_n^* into \mathbb{C} . $h_n \circ g_t^* = f \circ g_t^*$ for every $t \in T$. By Corollary 1.3 $\prod_{t \in S}^{*} g_t^*(d_t) = 0$. However $\prod_{t \in S}^{\mathbb{C}} h_n \circ g_t^*(d_t) = f(d) \neq 0$. Thus h_n is not m -complete and $\{\{f \circ g_t^*\}_{t \in T}, \mathbb{C}\} \notin E_n$.

In order to show that $\{\{f \circ g_t^*\}_{t \in T}, \mathbb{C}\} \notin P_{n^+}$ it suffices to show that $\prod_{t \in S} f \circ g_t^*(-d_t) = 0$. In particular suppose $b = \prod_{t \in S}^{*} g_t^*(-d_t) \neq 0$. Since $b \cdot d = 0$ the definition of \mathcal{F} enables us to write $b = \bigcup_{t \in S} b_1 \cdot g_t^*(-d_t)$ with $b_1 \in \mathcal{F}_n^*$. Choose $t_0 \in S$ such that $0 \neq b_1 \cdot g_{t_0}^*(-d_{t_0}) \leq b$. By Lemma 1.2 there is a nonzero element $a = \bigcap_{t \in S} g_t^*(a_t)$ of G_n such that $a \leq b_1 \cdot g_{t_0}^*(-d_{t_0})$. Now $\bar{S}' \leq n$ and $\bar{S}' = n^+$ and it follows that $a \not\leq b$. Thus $\prod_{t \in S}^{*} g_t^*(-d_t) = 0$ and since f is m -complete, $\prod_{t \in S}^{\mathbb{C}} f \circ g_t^*(-d_t) = 0$.

We now consider the case $n = m$. It is known that $E_m \neq P_m$ if $m = \aleph_0$ (see [2], p. 190, Example D). In this example T is the two element set $\{1, 2\}$, \mathfrak{A}_1 and \mathfrak{A}_2 are σ -complete Boolean algebras which satisfy the σ -chain condition. The Boolean σ -product $\{\{i_1, i_2\}, \mathcal{B}\}$ is such that the subalgebra \mathcal{B}_0 of \mathcal{B} which is generated by $i_1(\mathfrak{A}_1) \cup i_2(\mathfrak{A}_2)$

is not a σ -regular subalgebra of \mathcal{B} . Let $\{f, \mathbb{C}\}$ be any m -extension of \mathcal{B} . It follows, using the σ -chain condition on \mathfrak{A}_1 and \mathfrak{A}_2 , that $\{f \circ i_1, f \circ i_2, \mathbb{C}\} \in \mathbf{P}_m$. Since T is finite $\{\{g_1^*, g_2^*\}, \mathcal{F}_m^*\}$ is the Boolean product of $\{\mathfrak{A}_1, \mathfrak{A}_2\}$. Let h be the homomorphism from \mathcal{F}_m^* into \mathcal{B} such that $h \circ g_1^* = i_1$ and $h \circ g_2^* = i_2$. Then h is an isomorphism from \mathcal{F}_m^* onto \mathcal{B} . Consider the isomorphism h_m , from \mathcal{F}_m^* into \mathbb{C} , given by Theorem 1.4. $h_m = f \circ h$ since they agree on $g_1^*(\mathfrak{A}_1) \cup g_2^*(\mathfrak{A}_2)$. h_m is not m -complete because $f(\mathcal{B}_0)$ is not m -regular in \mathbb{C} . Thus $\{f \circ i_1, f \circ i_2, \mathbb{C}\} \notin \mathbf{E}_m$. We give a simple for the case $m \geq 2^{\aleph_0}$.

EXAMPLE 1.8. Assume $m \geq 2^{\aleph_0}$ and let T be a set of power \aleph_0 . For each $t \in T$ let \mathfrak{A}_t be a Boolean algebra having exactly four elements. Let \mathcal{B} be the free Boolean m -algebra on \aleph_0 m -generators, $(D_t: t \in T)$. \mathcal{B} is not m -representable (see [2], p. 134). For each $t \in T$ choose d_t to be one of the atoms of \mathfrak{A}_t . Let i_t be the isomorphism from \mathfrak{A}_t into \mathcal{B} such that $i_t(d_t) = D_t$. Then $\{\{i_t\}_{t \in T}, \mathcal{B}\} \in \mathbf{P}_m$. By Lemma 1.2 \mathcal{F}_m^* is atomic, the atoms being all sets of the form $\bigcap_{t \in T} g_t^*(a_t)$, where for each $t \in T$ a_t is an atom of \mathfrak{A}_t . Denote the set of atoms of \mathcal{F}_m^* by $\{C_r: r \in R\}$, then $\bar{R} = 2^{\aleph_0}$. We consider the isomorphism h_m from \mathcal{F}_m^* into \mathcal{B} . For each $r \in R$, $h_m(c_r)$ is an atom of \mathcal{B} . To show this we define

$$\mathfrak{A} = \{b \in \mathcal{B}: \text{for each } r \in R \text{ either } b \cdot h_m(c_r) = 0 \text{ or } h_m(c_r) \leq b\}.$$

It is easily seen that \mathfrak{A} is an m -subalgebra of \mathcal{B} which includes $\{D_t: t \in T\}$. Hence $\mathfrak{A} = \mathcal{B}$. Finally, h_m is not m -complete. For otherwise $\sum_{r \in R} h_m(c_r) = 1$, and \mathcal{B} would be atomic and hence isomorphic to an m -field of sets.

2. We now consider the problem of the existence of a smallest element of \mathbf{P} , relative to the quasi-ordering " \leq ". A minimal element of \mathbf{P} always exists and can be constructed as follows. Let $\{\{f_t\}_{t \in T}, \mathbb{C}\}$ be a Boolean product of $\{\mathfrak{A}_t\}_{t \in T}$ and let $\{h, \mathcal{B}\}$ be an m -completion of \mathbb{C} . Then $\{\{h \circ f_t\}_{t \in T}, \mathcal{B}\}$ is a minimal element of \mathbf{P} . We shall show that this product need not be a smallest element of \mathbf{P} . Hence \mathbf{P} need not have a smallest element.

EXAMPLE 2.1. Let m be any infinite cardinal. Let $\bar{T} = \aleph_0$ and suppose that for each $t \in T$ \mathfrak{A}_t is a four element Boolean algebra. For each $t \in T$ choose a_t to be one of the atoms of \mathfrak{A}_t . \mathbb{C} is a free Boolean algebra of power \aleph_0 , one set of free generators being $\{f_t(a_t): t \in T\}$. \mathcal{B} has a countable dense subset, in particular \mathcal{B} satisfies the countable chain condition. Thus \mathcal{B} is complete. It follows that \mathcal{B} is isomorphic to the quotient algebra \mathcal{F}/Δ_0 where \mathcal{F} is the σ -field

of Borel subsets of the unit interval $I = \{x: 0 < x \leq 1\}$ of real numbers and Δ_0 is the ideal consisting of those Borel sets which are of the first category.

To show that $\{\{h \circ f_i\}_{i \in T}, \mathcal{B}\}$ is not a smallest element of \mathbf{P} we construct another (m-0) product as follows. Let G be the set of all halfopen intervals of the form $\{x: 0 < x \leq r\}$ such that r is rational and $0 < r \leq 1$. \mathcal{F} is σ -generated by G . The subalgebra \mathcal{F}_0 of \mathcal{F} which is generated by G is denumerable and atomless. Hence \mathcal{F}_0 is isomorphic to \mathbb{C} (see [1], p. 54). Let g be an isomorphism from \mathbb{C} onto \mathcal{F}_0 . Let Δ_1 be the ideal of \mathcal{F} consisting of those Borel sets having Lebesgue measure 0. We note that $\mathcal{F}_0 \cap \Delta_1 = \{0\}$. Finally for each $t \in T$ let h_t be the isomorphism from \mathfrak{A}_t into \mathcal{F}/Δ_1 defined by $h_t(a_i) = [g \circ f_t(a_i)]\Delta_1$. It is easily seen that $\{\{h_t\}_{t \in T}, \mathcal{F}/\Delta_1\} \in \mathbf{P}$.

Now assume $\{\{h \circ f_i\}_{i \in T}, \mathcal{B}\} \leq \{\{h_t\}_{t \in T}, \mathcal{F}/\Delta_1\}$. Then there is an m-homomorphism p from \mathcal{F}/Δ_1 onto \mathcal{F}/Δ_0 . Since \mathcal{F}/Δ_1 satisfies the countable chain condition the kernel of p is a principal ideal. \mathcal{F}/Δ_0 is isomorphic to a principal ideal of \mathcal{F}/Δ_1 . However \mathcal{F}/Δ_1 is homogeneous (see [2], p. 105). Thus \mathcal{F}/Δ_0 is isomorphic to \mathcal{F}/Δ_1 , which is a contradiction.

Next we consider the problem of the existence of a smallest element of \mathbf{P}_n . Let $\{g, \mathcal{B}\}$ be an m-completion of \mathcal{F}_n^* . Then $\{\{g \circ g_i^*\}_{i \in T}, \mathcal{B}\}$ is a minimal element of \mathbf{P}_n . Also it is known (see [2], p. 183) that if all the \mathfrak{A}_i are m-representable then there is an (m-n) product $\{\{i_i\}_{i \in T}, \mathbb{C}\}$ for which \mathbb{C} is m-representable. We give an example of $\{\mathfrak{A}_i\}_{i \in T}$ for which \mathcal{B} is not m-representable and $\{\{g \circ g_i^*\}_{i \in T}, \mathcal{B}\}$ is not a smallest element of \mathbf{P}_n .

EXAMPLE 2.2. Assume that $m \geq 2^{(n^+)}$. Let $T = n^+$ and for each $t \in T$ let \mathfrak{A}_t be a four element Boolean algebra. We show that \mathcal{B} is not n^+ -distributive. Choose, for each $t \in T$, a_t to be one of the atoms of \mathfrak{A}_t . Then

$$\prod_{i \in T}^{\mathcal{B}} (g \circ g_i^*(a_i) + -g \circ g_i^*(a_i)) = 1 .$$

However for each function $\eta \in H^T$ (here $H = \{+1, -1\}$) we have

$$\prod_{i \in T}^{\mathcal{F}_n^*} \eta(t) \cdot g_i^*(a_i) = 0 .$$

This follows from Corollary 1.3. Thus $\prod_{i \in T}^{\mathcal{B}} \eta(t) \cdot g \circ g_i^*(a_i) = 0$. This proves \mathcal{B} is not n^+ -distributive and hence not m-representable.

To show that $\{\{g \circ g_i^*\}_{i \in T}, \mathcal{B}\}$ is not a smallest element of \mathbf{P}_n , let $\{\{i_i\}_{i \in T}, \mathbb{C}\}$ be any (m-n) product of $\{\mathfrak{A}_i\}_{i \in T}$ such that \mathbb{C} is m-representable. \mathcal{B} is not an m-homomorphic image of \mathbb{C} . Thus the inequality

$$\{\{g \circ g_i^*\}_{i \in T}, \mathcal{B}\} \cong \{\{i_i\}_{i \in T}, \mathcal{C}\}$$

does not hold.

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