CARTAN SUBALGEBRAS OF A LIE ALGEBRA AND ITS IDEALS

DAVID J. WINTER
The purpose of this paper is to describe, under suitable conditions which are always satisfied at characteristic 0, a close relationship between Cartan subalgebras of a Lie algebra $\mathfrak{L}$ and Cartan subalgebras of an ideal $\mathfrak{L}'$ of $\mathfrak{L}$. Under the conditions referred to, a mapping $\alpha^*$ from the set of Cartan subalgebras of $\mathfrak{L}$ onto the set of Cartan subalgebras of $\mathfrak{L}'$ is described and the fibres of $\alpha^*$ are determined.

The main tools for the paper are N. Jacobson’s generalization of Engel’s Theorem [2; p. 33], and Theorem 5 of [4] which deals with Cartan subalgebras of the Fitting zero space of a derivation of a Lie algebra $\mathfrak{L}$. In addition, general material on Lie algebras, to be found in [2], [3], is presupposed.

Throughout this paper, Lie algebras and vector spaces are finite dimensional.

If $V$ is an $\mathcal{N}$-module where $\mathcal{N}$ is a nilpotent Lie algebra over the field $F$, the null and one components of $V$ are denoted $V_0(\mathcal{N})$, $V_1(\mathcal{N})$ respectively [cf. 2; pp. 37-43] and, for $\alpha$ a function from $\mathcal{N}$ into $F$, $V_\alpha(\mathcal{N}) = \{ v \in V | \forall (I - \alpha(x))^0 \dim V = 0 \text{ for all } x \in \mathcal{N} \}$.

If $V$ is a vector space (respectively Lie algebra, respectively module for a Lie algebra, over $F$, then the extension $V \otimes_F K$ of $V$ to an extension field $K$ of $F$ is denoted $V_K$.

2. Cartan subalgebras of a Lie algebra and its ideals. Throughout this section, $\mathcal{L}$ denotes a Lie algebra over an arbitrary field $F$. The characteristic of $F$ is denoted $p$, $p = 0$ being permissible. Let $\mathcal{L}'$ be an ideal of $\mathcal{L}$ and let the canonical short exact sequence determined by $\mathcal{L}, \mathcal{L}'$ be denoted

$$0 \rightarrow \mathcal{L}' \xrightarrow{\alpha} \mathcal{L} \xrightarrow{\beta} \mathcal{F} = \mathcal{L}/\mathcal{L}' \rightarrow 0,$$

where $\alpha$ is the inclusion mapping. The set of Cartan subalgebras of $\mathcal{L}$ is denoted Cart $\mathcal{L}$. For $\mathcal{H} \in \text{Cart } \mathcal{L}$, $(\mathcal{L}')_0(\text{ad } (\mathcal{H} \cap \mathcal{L}'))$ is denoted $\alpha^*(\mathcal{H})$. Our main objective is to prove the following theorem.

**Theorem.** Suppose that either $p = 0$, or $p \neq 0$ and $(\text{ad } \mathcal{L}', \mathcal{L}')^p \subset \text{ad } \mathcal{L}'$ and $(\text{ad } \mathcal{L})^p \subset \text{ad } \mathcal{L}$. Then $\alpha^*(\text{Cart } \mathcal{L}) = \text{Cart } \mathcal{L}'$ and
\[ \alpha^{-1}(H') = \text{Cart} \mathcal{L}_0(\text{ad } H') \text{ for } H' \in \text{Cart } \mathcal{L}'. \]

We defer the proof for the moment, since it is convenient to have the following lemma at our disposal.

**Lemma.** Let \( V \) be a vector space over \( F \), \( \mathcal{L} \) a Lie subalgebra of \( \text{Hom}_F V \). If the characteristic of \( F \) is \( p \neq 0 \), suppose that \( \mathcal{L} \) is closed under \( p \)-th powers. Let \( \mathcal{N} \) be a nilpotent subalgebra of \( \text{Hom}_F V \) which normalizes \( \mathcal{L} \). Suppose that \( \mathcal{L}_0(\text{ad } \mathcal{N}) \) consists of nilpotent transformations of \( V \). Then \( \mathcal{L} \) consists of nilpotent transformations of \( V \).

**Proof of lemma.** Since \( \mathcal{L}_0(\text{ad } \mathcal{N}) \) consists of nilpotent transformations and is closed under brackets, \( \mathcal{L}_0(\text{ad } \mathcal{N})_K = (\mathcal{L}_K)_0(\text{ad } \mathcal{N}_K) \) consists of nilpotent transformations where \( K \) is the algebraic closure of \( F \). Moreover, if the characteristic of \( F \) is \( p \neq 0 \), \( \mathcal{L}_K \) is closed under \( p \)-th powers [cf. 2; p. 190]. Thus, we may assume without loss of generality that \( F \) is algebraically closed.

Now \( \mathcal{L} = \sum \mathcal{L}_n(\text{ad } \mathcal{N}) \) and \( V = \sum V_n(\mathcal{N}) \). For all \( \alpha, \beta \), we have \( V_\beta(\mathcal{N}) \mathcal{L}_n(\text{ad } \mathcal{N}) \subset V_{\beta + \alpha}(\text{ad } \mathcal{N}) \) [cf. 2; p. 63]. Thus, if the characteristic of \( F \) is 0, \( \mathcal{L}_0(\text{ad } \mathcal{N}) \) consists of nilpotent transformations for all \( \alpha \): for \( \alpha = 0 \) by hypothesis and for \( \alpha \neq 0 \) by the above observation. Suppose next that the characteristic of \( F \) is \( p \neq 0 \). Let \( x \in \mathcal{L}_n(\text{ad } \mathcal{N}) \). Then \( x^p \in \mathcal{L} \cap (\text{Hom}_F V)_n(\text{ad } \mathcal{N}) = \mathcal{L}_0(\text{ad } \mathcal{N}) \), for if \( t \) is the semi-simple part of an element \( y \) of \( \mathcal{N} \), \( t \text{ ad } x = -\alpha(y)x \) so that \( 0 = t(\text{ad } x)^p = \cdots = t(\text{ad } x)^2 = [t, x^p] \). Thus, \( x^p \), hence \( x \), is nilpotent. Thus, the \( \mathcal{L}_n(\text{ad } \mathcal{N}) \) again consist of nilpotent transformations for all \( \alpha \). We now can apply [2; p. 33] to the weakly closed set \( \cup \mathcal{L}_n(\text{ad } \mathcal{N}) \) of nilpotent transformations. This implies that the Lie algebra generated by \( \cup \mathcal{L}_n(\text{ad } \mathcal{N}) \), namely \( \mathcal{L} \) itself, consists of nilpotent transformations.

**Proof of theorem.** We first show that \( \alpha^*(\text{Cart } \mathcal{L}) \) \( \subset \text{Cart } \mathcal{L}' \). Thus, let, \( \mathcal{H} \in \text{Cart } \mathcal{L} \). Then \( \mathcal{H} \cap \mathcal{L}' = \mathcal{L}_0(\text{ad } \mathcal{H'}) = (\mathcal{L}'_0)(\text{ad } \mathcal{H}) \).

Now \( \mathcal{N} = \text{ad } \mathcal{H}|_{\mathcal{H}'} \) is a nilpotent Lie algebra of derivations of \( \mathcal{L}' \) and \( \mathcal{H} \cap \mathcal{L}' \) is trivially a Cartan subalgebra of \( (\mathcal{L}')_0(\mathcal{N}) = \mathcal{H} \cap \mathcal{L}' \). Thus, Theorem 5 of [4] applies and shows that \( (\mathcal{L}')_0(\text{ad } (\mathcal{H} \cap \mathcal{L}')) = \alpha^*(\mathcal{H}') \) is a Cartan subalgebra of \( \mathcal{L}' \).

Next suppose that \( \mathcal{H} \in \text{Cart } \mathcal{L}' \) and that \( \mathcal{N} \in \text{Cart } \mathcal{L}_0(\text{ad } \mathcal{H}) \). Since \( \mathcal{L}_0(\text{ad } \mathcal{H}') \) normalizes \( \mathcal{L}_0(\text{ad } \mathcal{H}') \cap \mathcal{L}' = (\mathcal{L}')_0(\text{ad } \mathcal{H}') = \mathcal{H}' \), we have:

\[ \mathcal{N} \text{ normalizes } \mathcal{H}'. \]

In view of (1), we have \( \mathcal{H}' = \mathcal{H}'_0 \oplus \mathcal{H}'_1 \) where \( \mathcal{H}'_0 = (\mathcal{H}')_0(\text{ad } \mathcal{N}) \)
and $\mathcal{H}' = (\mathcal{H}'')'$. Note that $\mathcal{H}'_0 = \mathcal{H}' \cap \mathcal{N}$ since $\mathcal{N} \in \text{Cart } L(\text{ad } \mathcal{H})$ and $\mathcal{H}' \subset L(\text{ad } \mathcal{H}'_0)$. Let $V = (L')_0(\text{ad } \mathcal{H}'_0)$. Since $\mathcal{H}'_0 \subset \mathcal{H}'$ and $\mathcal{H}' \subset \mathcal{N}$, $V$ is stable under $\text{ad } \mathcal{H}'$ and $\text{ad } \mathcal{N}$ [cf. 2; p. 58]. Now we prepare the way for applying the above lemma to $(V, \text{ad } \mathcal{H}'_0|_V, \text{ad } \mathcal{N}|_V)$. Thus, note that $\text{ad } \mathcal{H}'_0|_V$ is a subalgebra of $\text{Hom}_F V$ normalized by the nilpotent subalgebra $\mathcal{H}'_0$ and that, if the characteristic of $F$ is $p \neq 0$, $\text{ad } \mathcal{H}'_0|_V$ is closed under $p$-th powers. (In fact, $\text{ad } \mathcal{H}'_0|_V$ is closed under $p$-th powers since $\mathcal{H}'_0$ is a Cartan subalgebra of $L'$: for $x \in \mathcal{H}'$, $(ad x)^p = ad y$ for some $y \in L'$, and $y \in \mathcal{H}'$, since $\mathcal{H} \supset \mathcal{H}$ $(ad x)^p = [x, y])$. Moreover

$$(\text{ad } \mathcal{H}'_0|_V)((\text{ad } \mathcal{N}|_V)) = \text{ad } \mathcal{H}'_0|_V$$

and $\text{ad } \mathcal{H}'_0|_V$ consists of nilpotent transformations by the definition of $V$. Thus, by the lemma, $\text{ad } \mathcal{H}'_0|_V$ consists of nilpotent transformations. Thus, $(L'),(\text{ad } \mathcal{H}') = ((L'),(\text{ad } \mathcal{H}') = \mathcal{H}'$. We therefore have:

$$(2) \quad \mathcal{H}' = (L')_0(\text{ad } (\mathcal{H}' \cap \mathcal{N})).$$

We show that (2) implies $\mathcal{H}' = (L')_0(\text{ad } (\mathcal{H}' \cap \mathcal{H}))$ for substable $\mathcal{H} \in \text{Cart } L$. Thus, let $\mathcal{H} = L_0(\text{ad } \mathcal{N})$. Then $\mathcal{H} \in \text{Cart } L$, by Theorem 5 of [4], since $\mathcal{N}$ is a Cartan subalgebra of $L_0(\text{ad } \mathcal{H}')$. Since $\mathcal{H}' \cap \mathcal{N} \subset L'_0 \cap \mathcal{H}$, (2) implies that

$$\alpha^*(\mathcal{H}) = (L')_0(\text{ad } (L' \cap \mathcal{H})) (L')_0(\text{ad } (\mathcal{H}' \cap \mathcal{N})) = \mathcal{H}'.$$

Thus, since $\alpha^*(\mathcal{H}) \in \text{Cart } L'$, by the preceding paragraph, $\alpha^*(\mathcal{H}) = \mathcal{H}'$, by the maximal nilpotency of Cartan subalgebras. Thus, we have:

$$(3) \quad L_0(\text{ad } \mathcal{N}) \in \text{Cart } L'$$

We have $\alpha^*(\text{Cart } L) \subset \text{Cart } \mathcal{H}'$, from the first paragraph. Thus, it follows from (3) that $\alpha^*(\text{Cart } L) = \text{Cart } L'$. Note, however, that the existence of $\mathcal{N} \in \text{Cart } L_0(\text{ad } \mathcal{H}')$ is used for this conclusion. But $\text{ad } L_0(\text{ad } \mathcal{H}')$ is a linear Lie $p$-algebra for $p \neq 0$, as the null component of the linear Lie $p$-algebra $\text{ad } L$ with respect to the subalgebra $\text{ad } \mathcal{H}'$. Thus, $\text{ad } L_0(\text{ad } \mathcal{H}')$ has a Cartan subalgebra, by [3; p. 121], so that $L_0(\text{ad } \mathcal{H}')$ has a Cartan subalgebra.

Finally, we suppose that $\mathcal{H} \in \text{Cart } L'$. Let $\mathcal{H} \in \alpha^{*-1}(\mathcal{H}')$. Then $\mathcal{H} \subset L_0(\text{ad } (\mathcal{N} \cap L'))$, so that $\mathcal{H}$ normalizes

$$\mathcal{H}' = L_0(\text{ad } (\mathcal{H} \cap L')) \cap L'.$$

Thus, $\mathcal{H} \subset L_0(\text{ad } \mathcal{H}')$, so that $\mathcal{H} \in \text{Cart } L_0(\text{ad } \mathcal{H}')$. Suppose, conversely, that $\mathcal{N} \in \text{Cart } L_0(\text{ad } \mathcal{H}')$. By (3), $L_0(\text{ad } \mathcal{N}) \in \text{Cart } L'$.
and \( \alpha^*(L_3(\text{ad } N)) = \mathbb{H}' \). Thus, by first part of this paragraph, \( L_3(\text{ad } N) \subset L_3(\text{ad } \mathbb{H}) \). But then
\[
L_3(\text{ad } N) = L_3(\text{ad } \mathbb{H}')(\text{ad } N) = N,
\]
since \( N \in \text{Cart } L_3(\text{ad } \mathbb{H}') \) [cf. 2; p. 57–58]. Thus, \( N \) is a Cartan subalgebra of \( L \), by [2; p. 57–58]. Now
\[
\alpha^*(N) = \alpha^*(L_3(\text{ad } N)) = \mathbb{H}',
\]
by (3), and \( N \in \alpha^{-1}(\mathbb{H}') \). Thus, \( \alpha^{-1}(\mathbb{H}') = \text{Cart } L_3(\text{ad } \mathbb{H}) \).

We now turn to two related results. The first is concerned with the fibres of \( \alpha^* \). The second is concerned with the relations between the sequences
\[
\text{Cart } L' \leftarrow \alpha^* \rightarrow \text{Cart } L \leftarrow i \rightarrow \text{Cart } L_3(\text{ad } \mathbb{H}')
\]
\[
\text{Cart } \beta^{-1}(\mathbb{H}) \leftarrow i \rightarrow \text{Cart } L \leftarrow \beta^* \rightarrow \text{Cart } \mathbb{H}
\]
where \( i \) is inclusion, \( \mathbb{H} \in \text{Cart } L, \mathbb{H}' = \alpha^*(\mathbb{H}), \mathbb{H} = \beta(\mathbb{H}) \) and \( \beta^* \) is defined by \( \beta^*(\mathcal{I}) = \beta(\mathcal{I}) \) for \( \mathcal{I} \in \text{Cart } L \).

**Proposition 1.** Let the hypothesis be as in the theorem, and let \( \mathbb{H} \in \text{Cart } L, \mathbb{H}' \in \text{Cart } L' \). Then the following conditions are equivalent.

1. \( \mathbb{H}' = \alpha^*(\mathbb{H}) \);
2. \( \mathbb{H} \) normalizes \( \mathbb{H}' \);
3. \( \mathbb{H} \cap L' \subset \mathbb{H}' \).

**Proof.** If \( \mathbb{H}' = \alpha^*(\mathbb{H}) \), then \( \mathbb{H} \subset L_3(\text{ad } (L' \cap \mathbb{H}')) \) and \( \mathbb{H} \) normalizes \( \mathbb{H}' = L_3(\text{ad } (L' \cap \mathbb{H}')) \cap L' \). Thus, (1) implies (2). If \( \mathbb{H} \) normalizes \( \mathbb{H}' \), then \( \mathbb{H} \subset L_3(\text{ad } \mathbb{H}') \) and
\[
\mathbb{H} \cap L' \subset (L')_3(\text{ad } \mathbb{H}') = \mathbb{H}'.
\]
Thus, (2) implies (3). Suppose, finally, that \( \mathbb{H} \cap L' \subset \mathbb{H}' \). Then \( \mathbb{H}' = (L')_3(\text{ad } \mathbb{H}') \subset (L')_3(\text{ad } (\mathbb{H} \cap L')) = \alpha^*(\mathbb{H}) \). But \( \alpha^*(\mathbb{H}) \in \text{Cart } L' \) and \( \mathbb{H}' \) is a Cartan subalgebra of \( L' \), maximal nilpotent in \( L' \). Thus, \( \mathbb{H}' = L(\mathbb{H})^*(\mathbb{H}) \). Thus (3) implies (1), and the conditions (1)–(3) are equivalent.

**Proposition 2.** Let the hypothesis be as in the theorem. Let \( \mathbb{H} \in \text{Cart } L, \mathbb{H}' = \alpha^*(\mathbb{H}), \mathbb{H} = \beta(\mathbb{H}) \). Then \( \mathbb{H} \) normalizes \( \mathbb{H}' \), Cart \( L \) contains Cart \( (\mathbb{H} + \mathbb{H}') \), Cart \( L_3(\text{ad } \mathbb{H}') \) and Cart \( \beta^{-1}(\mathbb{H}) \), and (Cart \( L_3(\text{ad } \mathbb{H}')) \cap \text{Cart } \beta^{-1}(\mathbb{H}) = \text{Cart } (\mathbb{H} + \mathbb{H}') \).

**Proof.** \( \mathbb{H} \) normalizes \( \mathbb{H}' \), by Proposition 1. Since
Thus, it suffices to show that \( \text{Cart} \subseteq \text{Cart} \beta^{-1}(\mathcal{H}) \cap \text{Cart} (\mathcal{H} + \mathcal{H}') \) and \( \text{Cart} \beta^{-1}(\mathcal{H}) \). The first two sets are contained in \( \text{Cart} \subseteq \text{Cart} \beta^{-1}(\mathcal{H}) \cap \text{Cart} (\mathcal{H} + \mathcal{H}') \) by the theorem and \( \text{Cart} \beta^{-1}(\mathcal{H}) \) by [1]. Thus, it remains only to show that

\[
\text{Cart} (\mathcal{H} + \mathcal{H}') \subseteq \text{Cart} \beta^{-1}(\mathcal{H}).
\]

But \( \mathcal{H}' = \mathcal{L}(\text{ad} \mathcal{H}') \cap \mathcal{L} \) is an ideal of \( \mathcal{L}(\text{ad} \mathcal{H}') \) and \( \mathcal{H} \in \text{Cart} \mathcal{L}(\text{ad} \mathcal{H}') \). Thus, \( \text{Cart} (\mathcal{H} + \mathcal{H}') \subseteq \text{Cart} \mathcal{L}(\text{ad} \mathcal{H}') \), by [1], since \( \mathcal{H} + \mathcal{H}' \) is the preimage in \( \mathcal{L}(\text{ad} \mathcal{H}') \) of the Cartan subalgebra \( (\mathcal{H} + \mathcal{H}')/\mathcal{H}' \) of \( \mathcal{L}(\text{ad} \mathcal{H}')/\mathcal{H}' \).

**References**


Received February 11, 1969. This research was done while the author was a National Science Foundation Postdoctoral Fellow at the University of Bonn.
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PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), 7-17, Fujimi 2-chome, Chiyoda-ku, Tokyo, Japan.

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