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**ON $C, 1$ SUMMABILITY FACTORS OF FOURIER SERIES AT A
GIVEN POINT**

FU CHENG HSIANG

ON $|C, 1|$ SUMMABILITY FACTORS OF FOURIER SERIES AT A GIVEN POINT

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Let $f(x)$ be a function integrable in the sense of Lebesgue over the interval $(-\pi, \pi)$ and periodic with period 2π . Let its Fourier series be

$$\begin{aligned} f(x) &\sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \\ &\equiv \sum_{n=0}^{\infty} A_n(x). \end{aligned}$$

Whittaker proved that the series

$$\sum_{n=1}^{\infty} A_n(x)/n^\alpha \quad (\alpha > 0)$$

is summable $|A|$ almost everywhere. Prasad improved this result by showing that the series

$$\sum_{n=n_0}^{\infty} A_n(x) / \left(\prod_{\mu=1}^{k-1} \log^\mu n \right) (\log^k n)^{1+\varepsilon} \quad (\log^k n_0 > 0)$$

is summable $|A|$ almost everywhere.

In this note, the author is interested particularly in the $|C, 1|$ summability factors of the Fourier series at a given point x_0 .

Write

$$\begin{aligned} \varphi(t) &= f(x_0 + t) + f(x_0 - t) - 2f(x_0), \\ \Phi(t) &= \int_0^t |\varphi(u)| du. \end{aligned}$$

The author establishes the following theorems.

THEOREM 1. If

$$\Phi(t) = O(t) \quad (t \rightarrow +0),$$

then the series

$$\sum_{n=1}^{\infty} A_n(x_0)/n^\alpha$$

is summable $|C, 1|$ for every $\alpha > 0$.

THEOREM 2. If

$$\Phi(t) = O \left\{ \frac{t}{\prod_{\mu=1}^k \log^\mu \frac{1}{t}} \right\}$$

as $t \rightarrow +0$, then the series

$$\sum_{n=n_0}^{\infty} \frac{A_n(x_0)}{\left(\prod_{\mu=1}^{k-1} \log^\mu n \right) (\log^k n)^{1+\varepsilon}}$$

is summable $|C, 1|$ for every $\varepsilon > 0$.

A series $\sum a_n$ is said to be absolutely summable (A) or summable $|A|$, if the function

$$f(x) = \sum_{n=0}^{\infty} a_n x^n$$

is of bounded variation in the interval $\langle 0, 1 \rangle$. Let σ_n^α denote the n th Cesàro mean of order α of the series $\sum a_n$, i.e.,

$$\sigma_n^\alpha = \frac{1}{(\alpha)_n} \sum_{k=0}^n (\alpha)_k a_{n-k}, \quad (\alpha)_k = \Gamma(k + \alpha + 1) / \Gamma(k + 1) \Gamma(\alpha + 1).$$

If the series

$$\sum |\sigma_n^\alpha - \sigma_{n-1}^\alpha|$$

converges, then we say that the series $\sum a_n$ is absolutely summable (C, α) or summable $|C, \alpha|$. It is known that [2] *if a series is summable $|C|$, it is also summable $|A|$, but not conversely.*

2. Suppose that $f(x)$ is a function integrable in the sense of Lebesgue and periodic with period 2π . Let its Fourier series be

$$\begin{aligned} f(x) &\sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \\ &\equiv \sum A_n(x). \end{aligned}$$

Whittaker [4] proved that the series

$$\sum_{n=1}^{\infty} A_n(x) / n^\alpha \quad (\alpha > 0)$$

is summable $|A|$ almost everywhere. Prasad [4] improved this result by showing that the series

$$\sum_{n=n_0}^{\infty} A_n(x) / \left(\prod_{\mu=1}^{k-1} \log^\mu n \right) (\log^k n)^{1+\varepsilon} (\log^k n_0 > 0),$$

where $\log^k n = \log(\log^{k-1} n)$, $\log^2 = \log(\log n)$, is summable $|A|$ almost everywhere.

Let (λ_n) be a convex and bounded sequence, Chow [1] demonstrated that the series

$$\sum A_n(x) \lambda_n$$

is summable $|C, 1|$ almost everywhere, if the series $\sum n^{-1} \lambda_n$ converges.

In this note, we are interested particularly in the $|C, 1|$ summability factors of the Fourier series at a given point. For a fixed point x_0 , we write

$$\varphi(t) = \varphi_{x_0}(t) = f(x_0 + t) + f(x_0 - t) - 2f(x_0),$$

and

$$\Phi(t) = \int_0^t |\varphi(u)| du .$$

We are going to establish the following

THEOREM 1. *If*

$$(i) \quad \Phi(t) = O(t)$$

as $t \rightarrow +0$, then the series

$$\sum_{n=1}^{\infty} \frac{A_n(x_0)}{n^\alpha}$$

is summable $|C, 1|$ for every $\alpha > 0$.

3. The following lemmas are required.

LEMMA 1 [3]. *Let $\alpha > -1$ and let τ_n^α be the n th Cesàro mean of order α of the sequence $\{na_n\}$, then*

$$\tau_n^\alpha = n(\sigma_n^\alpha - \sigma_{n-1}^\alpha) .$$

LEMMA 2. *Write*

$$S_n(t) = \sum_{k=0}^n (n+2-k) \cos(n+2-k)t ,$$

then

$$S_n(t) = O \begin{cases} nt^{-1} & (nt \geq 1) , \\ n^2 & (\text{for all } t) . \end{cases}$$

In fact, we have

$$\begin{aligned} S_n(t) &= I \left\{ \frac{d}{dt} e^{i(n+2)t} \sum_{k=0}^n e^{-ikt} \right\} \\ &= I \left\{ \frac{d}{dt} \left(\frac{e^{i(n+2)t}}{1 - e^{-it}} - \frac{e^{it}}{1 - e^{-it}} \right) \right\} \\ &= I \left\{ (n+2) \frac{ie^{i(n+2)t}}{1 - e^{-it}} - \frac{ie^{i(n+2)t}}{(1 - e^{-it})^2} \right. \\ &\quad \left. - \frac{ie^{it}}{1 - e^{-it}} + \frac{i}{(1 - e^{-it})^2} \right\} \\ &= O(nt^{-1}) + O(t^{-2}) \\ &= O(nt^{-1}) , \end{aligned}$$

if $nt \leq 1$. This proves the lemma. From this lemma, we can easily derive the following

LEMMA 3.

$$\left| \frac{1}{n+1} \left\{ \sum_{\nu=1}^n S_{\nu}(t) \Delta \frac{1}{(\nu+2)^{\alpha}} \right\} \right| \leq \begin{cases} \frac{A}{th^{\alpha}} + \frac{A}{nt^{2-\alpha}} & (t \geq 1), \\ An^{1-\alpha} & (\text{for all } t). \end{cases}$$

By Lemma 2, for $nt \geq 1$, we write

$$\begin{aligned} \frac{1}{n+1} \left\{ \sum_{\nu=1}^n S_{\nu}(t) \Delta \frac{1}{(\nu+2)^{\alpha}} \right\} &= \frac{1}{n+1} \left\{ \sum_{\nu=1}^{[t^{-1}]-1} + \sum_{\nu=[t^{-1}]+1}^n \right\} + O\left(\frac{1}{nt^{2-\alpha}}\right) \\ &= \frac{1}{n} O\left(\sum_{\nu=1}^{[t^{-1}]} \nu^{1-\alpha}\right) + \frac{1}{nt} O\left(\sum_{\nu=1}^n \frac{1}{\nu^{\alpha}}\right) \\ &\quad + O\left(\frac{1}{nt^{2-\alpha}}\right) \\ &= O\left(\frac{1}{nt^{2-\alpha}}\right) + O\left(\frac{1}{tn^{\alpha}}\right), \end{aligned}$$

and for all t ,

$$\begin{aligned} \frac{1}{n+1} \left\{ \sum_{\nu=1}^n S_{\nu}(t) \Delta \frac{1}{(\nu+2)^{\alpha}} \right\} &= \frac{1}{n+1} O\left\{ \sum_{\nu=1}^n \nu^2 \frac{1}{\nu^{1+\alpha}} \right\} \\ &= \frac{1}{n+1} O\left\{ \sum_{\nu=1}^n \nu^{1-\alpha} \right\} \\ &= O(n^{1-\alpha}). \end{aligned}$$

This proves the lemma.

4. We have

$$A_n(x_0) = \frac{2}{\pi} \int_0^{\pi} \varphi(t) \cos ntdt.$$

Let $\tau_n(x_0)$ be the n th Cesàro mean of first order of the sequence $\{nA_n(x_0)/n^{\alpha}\}$, then

$$\frac{\pi}{2} \tau_n(x_0) = \int_0^{\pi} \varphi(t) \frac{1}{n+1} \sum_{\nu=0}^n \frac{(\nu+2) \cos(\nu+2)t}{(\nu+2)^{\alpha}} dt.$$

Abel's transformation gives

$$\begin{aligned} \frac{\pi}{2} \tau_n(x_0) &= \int_0^{\pi} \varphi(t) \frac{1}{n+1} \left\{ \sum_{\nu=0}^n S_{\nu}(t) \Delta \frac{1}{(\nu+2)^{\alpha}} \right\} dt \\ &\quad + \int_0^{\pi} \varphi(t) \frac{1}{n+1} \cdot \frac{S_n(t)}{(n+3)^{\alpha}} dt \\ &= I_{1n} + I_{2n}, \end{aligned}$$

say. Thus, on writing

$$I_{1n} = \int_0^{1/n} + \int_{1/n}^{\pi} = I_{3n} + I_{4n},$$

say, we see that

$$I_{3n} = O\left(n^{1-\alpha} \int_0^{1/n} |\varphi| dt\right) = O(n^{-\alpha}),$$

by condition (i) of the theorem.

$$I_{4n} = O\left\{\frac{1}{n^\alpha} \int_{1/n}^{\pi} \frac{|\varphi|}{t} dt\right\} + O\left\{\frac{1}{n} \int_{1/n}^{\pi} \frac{|\varphi|}{t^{2-\alpha}} dt\right\}.$$

Now,

$$\int_{1/n}^{\pi} \frac{|\varphi|}{t} dt = \left(\frac{\Phi}{t}\right)_{1/n}^{\pi} + \int_{1/n}^{\pi} \frac{\Phi}{t^2} dt = O(1) + O\left\{\int_{1/n}^{\pi} \frac{dt}{t}\right\} = O(\log n),$$

and

$$\int_{1/n}^{\pi} \frac{|\varphi|}{t^{2-\alpha}} dt \leq n^{1-\alpha} \int_{1/n}^{\pi} \frac{|\varphi|}{t} dt = O(n^{1-\alpha} \log n).$$

It follows that

$$I_{4n} = O\{\log n/n^\alpha\}.$$

As before, we write

$$I_{2n} = \int_0^{1/n} + \int_{1/n}^{\pi} = I_{5n} + I_{6n},$$

say. Then,

$$I_{5n} = O\left(n^{1-\alpha} \int_0^{1/n} |\varphi| dt\right) = O(n^{-\alpha}).$$

And

$$I_{6n} = O\left\{n^{-\alpha} \int_{1/n}^{\pi} \frac{|\varphi|}{t} dt\right\} = O\{\log n/n^\alpha\},$$

by the similar arguments as in the estimation of the integral I_{4n} . By Lemma 1, we have to establish the convergence of $\sum |\tau_n(x_0)|/n$. And from the above analysis, it concludes that

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{|\tau_n(x_0)|}{n} &\leq \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \{|I_{3n}| + |I_{4n}| + |I_{5n}| + |I_{6n}|\} \\ &= O\left\{\sum_{n=1}^{\infty} \frac{\log n}{n^{1+\alpha}}\right\} = O(1). \end{aligned}$$

This proves Theorem 1.

5. Let $\tau_n(x_0)$ be the n th Cesàro mean of first order of the sequence

$$\left\{ nA_n(x_0) / \left(\prod_{\mu=1}^{k-1} \log^\mu n \right) (\log^k n)^{1+\varepsilon} \right\} \quad (\varepsilon > 0),$$

where k is a positive integer. Abel's transformation gives

$$\begin{aligned} \frac{\pi}{2} \tau_n(x_0) &= \int_0^\pi \varphi(t) \frac{1}{n+1} \left\{ \sum_{\nu=0}^n S_\nu(t) \Delta \frac{1}{\left\{ \prod_{\mu=1}^{k-1} \log^\mu (\nu+2) \right\} \{\log^k (\nu+2)\}^{1+\varepsilon}} \right\} dt \\ &\quad + \int_0^\pi \varphi(t) \frac{1}{n+1} \cdot \frac{S_n(t)}{\left\{ \prod_{\mu=1}^{k-1} \log^\mu (n+3) \right\} \{\log^k (n+3)\}^{1+\varepsilon}} dt \\ &= I_{1n} + I_{2n}, \end{aligned}$$

say. As before, we write

$$I_{1n} = \int_0^{1/n} + \int_{1/n}^\pi = I_{3n} + I_{4n},$$

say, and

$$I_{2n} = \int_0^{1/n} + \int_{1/n}^\pi = I_{5n} + I_{6n},$$

say. Since, for $\nu \geq n_0$,

$$\left| \Delta \frac{1}{\left(\prod_{\mu=1}^{k-1} \log^\mu \nu \right) (\log^k \nu)^{1+\varepsilon}} \right| \leq \frac{A}{\nu \left(\prod_{\mu=1}^{k-1} \log^\mu \nu \right) (\log^k \nu)^{1+\varepsilon}},$$

we obtain

$$\begin{aligned} &\left| \frac{1}{n+1} \left\{ \sum_{\nu=0}^n S_\nu(t) \Delta \frac{1}{\left(\prod_{\mu=1}^{k-1} \log^\mu (\nu+2) \right) (\log^k (\nu+2))^{1+\varepsilon}} \right\} \right| \\ &\leq \begin{cases} \frac{A}{t \left(\prod_{\mu=0}^{k-1} \log^\mu n \right) (\log^k n)^{1+\varepsilon}} + \frac{A}{t^2 \left(\prod_{\mu=1}^{k-1} \log^\mu \frac{1}{t} \right) (\log^k \frac{1}{t})^{1+\varepsilon}} & (nt \geq 1), \\ \frac{An}{\left(\prod_{\mu=1}^{k-1} \log^\mu n \right) (\log^k n)^{1+\varepsilon}} & (\text{for all } t). \end{cases} \end{aligned}$$

Now, if

$$\Phi(t) = O \left\{ \frac{t}{\left(\prod_{\mu=1}^k \log^\mu \frac{1}{t} \right)} \right\}$$

as $t \rightarrow +0$, then

$$\begin{aligned} I_{3n} &= O\left\{\frac{n}{\left(\prod_{\mu=1}^{k-1} \log^{\mu} n\right)(\log^k n)^{1+\varepsilon}} \int_0^{1/n} |\varphi| dt\right\} \\ &= O\left\{\frac{1}{\left(\prod_{\mu=1}^{k-1} \log^{\mu} n\right)(\log^k n)^{1+\varepsilon}}\right\}. \end{aligned}$$

$$\begin{aligned} I_{4n} &= O\left\{\frac{1}{\left(\prod_{\mu=1}^{k-1} \log^{\mu} n\right)(\log^k n)^{1+\varepsilon}} \int_{1/n}^{\pi} \frac{|\varphi|}{t} dt\right\} \\ &+ O\left\{\frac{1}{n} \int_{1/n}^{\pi} \frac{|\varphi|}{t^2 \left(\prod_{\mu=1}^{k-1} \frac{1}{t}\right) (\log^k \frac{1}{t})^{1+\varepsilon}} dt\right\}. \end{aligned}$$

But since

$$\begin{aligned} \int_{1/n}^{\pi} \frac{|\varphi|}{t} dt &= \left(\frac{\Phi}{t}\right)_{1/n}^{\pi} + \int_{1/n}^{\pi} \frac{\Phi}{t^2} dt \\ &= O(1) + O\left\{\int_{1/n}^{\pi} \frac{dt}{t \left(\prod_{\mu=1}^k \log^{\mu} \frac{1}{t}\right)}\right\} \\ &= O(1) + O\{\log^{k+1} n\}, \end{aligned}$$

and

$$\begin{aligned} \int_{1/n}^{\pi} \frac{|\varphi|}{t^2 \left(\prod_{\mu=1}^{k-1} \log^{\mu} \frac{1}{t}\right) (\log^k \frac{1}{t})^{1+\varepsilon}} dt &= O\left\{\frac{n}{\left(\prod_{\mu=1}^{k-1} \log^{\mu} n\right)(\log^k n)^{1+\varepsilon}} \int_{1/n}^{\pi} \frac{|\varphi|}{t} dt\right\} \\ &= O\left\{\frac{n \log^{k+1} n}{\left(\prod_{\mu=1}^{k-1} \log^{\mu} n\right)(\log^k n)^{1+\varepsilon}}\right\}, \end{aligned}$$

we obtain

$$I_{4n} = O\left\{\frac{\log^{k+1} n}{\left(\prod_{\mu=1}^{k-1} \log^{\mu} n\right)(\log^k n)^{1+\varepsilon}}\right\}.$$

Finally,

$$\begin{aligned} I_{5n} &= O\left\{\frac{n}{\left(\prod_{\mu=1}^{k-1} \log^{\mu} n\right)(\log^k n)^{1+\varepsilon}} \int_0^{1/n} |\varphi| dt\right\} \\ &= O\left\{\frac{1}{\left(\prod_{\mu=1}^{k-1} \log^{\mu} n\right)(\log^k n)^{1+\varepsilon}}\right\}, \end{aligned}$$

$$\begin{aligned}
 I_{6n} &= O\left\{\frac{1}{\left(\prod_{\mu=1}^{k-1} \log^{\mu} n\right)(\log^k n)^{1+\varepsilon}} \int_{1/n}^{\pi} \frac{|\varphi|}{t} dt\right\} \\
 &= O\left\{\frac{\log^{k+1} n}{\left(\prod_{\mu=1}^{k-1} \log^{\mu} n\right)(\log^k n)^{1+\varepsilon}}\right\}.
 \end{aligned}$$

Thus,

$$\begin{aligned}
 \sum_{n=1}^{\infty} \frac{|\tau_n(x_0)|}{n} &= O\left\{\sum_{n=n_0}^{\infty} \frac{\log^{k+1} n}{n\left(\prod_{\mu=1}^{k-1} \log^{\mu} n\right)(\log^k n)^{1+\varepsilon}}\right\} + O(1) \\
 &= O(1).
 \end{aligned}$$

Hence, we establish

THEOREM 2. *If*

$$(ii) \quad \Phi(t) = O\left\{\frac{t}{\prod_{\mu=1}^k \log^{\mu} \frac{1}{t}}\right\}$$

as $t \rightarrow +0$, then the series

$$\sum_{n=n_0}^{\infty} \frac{A_n(x_0)}{\left(\prod_{\mu=1}^{k-1} \log^{\mu} n\right)(\log^k n)^{1+\varepsilon}} \quad (\log^k n_0 > 0)$$

is summable $|C, 1|$ for every $\varepsilon > 0$.

6. For the conjugate series

$$\sum_{n=1}^{\infty} (b_n \cos nx - a_n \sin nx) \equiv \sum B_n(x),$$

we can derive two analogous theorems. Write, for a fixed $x = x_0$,

$$\Psi(t) = \int_0^t |\gamma_r(u)| du \equiv \int_0^t |f(x_0 + u) - f(x_0 - u)| du.$$

We have the following

THEOREM 3. *If*

$$(iii) \quad \Psi(t) = O(t)$$

as $t \rightarrow +0$, then the series

$$\sum_{n=1}^{\infty} \frac{B_n(x_0)}{n^{\alpha}}$$

is summable $|C, 1|$ for every $\alpha > 0$.

THEOREM 4. *If*

$$(iv) \quad \Psi(t) = O\left\{\frac{t}{\prod_{\mu=1}^k \log^{\mu} \frac{1}{t}}\right\}$$

as $t \rightarrow +0$, then the series

$$\sum_{n=n_0}^{\infty} \frac{B_n(x_0)}{\left(\prod_{\mu=1}^{k-1} \log^{\mu} n\right) (\log^k n)^{1+\varepsilon}} \quad (\log^k n_0 > 0)$$

is summable $|C, 1|$ for every $\varepsilon > 0$.

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