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VINCENT MANCUSO

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V. J. MANCUSO

This paper is concerned with some of those generalizations of paracompactness which can arise by broadening the concept of local finiteness, e.g., metacompactness, in contrast to those which come about by varying the power of an open cover, e.g., countable paracompactness. Quite recently, several generalizations of the first type have been studied. These include mesocompactness and sequential mesocompactness, strong and weak cover compactness, and Property Q.

In § 1, the notion of metacompactness (=pointwise paracompactness) is used to establish a hierarchy among these concepts, and in regular r-spaces, some of these notions are shown to be equivalent to paracompactness. In § 2, it is shown that mesocompactness is an invariant, in both directions, of perfect maps and that unlike paracompact spaces, there exists a mesocompact T_3 space which is not normal, and a mesocompact T_2 space which is not regular.

Throughout this paper, a space will mean a *Hausdorff space*. A convergent sequence in a space will mean the sequence and its limit, and we will use $\mathrm{Cl}(A)$ to denote the closure of A.

1. Related properties.

We will begin with some pertinent definitions.

DEFINITION 1.1. A family $\mathscr{F}$ of sets in a space X is called *compact-finite* (*convergent sequence finite*) if every compact set (convergent sequence) in X meets at most finitely many members of $\mathscr{F}$. A space X is called *mesocompact* (*sequentially mesocompact*) if every open cover of X has a compact-finite (convergent sequence finite) open refinement (see [3]). We will use the abbreviation cs-finite for convergent sequence finite.

DEFINITION 1.2. A cover $\mathscr{V}$ of a space X is called *strong cover compact* if whenever $\{V_i; i \in N\}$ is a countably infinite subcollection of distinct elements of $\mathscr{V}$, p_i and $q_i \in V_i$ for each i, with $p_i \neq p_j$ and $q_i \neq q_j$ for $i \neq j$ and the point set $\{p_i; i \in N\}$ has a limit point in X, then the point set $\{q_i; i \in N\}$ has a limit point in X.

DEFINITION 1.3. A space X is *strong cover compact* if every open cover of X has a strong cover compact open refinement. We will use the abbreviation scc for strong cover compact.

If "countably" is replaced by "uncountably" in Definition 1.2, we obtain the notion of a weak cover compact cover. Then if every open cover of X has a weak cover compact open refinement, we say that X is *weak cover compact*, denoted by wcc (see [4]).

DEFINITION 1.4. A cover $\mathscr{V}$ of a space X is said to have *Property Q* if whenever $\{V_i; i \in N\}$ is a countably infinite subcollection of distinct elements of $\mathscr{V}$, with p_i and $q_i \in V_i$ for each i and $\{p_i; i \in N\}$ converges to p in X, then $\{q_i; i \in N\}$ converges to p in X.

DEFINITION 1.5. A space X has *Property Q* if every open cover of X has an open refinement with Property Q (see [16]).

REMARK 1.6. It is immediate that paracompact implies mesocompact implies sequentially mesocompact implies metacompact. Moreover, since a locally finite cover vacuously satisfies scc, wcc and Property Q, a paracompact space is scc, wcc and has Property Q.

We will often refer to a minimality property of point-finite covers [5, p. 160]: If $\mathscr{A}$ is a point-finite open cover of a space X, then there exists an irreducible sub-cover, i.e., a sub-cover that, when any single set is removed, is no longer a cover of X. This property was especially utilized in [4].

PROPOSITION 1.7. *A space X has Property Q if and only if X is sequentially mesocompact.*

Proof. Since a *cs*-finite cover vacuously satisfies Property Q, we need only establish the necessity. Let $\mathscr{U}$ be an open cover of X and let $\mathscr{V}$ be an open refinement with Property Q. Now some subcollection $\mathscr{V}'$ of $\mathscr{V}$ is point-finite and covers X [4, Th. 1]. Let $\mathscr{V}''$ be an irreducible sub-cover of $\mathscr{V}'$. We claim $\mathscr{V}''$ is cs-finite. For suppose $\{p_i; i \in N\} \cup \{p\}$ is a convergent sequence in X which meets infinitely many members of $\mathscr{V}''$, say $\{V_i; i \in N\}$. Since $\mathscr{V}''$ is point-finite, there exist subsequences $\{p_{i_n}; n \in N\}$ of $\{p_i; i \in N\}$ and $\{V_{i_n}; i \in N\}$ of $\{V_i; i \in N\}$ such that $p_{i_n} \in V_{i_n}$ for each n. If we let V_p be a member of $\mathscr{V}''$ containing p, we can use the minimality of $\mathscr{V}''$ to pick a sequence $\{q_{i_n}; n \in N\}$ such that $q_{i_n} \in V_{i_n} - V_p$ for each n. Clearly, $\{q_{i_n}; n \in N\}$ does not converge to p and yet $\{p_{i_n}; n \in N\}$ does. This is a contradiction and the proof is complete.

COROLLARY 1.8. *A developable space is metrizable if and only if it has Property Q.*

Proof. In a developable space, metrizability is equivalent to sequential mesocompactness [3, Th. 4.2], which is equivalent to Property Q by Proposition 1.7.

REMARK 1.9. Corollary 1.8 is obtained in [4, Corollary 5] by different means.

Both parts of the following proposition have similar proofs, each using the essential features of the proof of Theorem 8 or Theorem 13 in [4].

NOTATION. If $\mathscr{H}$ is a collection of sets, $\mathscr{H}^* = \cup\{h; h \in \mathscr{H}\}$.

PROPOSITION 1.10. (a) *If X is regular wcc and metacompact, X is mesocompact.* (b) *If X is scc and metacompact, X is mesocompact.*

Proof. To prove (a), we may assume X has an open cover $\mathscr{G}$ which has no countable sub-cover, or else X is Lindelof and hence paracompact. We can obtain an open refinement $\mathscr{V}$ of $\mathscr{G}$ which is wcc, point-finite and irreducible. We claim $\mathscr{V}$ is compact-finite. Suppose there exists a compact subset K of X which meets infinitely many members of $\mathscr{V}$ say $\{V_i; i \in N\}$. Then pick a distinct sequence $\{p_i; i \in N)$ such that $p_i \in K \cap V_i$ for each i. Since K is compact, the point set $\{p_i; i \in N\}$ has a limit point in X.

Let $\{V_\beta; \beta \in B\}$ be an uncountable subcollection of $V - \{V_i; i \in N\}$, which exists since $\mathscr{V}$ refines $\mathscr{G}$. Since $\mathscr{V}$ is minimal, for each $i \in N$ and $\beta \in B$, let $q_i \in V^* - V_i$ and $q_\beta \in \mathscr{V}^* - V_\beta$. Then the subcollection $\{V_i; i \in N\} \cup \{V_\beta; \beta \in B\}$ and the point sets $\{p_i; i \in N\} \cup \{q_\beta; \beta \in B\}$ and $\{q_i; i \in N\} \cup \{q_\beta; \beta \in B\}$ contradict the wcc property of $\mathscr{V}$, and so $\mathscr{V}$ is compact-finite.

To prove (b), we let $\mathscr{U}$ be an open cover of X. Without the regularity of X, Theorem 8 in [4] can still be used to obtain an open refinement $\mathscr{V}$ of $\mathscr{U}$ which is scc, point-finite and irreducible. A slight modification of the above indirect argument gives that $\mathscr{V}$ is compact-finite.

PROPOSITION 1.11. *If X has Property Q, X is metacompact.*

Proof. This is Corollary 1 in [4].

By Remark 1.6, Propositions 1.7, 1.10(a) and 1.11 we have the following:

THEOREM 1.12. *For any space X, the following implications* *hold*: (a) *implies* (b) *implies* (c) *implies* (d) *is equivalent to* (e) *implies* (*f*), *where*

(a) X *is paracompact*
(b) X *is* scc *and metacompact*
(c) X *is mesocompact*
(d) X *is sequentially mesocompact*
(e) X *has Property* Q
(f) X *is metacompact.*

REMARK 1.13. In view of a result of Michael [11, Th. 2] (a)-(f) are equivalent in collectionwise normal spaces. Examples 1.15 and 1.16 below show that (a) implies (b) and (e) implies (f) cannot be reversed. As yet, the author has no examples showing that (b) implies (c) or (c) implies (d) cannot be reversed.

The following theorem is fundamental and will be used in some of the examples.

THEOREM 1.14. (a) *A locally compact space is paracompact if and only if it is mesocompact.* (b) *A first countable space is paracompact if and only if it is sequentially mesocompact.*

Proof. See Theorems 3.7 and 3.10 in [3].

In our examples, Ω will denote the first uncountable ordinal, and unless otherwise specified, a given set of ordinals will carry the usual order topology and will be called an *ordinal space.*

EXAMPLE 1.15. A normal mesocompact space which is not collectionwise normal. Michael's subspace [12, Example 2] of Bing's space F [2, Example G]. For the proof that this space has the desired properties see [3, Example 5.1]. A slight modification of this space [4, Th. 18] yields a scc space with Property Q (hence metacompact by Proposition 1.11) which is not collectionwise normal.

EXAMPLE 1.16. A metacompact Moore space which is not sequentially mesocompact. R. W. Heath [7, Example 1] gives a metacompact Moore space which is not screenable and so not metrizable. Such a space could not be sequentially mesocompact by [3, Th. 4.2] or our Corollary 1.8.

EXAMPLE 1.17. A normal space which is not mesocompact. The ordinal space $X = \{x; x < \Omega\}$ is in fact a collectionwise normal locally compact space which is not paracompact. This space can't be mesocompact by Theorem 1.14(a). Moreover, X is first countable and so can't be sequentially mesocompact by Theorem 1.14(b).

DEFINITION 1.18. A space X is called an *r-space* [13, p. 985] if each $x \in X$ has a sequence of neighborhoods $\{U_i; i \in N\}$ such that if $x_i \in U_i$ for each i, $\{x_i; i \in N\}$ is contained in a compact subset of X.

REMARK 1.19. Clearly, locally compact spaces and first countable spaces are r-spaces.

PROPOSITION 1.20. (a) *If X is a mesocompact r-space, X is paracompact.* (b) *If X is a* scc *r-space, X is* wcc.

Proof. To prove (a), it suffices to show that a compactfinite family $\mathscr{F}$ in X is locally finite. Suppose there exists a point p in X every neighborhood of which meets infinitely many members of $\mathscr{F}$. Let $\{U_i; i \in N\}$ be a sequence of neighborhoods of p guaranteed by the definition of an r-space. We can then find an infinite subcollection $\{F_i; i \in N\}$ of $\mathscr{F}$ and a sequence of points $\{p_i; i \in N\}$ such that $p_i \in U_i \cap F_i$ for each i. Now $\{p_i; i \in N\}$ is contained in some compact subset of X, which contradicts the compact-finiteness of $\mathscr{F}$.

To prove (b), we modify the proof of Theorem 3 in [4]. Let $\mathscr{U}$ be an open cover of X and $\mathscr{V}$ a scc open refinement of $\mathscr{U}$. If V is not wcc, there exists an uncountable subcollection $\{V_\alpha; \alpha \in A\}$ of distinct members of $\mathscr{V}$ and uncountable point sets $\{p_\alpha; \alpha \in A\}$ and $q_\alpha; \alpha \in A\}$ such that $p_\alpha, q_\alpha \in V_\alpha$, $p_\alpha \neq p_\beta$ and $p_\alpha \neq q_\beta$ for $\alpha \neq \beta$. The point set $\{p_\alpha; \alpha \in A\}$ has a limit point in X but $\{q_\alpha; \alpha \in A\}$ does not. Since X is an r-space, let $\{U_i; i \in N\}$ be a sequence of neighborhoods of p. Then there exists an infinite subset $\{p_i; i \in N\}$ of $\{p_\alpha; \alpha \in A\}$ such that $p_i \neq p_j$ for $i \neq j$ and $p_i \in U_i \cap V_i$. Now $\{p_i; i \in N\}$ is contained in a compact subset of X and so has a limit point in X. Let q_i be the member of $\{q_\alpha; \alpha \in A\}$ corresponding to p_i. Since $\mathscr{V}$ is scc, $\{q_i; i \in N\}$ has a limit point in X, contradicting the fact that $\{q_\alpha; \alpha \in A\}$ does not.

REMARK 1.21. (1) By slightly modifying the proof of (a) in Proposition 1.20, we can show that a cs-finite family in a sequentially compact space is compact-finite, and so sequential mesocompactness reduces to mesocompactness in sequentially compact spaces. Proposition 1.20(b) slightly generalizes Theorems 3 and 11 in [4] (see Remark 1.19).

(2) A space X is called a *k'-space* if whenever $x \in \mathrm{Cl}(A)$, there exists a compact subset K of X such that $x \in \mathrm{Cl}(K \cap A)$ (see [1]). The k'-spaces include the locally compact spaces and the Frechet spaces. Our interest in k'-spaces arises from the fact that a regular

k'-space is paracompact if and only if it is mesocompact [3, Corollary 3.6].

This result, however, does not include Proposition 1.20(a) (see also Example 2.15), since E. Michael has kindly furnished us with an example of an r-space which is not a k'-space:

EXAMPLE 1.22. A separable metric space X and a compact space Y such that $X \times Y$ is not a k'-space. Let X be an uncountable subset of the reals all of whose compact subsets are countable. Let $Z = X' \cup \{z_0\}$ where X' is a discrete space of the same cardinality as X, and whose neighborhoods of z_0 in Z are the sets with countable complement in Z. Let $Y = \beta Z$. To show that $X \times Y$ is not a k'-space, pick an $x_0 \in X$ such that no neighborhood of x_0 in X is countable and let $h: X \to Z$ be a one-to-one correspondence with $h(x_0) = z_0$. Finally, let $A = \{(x, h(x)) \in X \times Y; x \neq x_0\}$. Then it can be shown that $(x_0, z_0) \in \mathrm{Cl}(A)$ but $(x_0, z_0) \notin \mathrm{Cl}(K \cap A)$ for any compact $K \subset X \times Y$. Now $X \times Y$ is an r-space since it is not difficult to show that the product of an r-space and a locally compact space is an r-space.

By Remark 1.6, Propositions 1.10(a) and 1.20 we have:

THEOREM 1.23. *Let X be a regular r-space. Then the following statements are equivalent*:
(a) *X is paracompact*
(b) *X is* scc *and metacompact*
(c) *X is* wcc *and metacompact*
(d) *X is mesocompact.*

REMARK 1.24. In [8, Th. 3] it was announced that in a q-space[1] the concepts of paracompactness, Q_0^* and Q_1^*[2] are equivalent. Since an r-space is a q-space, we may add (e) X is a Q_0^*-space and (f) X is a Q_1^*-space to Theorem 1.23.

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[1] X is called a *q-space* if every $x \in X$ has a sequence of neighborhoods $\{U_i; i \in N\}$ such that if $x_i \in U_i$ for each i and the set $\{x_i; i \in N\}$ is infinite, then the set $x_i; i \in N\}$ has a limit point in X.

[2] A cover $\mathscr{U}$ of X has *property* Q_0^* if no indexed point set $\{p_\alpha; \alpha \in A\}$, $p_\alpha \in U_\alpha$, has a limit point where $\mathscr{U} = \{U_\alpha; \alpha \in A\}$. A cover $\mathscr{U} = \{U_\alpha; \alpha \in A\}$ has *property* Q_1^* provided for any subcollection $\{U_\alpha; \alpha \in A^*\}$, with $A^* \subset A$, if there is an indexed point set $\{p_\alpha; \alpha \in A^*\}$, $p_\alpha \in U_\alpha$, having a limit point then there exists a compact set B such that if $\{q_\alpha; \alpha \in A^*\}$, $q_\alpha \in U_\alpha$, is any indexed point set then $\{q_\alpha; \alpha \in A^*\}$ has a limit point $p \in B$. A space X is called a $Q_0^*(Q_1^*)$-*space* if every open cover of X has an open refinement with property $Q_0^*(Q_1^*)$.

2. Invariance of mesocompactness. In this section, we establish some permanence properties which mesocompact spaces share with paracompact spaces.

Throughout this section, a map will mean a *continuous surjection.*

DEFINITION 2.1. A closed map $f: X \to Y$ is called *perfect* (or proper) if $f^{-1}(y)$ is compact in X for each $y \in Y$. A set A in X is *saturated* if $A = f^{-1}f(A)$.

EXAMPLE 2.2. A quotient space of a mesocompact space need not be mesocompact. The ordinal space $X = \{x; x \angle \Omega\}$ is locally compact and locally metrizable and so is the continuous open image of a locally compact metric space [14, Th. 3]. However, we have seen in Example 1.17 that X isn't even sequentially mesocompact.

REMARK 2.3. It is well known that paracompactness is an invariant of a closed map [11]. Less well known, perhaps, is that metacompactness also has this property [15]. We do not know if mesocompactness (or sequential mesocompactness) is an invariant of a closed map. However, for perfect maps we have:

THEOREM 2.4. *Let $f: X \to Y$ be a perfect map. Then X is mesocompact (sequentially mesocompact) if and only if Y is mesocompact (sequentially mesocompact).*

Proof. Let X be mesocompact and $\mathscr{V} = \{V_\alpha; \alpha \in A\}$ an open cover of Y. Then $\{f^{-1}(V_\alpha); \alpha \in A\}$ has a precise[3] compact-finite open refinement $\mathscr{U} = \{U_\alpha; \alpha \in A\}$.

For each $y \in Y$, let G_y be a finite union of members of $\mathscr{U}$ covering the compact set $f^{-1}(y)$. Since $f^{-1}(y)$ is saturated and f is a closed map, we can find a saturated neighborhood H_y of $f^{-1}(y)$ in X such that $H_y \subset G_y$. For each $y \in Y$, let $V_\alpha(y)$ be a member of $\mathscr{V}$ containing y. Since H_y is saturated and f is closed, $f(H_y)$ is open in Y for each y and so $\mathscr{V}' = \{f(H_y) \cap V_\alpha(y); y \in Y\}$ is an open cover of Y which clearly refines $\mathscr{V}$. Moreover, $\mathscr{V}'$ is compact-finite, for if K is compact in Y, $f^{-1}(K)$ is compact in X [6, Th. 1] and so meets at most finitely many members of $\mathscr{U}$ and hence at most finitely many H_y. Clearly then, K can meet at most finitely many members of $\mathscr{V}'$.

Conversely, let Y be mesocompact and $\mathscr{U} = \{U_\alpha; \alpha \in A\}$ be an open cover of X. For each $y \in Y$, let G_y be a finite union of members of $\mathscr{U}$ containing the compact set $f^{-1}(y)$. Since f is closed, there exists

[3] A refinement $\mathscr{V} = \{V_\beta; \beta \in B\}$ of $\mathscr{U} = \{U_\alpha; \alpha \in A\}$ is called *precise* if $B = A$ and $V_\alpha \subset U_\alpha$ for each α. If a cover has a compact-finite open refinement, it has a precise compact-finite open refinement (see the proof of Theorem 1.4 in [5, p. 162]).

a neighborhood W_y of y in Y such that $f^{-1}(W_y) \subset G_y$ for each y. Let $\{V_y; y \in Y\}$ be a precise compact-finite open refinement of $\{W_y; y \in Y\}$. Then $\mathscr{U}' = \{f^{-1}(V_y) \cap U_\alpha; U_\alpha \subset G_y, y \in Y\}$ is a compact-finite open refinement of $\mathscr{U}$.

Replacing compact-finite by cs-finite in the above argument, gives the proof in the sequential mesocompact case, and the theorem is proved.

Since projections parallel to compact factors are perfect maps, the following corollary is immediate.

COROLLARY 2.5. *If X is mesocompact (sequentially mesocompact) and Y is compact, then $X \times Y$ is mesocompact (sequentially mesocompact).*

EXAMPLE 2.6. The product of paracompact spaces need not be sequentially mesocompact. Let X be the reals with the upper limit topology [5, p. 66]. Then X is a first countable paracompact space such that $X \times X$ is not normal. Clearly, $X \times X$ can't be sequentially mesocompact by Theorem 1.14(b).

EXAMPLE 2.7. The product of a compact space and a locally compact space need not be mesocompact. Consider the ordinal spaces $X = \{x; x \leqq \Omega\}$ and $Y = \{y; y \angle \Omega\}$. Then $X \times Y$ is locally compact and not normal and so can't be mesocompact by Theorem 1.14(a) or Theorem 1.23.

COROLLARY 2.8. *Let X be mesocompact and Y locally compact and mesocompact. Then $X \times Y$ is mesocompact.*

Proof. By hypothesis, Y has a compact-finite (in fact a star finite) open cover $\mathscr{V} = \{V_\alpha; \alpha \in A\}$ such that $\mathrm{Cl}(V_\alpha)$ is compact for each $\alpha \in A$. Let $\mathscr{U}$ be an open cover of $X \times Y$. Then for each $\alpha \in A$, $X \times \mathrm{Cl}(V_\alpha)$ is mesocompact by Corollary 2.5 and so we can find a (relatively) open cover $\mathscr{U}_\alpha$ of $X \times \mathrm{Cl}(V_\alpha)$ which refines $\mathscr{U}$ and is compact-finite in $X \times \mathrm{Cl}(V_\alpha)$ (and hence in $X \times Y$). Let

$$\mathscr{W}_\alpha = \{(X \times V_\alpha) \cap U; U \in \mathscr{U}_\alpha\}$$

and let $\mathscr{W} = \{\mathscr{W}_\alpha; \alpha \in A\}$. Since $\{X \times V_\alpha; \alpha \in A\}$ is also compact-finite, $\mathscr{W}$ is a compact-finite open cover of $X \times Y$ which refines $\mathscr{U}$.

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[4] A map $f: X \to Y$ is called *compact* if $f^{-1}(K)$ is compact in X whenever K is compact in Y.

REMARK 2.9. Examples 2.6 and 2.7 show that neither hypothesis for Y in Corollary 2.8 can be dropped.

For compact maps[4], we have the following result.

PROPOSITION 2.10. *Let $f: X \to Y$ be a compact map and Y a k'-space* (see Remark 1.21(2)). *The X is mesocompact if and only if Y is mesocompact.*

Proof. In view of Theorem 2.4, we need only show that f is a closed map. Suppose $f(A)$ is not closed in Y for some closed A in X. Then there exists a $y \in \mathrm{Cl}(f(A)) - f(A)$. There exists a compact subset K of Y such that $y \in \mathrm{Cl}(K \cap f(A))$. Then $f^{-1}(K) \cap A$ is compact in X since A is closed and f is a compact map. Applying f, we have that $K \cap f(A)$ is compact. But then there exist disjoint open sets containing y and $K \cap f(A)$ respectively, a contradiction since $y \in \mathrm{Cl}(K \cap f(A))$.

The following proposition will be used in Example 2.15.

PROPOSITION 2.11. *If $f: X \to Y$ is a closed map and no countably infinite subset of X has a limit point in X, then no countably infinite subset of Y has a limit point in Y.*

Proof. Suppose $\{y_i; i \in N\}$ is a countably infinite subset of Y with limit point y. Pick an $x_i \in f^{-1}(y_i)$ for each $i \in N$. The proposition will be proved if we can show that $\{x_i; i \in N\}$ has a limit point in $f^{-1}(y)$. If not, each $x \in f^{-1}(y)$ has a neighborhood G_x which misses $\{x_i; i \in N\} - \{x\}$. Then $f^{-1}(y) \subset G = \cup \{G_x; x \in f^{-1}(y)\}$ and so, since f is closed, there exists a neighborhood V of y in Y such that $f^{-1}(V) \subset G$. There exists a $y_i \in V$ such that $y_i \neq y$, and so $x_i \in G$ and $x_i \notin f^{-1}(y)$. This is a contradiction which completes the proof.

REMARK 2.12. A standard type argument shows that a closed subspace of a mesocompact space is mesocompact. In view of Proposition 1.7, this also holds for sequentially mesocompact spaces [4, Th. 25]. Arbitrary subspaces of even compact spaces may not be sequentially mesocompact as the subspace $X' = \{x; x < \Omega\}$ of the ordinal space $X = \{x; x \leqq \Omega\}$ shows (see Example 1.17).

REMARK 2.13. In [10] we asked whether a mesocompact space need be normal or regular. The final two examples answer this question in the negative.

EXAMPLE 2.14. A mesocompact regular space which is not normal. Briggs [4, Th. 16] gives a scc and metacompact (hence mesocompact by Proposition 1.10(b)) regular space which is not normal. Let $X = \{x; x \leqq \Omega\}$, with the discrete topology except at Ω where it has the order topology. Let Ω' denote the first ordinal such that if $Y = \{y; y \leqq \Omega'\}$, the cardinality of Y is greater than that of X. Let Y have the discrete topology except at Ω' where it has the order topology. Let $Z = X \times Y - \{(\Omega, \Omega')\}$ with the product topology. We record here that Z is scc since no countable subset of Z has a limit point.

EXAMPLE 2.15. A mesocompact space which is not regular. Let Z be as in Example 2.14. Since Z is not normal, there are disjoint closed subsets A and B of Z such that each neighborhood of A meets each neighborhood of B. Indentify say A to a point, and denote the resulting quotient space by Z/A. Then Z/A is Hausdorff but not regular [9, p. 132G]. The quotient map $f: Z \rightarrow Z/A$ is closed, and so by Remark 2.3 and Proposition 2.11, Z/A is metacompact and scc. Again by Proposition 1.10(b), Z/A is mesocompact.

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REFERENCES

1. A. Arhangelskii, *Bicompact sets and the topology of spaces*, Soviet Math. Dokl. **4** (1963), 561-564.
2. R. H. Bing, *Metrization of topological spaces*, Canad. J. Math. **3** (1951), 175-186.
3. J. R. Boone, *Some characterizations of paracompactness in k-spaces* (to appear).
4. R. C. Briggs, III, *A comparison of covering properties in T_3 and T_4 spaces*, Doctoral Dissertation, Univ. of Houston, 1968.
5. J. Dugundji, *Topology*, Allyn and Bacon Inc., Boston, 1966.
6. E. Halfar, *Compact mappings*, Proc. Amer. Math. Soc. **8** (1957), 828-830.
7. R. W. Heath, *Screenability, pointwise paracompactness, and metrization of Moore spaces*, Canad. J. Math. **16** (1964), 763-770.
8. S. B. Higgins, *Some generalizations of paracompactness*, Notices Amer. Math. Soc. **16** (1969), 305.
9. J. L. Kelley, *General topology*, Van Nostrand, Princeton, 1955.
10. V. J. Mancuso, *Some properties of mesocompact spaces*, Notices Amer. Math. Soc. **16** (1969), 527.
11. E. Michael, *Another note on paracompact spaces*, Proc. Amer. Math. Soc. **8** (1957), 822-828.
12. ———, *Point-finite and locally finite coverings*, Canad. J. Math. **7** (1955), 275-279.
13. ———, *$\aleph_0$-spaces*, J. Math. Mech. **15** (1966), 983-1002.
14. K. Morita, *On decomposition spaces of locally compact spaces*, Proc. Japan Acad. **32** (1956), 544-548.
15. J. M. Worrell, Jr., *The closed continuous images of metacompact spaces*, Port. Math. **25** (1966), 175-179.

16. J. N. Younglove, *Concerning dense metric subspaces of certain non-metric spaces*, Fund. Math. **48** (1960), 15–25.

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