

# Pacific Journal of Mathematics

**SEMI-SIMPLE RADICAL CLASSES**

PATRICK NOBLE STEWART

## SEMI-SIMPLE RADICAL CLASSES

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**The purpose of this paper is to characterize all semi-simple radical classes (those classes of rings which are semi-simple classes and at the same time radical classes).**

Andrunakievic has shown that the class of Boolean rings is a semi-simple radical class. More recently, Armendariz has considered such classes.

For “ $I$  is an ideal of the ring  $R$ ” we shall write “ $I \triangleleft R$ ”.

Following Divinsky [6], but substituting classes of rings for ring properties, we define:

(i) A nonempty class of rings  $\mathcal{C}$  is a *radical class* if and only if  $\mathcal{C}$  satisfies the following conditions:

(A) Homomorphic images of rings in  $\mathcal{C}$  are in  $\mathcal{C}$ .

(B) Every ring  $R$  has an ideal  $\mathcal{C}(R) \in \mathcal{C}$  such that if  $I \triangleleft R$  and  $I \in \mathcal{C}$  then  $I \subseteq \mathcal{C}(R)$ .

(C) The only ideal of the factor ring  $R/\mathcal{C}(R)$  which is in  $\mathcal{C}$  is the zero ideal.

(ii) If  $\mathcal{C}$  is a radical class, a ring  $R$  is  $\mathcal{C}$  *semi-simple* if and only if  $\mathcal{C}(R) = (0)$ .

(iii) A nonempty class of rings  $\mathcal{C}$  is a *semi-simple class* if and only if  $\mathcal{C}$  satisfies the following conditions:

(E) Every nonzero ideal of a ring in  $\mathcal{C}$  can be homomorphically mapped onto a nonzero ring in  $\mathcal{C}$ .

(F) If every nonzero ideal of a ring  $R$  can be homomorphically mapped onto a nonzero ring in  $\mathcal{C}$  then  $R \in \mathcal{C}$ .

2. Rings without nilpotent elements. Our purpose in this section is to establish:

**THEOREM 2.1.**<sup>1</sup> *A ring  $R$  without nilpotent elements is isomorphic to a subdirect sum of rings without proper divisors of zero.*

It will be convenient to first prove:

**LEMMA 2.2.** *If  $R$  has no nilpotent elements and  $0 \neq x \in R$  then*

(i)  $x_r = \{y \in R: xy = 0\} \triangleleft R$  and  $x_r = x_l = \{y \in R: yx = 0\}$ ,

(ii)  $x \notin x_l$ ,

<sup>1</sup> The author wishes to thank the referee for pointing out that this result has also been obtained by V. Andrunakievic and Ju. M. Rjabuhin, *Rings without nilpotent elements, and completely simple ideals*, Dokl. Akad. Nauk. SSR. **180**, 9-11 (Translation, Soviet Mathematics **9** (1968), 565-568).

- (iii) if  $r \in R$  and  $rx \in x_i$  then  $r \in x_i$ ,
- (iv) the factor ring  $R/x_i$  has no nilpotent elements.

*Proof.* Let  $R$  be a ring with no nilpotent elements and  $0 \neq x \in R$ . If  $a \in R$  and  $ax = 0$  then  $(xa)^2 = 0$  so  $xa = 0$ . Similarly if  $xa = 0$  then  $ax = 0$ . This establishes (i). Since  $x^2 \neq 0$ , (ii) is clear. If  $a, b \in R$  and  $ab^2 = 0$  then  $(bab)^2 = 0$  so  $bab = 0$ , but then  $(ab)^2 = 0$  so  $ab = 0$ . From this (iii) and (iv) follow immediately.

To prove the theorem it is sufficient to find, for each  $0 \neq x \in R$ , an ideal  $I(x)$  of  $R$  for which  $R/I(x)$  has no proper divisors of zero and  $x \notin I(x)$ . Let  $Z(x) = \{I \triangleleft R: x \notin I, \text{ if } rx \in I \text{ then } r \in I, \text{ and } R/I \text{ has no nilpotent elements}\}$ . By 2.2  $x_i \in Z(x)$  so  $Z(x) \neq \emptyset$  and it is clear that the union of an ascending chain in  $Z(x)$  is also in  $Z(x)$ . Thus we may choose, by Zorn's Lemma,  $I(x)$  maximal in  $Z(x)$ .

If  $a \in R$  and  $a \in I(x)$  let  $J = \{y \in R: ay \in I(x)\} \supseteq I(x)$ . Then  $J/I(x) = (a + I(x))_r$  in  $R/I(x)$  and by 2.2 (i)  $(a + I(x))_i = (a + I(x))_r \triangleleft R/I(x)$ . Since  $a \notin I(x)$ ,  $ax \notin I(x)$  so  $x \notin J$ . If  $rx \in J$  then  $arx \in I(x)$  so  $ar \in I(x)$ , hence  $r \in J$ . Finally by 2.2 (iv)  $R/J \cong R/I(x)/J/I(x)$  has no nilpotent elements, so  $J \in Z(x)$ . Hence  $J = I(x)$  so  $R/I(x)$  has no proper divisors of zero.

*Note 2.3.* The generalized nil radical  $Ng$  of Andrunakievic [4] and Thierrin [10] (see also [6]) is the upper radical with respect to the class of rings without proper divisors of zero. A ring  $R$  is  $Ng$  semi-simple if and only if  $R$  is isomorphic to a subdirect sum of rings without proper divisors of zero. In this context, 2.1 can be restated as: A ring  $R$  is  $Ng$  semi-simple if and only if  $R$  has no nilpotent elements.

3.  $\mathcal{B}_1$ -rings. If  $x \in R$ , let  $[x]$  = the subring of  $R$  generated by  $x$ .

DEFINITION 3.1.  $R$  is a  $\mathcal{B}_1$ -ring  $\equiv$  for all  $x \in R$ ,  $[x] = [x]^2$ .

Let  $R$  be a ring and  $x \in R$ . Clearly  $[x] = [x]^2$  if and only if  $x \in [x]^2$  if and only if there are integers  $a_2, \dots, a_k$  such that  $x = \sum_{i=2}^k a_i x^i$ . Using this it is clear that homomorphic images of  $\mathcal{B}_1$ -rings are  $\mathcal{B}_1$ -rings and that if  $A/B$  and  $B$  are  $\mathcal{B}_1$ -rings then  $A$  is a  $\mathcal{B}_1$ -ring. It then easily follows that the class of  $\mathcal{B}_1$ -rings (which we shall denote by  $\mathcal{B}_1$ ) is a radical class.

LEMMA 3.2. A nonzero  $\mathcal{B}_1$ -ring without proper divisors of zero is a field of prime characteristic which is algebraic over its prime subfield.

*Proof.* Let  $R$  be a nonzero  $\mathcal{B}_1$ -ring without proper divisors of

zero. If  $x$  is a nonzero element of  $R$  there are integers  $a_2, \dots, a_k$  such that  $x = \sum_{i=2}^k a_i x^i$ , hence  $e_x = \sum_{i=2}^k a_i x^{i-1}$  is an identity for  $[x]$ . Since  $x$  is not a zero divisor  $e_x$  is an identity for  $R$ . If  $w \in R, w \neq 0, e_w \in [w] = [w]^2$  so  $e_w \in [w] \cdot w \subseteq Rw$  thus  $R = Rw$ . Since  $R$  is nonzero,  $R$  is a division ring.

Let  $e$  be the identity of  $R$ . Then  $[2e] = [2e]^2 = [4e]$  so  $Ne = 0$  for some positive integer  $N$ . Consequently the characteristic of  $R$  is a prime and since  $e = e_w \in [w]$  for all nonzero  $w \in R, R$  is algebraic over its prime subfield. Therefore, by Theorem 2, page 183 of Jacobson [7]  $R$  is a field.

**COROLLARY 3.3.** *If  $R$  is a  $\mathcal{B}_1$ -ring then  $R$  is isomorphic to a subdirect sum of algebraic fields of prime characteristic. So, in particular,  $R$  is commutative.*

*Proof.* If  $x \in R, x^N = 0$  and  $R \in \mathcal{B}_1$ , then  $[x] = [x]^2 = \dots = [x]^N = (0)$  so  $x = 0$ . Hence  $\mathcal{B}_1$ -rings do not have nilpotent elements so the corollary follows from 2.1 and 3.2.

**THEOREM 3.4.** *A ring  $R$  is a  $\mathcal{B}_1$ -ring if and only if every finitely generated subring of  $R$  is isomorphic to a finite direct sum of finite fields.*

*Proof.* Let  $R \in \mathcal{B}_1$  and  $R'$  be a finitely generated subring of  $R$ . Then  $R' \in \mathcal{B}_1$  and hence is commutative, so by the Hilbert Basis Theorem  $R'$  has maximum condition on ideals. If  $P' \neq R'$  and  $P'$  is a prime ideal of  $R'$  then  $P'$  is a maximal ideal of  $R'$  since by 3.2  $R'/P'$  is a field. Since  $R'$  is finitely generated, commutative, and  $[g]$  has an identity for each generator  $g$  of  $R', R'$  has an identity. Then by Theorem 2, page 203 of [11]  $R'$  has minimum condition on ideals. But then  $R'$  is a commutative Wedderburn ring so  $R'$  is isomorphic to a finite direct sum of fields each of which must be finite since they are finitely generated, algebraic and of prime characteristic.

The converse is obvious; in fact, if  $x \in R'$  and  $R'$  is isomorphic to a finite direct sum of finite fields then there is an integer  $n(x) \geq 2$  such that  $x^{n(x)} = x$ . Thus we have:

**COROLLARY 3.5.**  *$R$  is a  $\mathcal{B}_1$ -ring if and only if for each  $x \in R$  there exists an integer  $n(x) \geq 2$  such that  $x^{n(x)} = x$ .*

A class of rings  $\mathcal{C}$  is said to be *hereditary* if  $I \triangleleft R \in \mathcal{C}$  implies that  $I \in \mathcal{C}$ . Analogously we say:

**DEFINITION 3.6.** A class of rings  $\mathcal{C}$  is *strongly hereditary*  $\equiv$  if  $S$  is a subring of  $R \in \mathcal{C}$  then  $S \in \mathcal{C}$ .

**PROPOSITION 3.7.** *If  $\mathcal{F}$  is a strongly hereditary finite set of finite fields then a ring  $R$  is isomorphic to a subdirect sum of fields in  $\mathcal{F}$  if and only if every finitely generated subring of  $R$  is isomorphic to a finite direct sum of fields in  $\mathcal{F}$ .*

*Proof.* Since  $\mathcal{F}$  is a finite set of finite fields there exists an integer  $N \geq 2$  such that  $x^N = x$  for all  $x \in F \in \mathcal{F}$ .

Let  $R$  have ideals  $I_\alpha: \alpha \in A$  such that  $R/I_\alpha \cong F_\alpha \in \mathcal{F}$  and  $\cap \{I_\alpha: \alpha \in A\} = (0)$ . Let  $R'$  be a finitely generated subring of  $R$ . Then  $R' \in \mathcal{B}_1$  since  $x^N = x$  for all  $x \in R \supseteq R'$ , so by 3.4  $R' \cong A_1 \oplus \cdots \oplus A_n$  and the  $A_i$  are finite fields. Choose  $a_i \in R'$  such that  $[a_i] \cong A_i$ . Then  $a_i \neq 0$  so  $a_i \in I_{\beta_i}$  for some  $\beta_i \in A$  but  $I_{\beta_i} \cap [a_i] \triangleleft [a_i]$  so  $I_{\beta_i} \cap [a_i] = (0)$ . Therefore  $A_i \cong [a_i] \cong [a_i] + I_{\beta_i}/I_{\beta_i}$  is isomorphic to a subring of  $F_{\beta_i}$ . Since  $\mathcal{F}$  is strongly hereditary  $R'$  is isomorphic to a finite direct sum of fields in  $\mathcal{F}$ .

Conversely, if every finitely generated subring of  $R$  is isomorphic to a finite direct sum of fields in  $\mathcal{F}$ ,  $R$  must be a  $\mathcal{B}_1$ -ring since again  $x^N = x$  for all  $x \in R$ . Thus by 3.3 there are ideals  $I_\alpha: \alpha \in A$  of  $R$  such that  $\cap \{I_\alpha: \alpha \in A\} = (0)$  and  $R/I_\alpha$  is a field of prime characteristic; moreover,  $R/I_\alpha$  must be a finite field since  $x^N - x = 0 \in I_\alpha$  for all  $x \in R$ . Therefore, for each  $\alpha \in A$ , there exists  $x_\alpha \in R$  such that  $[x_\alpha] + I_\alpha/I_\alpha = R/I_\alpha$ . But then  $R/I_\alpha$  is a homomorphic image of  $[x_\alpha]$  so  $R/I_\alpha$  is isomorphic to a field in  $\mathcal{F}$ .

#### 4. Semi-simple radical classes.

**LEMMA 4.1.** *If  $\mathcal{C}$  is a class of rings such that subdirect sums of rings in  $\mathcal{C}$  are in  $\mathcal{C}$  and  $\mathcal{C}$  satisfies (A) then  $\mathcal{C}$  is strongly hereditary.*

*Proof.* Let  $R \in \mathcal{C}$  and  $S$  be a subring of  $R$ .

Set  $R_i = R$  for all  $i \in \mathbb{Z}^+ =$  the set of positive integers. Now the (discrete) direct sum  $\sum \{R_i: i \in \mathbb{Z}^+\}$  is an ideal of the direct product (complete direct sum)  $\prod \{R_i: i \in \mathbb{Z}^+\}$ . If  $s \in S$  let  $\hat{s}(i) = s$  for all  $i \in \mathbb{Z}^+$ . Then  $S \rightarrow \mathcal{A}(S) = \{\hat{s}: s \in S\}$  is an embedding of  $S$  into  $\prod \{R_i: i \in \mathbb{Z}^+\}$ .  $\mathcal{A}(S) + \sum \{R_i: i \in \mathbb{Z}^+\}$  is a subdirect sum of copies of  $R$  and hence is in  $\mathcal{C}$ , so

$$S \cong \mathcal{A}(S) \cong \frac{\mathcal{A}(S) + \sum \{R_i: i \in \mathbb{Z}^+\}}{\sum \{R_i: i \in \mathbb{Z}^+\}} \in \mathcal{C}.$$

Using a theorem of Amitsur [1] which states that every ring is a homomorphic image of a subdirect sum of total matrix rings of finite order over the ring of all integers, Armendariz in [5] proves

that if a hypernilpotent radical class  $\mathcal{C}$  is a semi-simple class, then  $\mathcal{C}$  contains all rings. A hypernilpotent radical class is a hereditary radical class which contains all nilpotent rings.

**THEOREM 4.2.** *If  $\mathcal{C}$  is a semi-simple radical class and  $\mathcal{C} \not\subseteq \mathcal{B}_1$ , then  $\mathcal{C}$  consists of all rings.*

*Proof.* Let  $\mathcal{C}$  be a semi-simple radical class. If  $\mathcal{C} \not\subseteq \mathcal{B}_1$  then there is a  $R \in \mathcal{C}$  and  $x \in R$  such that  $[x] \neq [x]^2$ . In [8] Kurosh shows that for any semi-simple class  $\mathcal{S}$ , subdirect sums of rings in  $\mathcal{S}$  are in  $\mathcal{S}$ . Thus, by 4.1,  $[x] \in \mathcal{C}$  and since  $[x]^2 \triangleleft [x]$ ,  $[x]/[x]^2 \in \mathcal{C}$ . Now  $[x]/[x]^2$  is a zero ring on a cyclic group and since  $\mathcal{C}$  satisfies (F),  $C^\infty =$  the zero ring on the infinite cyclic group is in  $\mathcal{C}$ . This implies (see [3] and [6]) that  $\mathcal{C}$  contains all nilpotent rings. Since  $\mathcal{C}$  is a semi-simple class (see [2] and [6])  $\mathcal{C}$  is hereditary, hence  $\mathcal{C}$  is hypernilpotent. Therefore, by [5],  $\mathcal{C}$  is the class of all rings.

**THEOREM 4.3.** *If  $\mathcal{C}$  is not the class of all rings then the following are equivalent:*

- (1)  $\mathcal{C}$  is a semi-simple radical class,
- (2) there is a strongly hereditary finite set  $\mathcal{C}(F)$  of finite fields such that:  $R \in \mathcal{C}$  if and only if  $R$  is isomorphic to a subdirect sum of fields in  $\mathcal{C}(F)$ ,
- (3) there is a strongly hereditary finite set  $\mathcal{C}(F)$  of finite fields such that:  $R \in \mathcal{C}$  if and only if every finitely generated subring of  $R$  is isomorphic to a finite direct sum of fields in  $\mathcal{C}(F)$ .

*Proof.* By 3.7 we have that (2) and (3) are equivalent.

Assume that  $\mathcal{C}$  satisfies condition (3). Clearly  $\mathcal{C}$  satisfies (A) and (E).

If  $B \triangleleft A$  and both  $A/B$  and  $B$  are in  $\mathcal{C}$  and  $A'$  is a finitely generated subring of  $A$  then  $A' + B/B \cong A'/A' \cap B$  is isomorphic to a finite direct sum of fields in  $\mathcal{C}(F)$ . A slight modification of the proof given for Proposition 1 on page 241 of Jacobson [7] shows that  $A' \cap B$  is finitely generated as a ring. Thus  $A' \cap B$  is also isomorphic to a finite direct sum of fields in  $\mathcal{C}(F)$  and so  $A' \cong A'/A' \cap B \oplus A' \cap B$ . Therefore  $A \in \mathcal{C}$ . From this it is easy to show that if  $\mathcal{C}(R) =$  the sum of all ideals of  $R$  which are in  $\mathcal{C}$  then  $\mathcal{C}(R) \in \mathcal{C}$  and  $\mathcal{C}(R/\mathcal{C}(R)) = (0)$ . Thus,  $\mathcal{C}$  satisfies (B) and (C).

If every nonzero ideal of a ring  $R$  can be homomorphically mapped onto a nonzero ring in  $\mathcal{C}$  then by 3.7, every nonzero ideal of  $R$  can be homomorphically mapped onto a ring in  $\mathcal{C}(F)$ . Sulinski [9] (see also [6], Theorem 46) shows that this implies that  $R$  is isomorphic to a subdirect sum of rings in  $\mathcal{C}(F)$  and hence by 3.7 again,  $R \in \mathcal{C}$ . So

$\mathcal{C}$  satisfies (F) and hence  $\mathcal{C}$  is a semi-simple radical class.

Conversely, suppose  $\mathcal{C}$  satisfies condition (1). Let  $\mathcal{C}(F)$  = the class of all fields which are in  $\mathcal{C}$  and define  $A = \coprod \{R: R \in \mathcal{C}(F)\}$ . Since  $\mathcal{C}$  is a semi-simple class subdirect sums of rings in  $\mathcal{C}$  are in  $\mathcal{C}$ ; thus  $A \in \mathcal{C}$ . By hypothesis,  $\mathcal{C} \subseteq \mathcal{B}_1$  so by 3.4 all elements of  $A$  must be torsion. From this it follows that there is a finite number of primes  $p_1, \dots, p_N$  such that every field in  $\mathcal{C}(F)$  is of characteristic  $p_i$  for some  $1 \leq i \leq N$ . For each finite field  $R \in \mathcal{C}(F)$  choose  $a(R)$  such that  $[a(R)] = R$  and for each infinite field  $R \in \mathcal{C}(F)$  set  $a(R) = 0$ . Then  $a = \{a(R)\}_{R \in \mathcal{C}(F)}$  is in  $A$  and by 3.5  $a^K = a$  for some integer  $K \geq 2$ . Thus, for all finite fields  $R$  in  $\mathcal{C}(F)$ , the dimension of  $R$  over its prime subfield is  $\leq K - 1$ . Hence there is only a finite number of finite fields in  $\mathcal{C}(F)$ . Suppose there is an infinite field  $R \in \mathcal{C}(F)$ . By 3.2  $R$  is of prime characteristic and is algebraic over its prime subfield so  $R$  has an infinite number of non-isomorphic finite subfields. All these subfields are in  $\mathcal{C}(F)$  since  $\mathcal{C}$  is strongly hereditary by 4.1. This is impossible since there is only a finite number of finite fields in  $\mathcal{C}(F)$ . Therefore  $\mathcal{C}(F)$  is a strongly hereditary finite set of finite fields. If  $R \in \mathcal{C}$  then  $R \in \mathcal{B}_1$  so by 3.3  $R$  is isomorphic to a subdirect sum of fields all of which are in  $\mathcal{C}(F)$  since  $\mathcal{C}$  satisfies (A). Conversely, any ring isomorphic to a subdirect sum of rings in  $\mathcal{C}(F)$  is in  $\mathcal{C}$  since  $\mathcal{C}$  is semi-simple class. Thus  $\mathcal{C}$  satisfies (2).

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