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CARTAN SUBALGEBRAS OF A LIE ALGEBRA AND ITS IDEALS

DAVID J. WINTER

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The purpose of this paper is to describe, under suitable conditions which are always satisfied at characteristic 0, a close relationship between Cartan subalgebras of a Lie algebra \mathcal{L} and Cartan subalgebras of an ideal \mathcal{L}' of \mathcal{L} . Under the conditions referred to, a mapping α^* from the set of Cartan subalgebras of \mathcal{L} onto the set of Cartan subalgebras of \mathcal{L}' is described and the fibres of α^* are determined.

The main tools for the paper are N. Jacobson's generalization of Engel's Theorem [2; p. 33], and Theorem 5 of [4] which deals with Cartan subalgebras of the Fitting zero space of a derivation of a Lie algebra \mathcal{L} . In addition, general material on Lie algebras, to be found in [2], [3], is presupposed.

Throughout this paper, Lie algebras and vector spaces are finite dimensional.

If V is an \mathcal{N} -module where \mathcal{N} is a nilpotent Lie algebra over the field F , the null and one components of V are denoted $V_0(\mathcal{N})$, $V_*(\mathcal{N})$ respectively [cf. 2; pp. 37-43] and, for α a function from \mathcal{N} into F , $V_\alpha(\mathcal{N}) = \{v \in V \mid v(I - \alpha(x))^{\dim V} = 0 \text{ for all } x \in \mathcal{N}\}$.

If V is a vector space (respectively Lie algebra, respectively module for a Lie algebra, over F , then the extension $V \otimes_F K$ of V to an extension field K of F is denoted V_K .

2. Cartan subalgebras of a Lie algebra and its ideals. Throughout this section, \mathcal{L} denotes a Lie algebra over an arbitrary field F . The characteristic of F is denoted p , $p = 0$ being permissible. Let \mathcal{L}' be an ideal of \mathcal{L} and let the canonical short exact sequence determined by $\mathcal{L}, \mathcal{L}'$ be denoted

$$0 \longrightarrow \mathcal{L}' \xrightarrow{\alpha} \mathcal{L} \xrightarrow{\beta} \overline{\mathcal{L}} = \mathcal{L}/\mathcal{L}' \longrightarrow 0,$$

where α is the inclusion mapping. The set of Cartan subalgebras of \mathcal{L} is denoted $\text{Cart } \mathcal{L}$. For $\mathcal{H} \in \text{Cart } \mathcal{L}$, $(\mathcal{L}')_0(\text{ad } (\mathcal{H} \cap \mathcal{L}'))$ is denoted $\alpha^*(\mathcal{H})$. Our main objective is to prove the following theorem.

THEOREM. *Suppose that either $p = 0$, or $p \neq 0$ and $(\text{ad}_{\mathcal{L}'} \mathcal{L}')^p \subset \text{ad}_{\mathcal{L}'} \mathcal{L}'$ and $(\text{ad}_{\mathcal{L}} \mathcal{L})^p \subset \text{ad}_{\mathcal{L}} \mathcal{L}$. Then $\alpha^*(\text{Cart } \mathcal{L}) = \text{Cart } \mathcal{L}'$ and*

$\alpha^{*-1}(\mathcal{H}') = \text{Cart } \mathcal{L}_0(\text{ad } \mathcal{H}')$ for $\mathcal{H}' \in \text{Cart } \mathcal{L}'$.

We defer the proof for the moment, since it is convenient to have the following lemma at our disposal.

LEMMA. *Let V be a vector space over F , \mathcal{L} a Lie subalgebra of $\text{Hom}_F V$. If the characteristic of F is $p \neq 0$, suppose that \mathcal{L} is closed under p -th powers. Let \mathcal{N} be a nilpotent subalgebra of $\text{Hom}_F V$ which normalizes \mathcal{L} . Suppose that $\mathcal{L}_0(\text{ad } \mathcal{N})$ consists of nilpotent transformations of V . Then \mathcal{L} consists of nilpotent transformations of V .*

Proof of lemma. Since $\mathcal{L}_0(\text{ad } \mathcal{N})$ consists of nilpotent transformations and is closed under brackets, $\mathcal{L}_0(\text{ad } \mathcal{N})_K = (\mathcal{L}_K)_0(\text{ad } \mathcal{N}_K)$ consists of nilpotent transformations where K is the algebraic closure of F . Moreover, if the characteristic of F is $p \neq 0$, \mathcal{L}_K is closed under p -th powers [cf. 2; p. 190]. Thus, we may assume without loss of generality that F is algebraically closed.

Now $\mathcal{L} = \sum \mathcal{L}_\alpha(\text{ad } \mathcal{N})$ and $V = \sum V_\beta(\mathcal{N})$. For all α, β , we have $V_\beta(\mathcal{N})\mathcal{L}_\alpha(\text{ad } \mathcal{N}) \subset V_{\beta+\alpha}(\text{ad } \mathcal{N})$ [cf. 2; p. 63]. Thus, if the characteristic of F is 0, $\mathcal{L}_\alpha(\text{ad } \mathcal{N})$ consists of nilpotent transformations for all α : for $\alpha = 0$ by hypothesis and for $\alpha \neq 0$ by the above observation. Suppose next that the characteristic of F is $p \neq 0$. Let $x \in \mathcal{L}_\alpha(\text{ad } \mathcal{N})$. Then $x^p \in \mathcal{L} \cap (\text{Hom}_F V)_0(\text{ad } \mathcal{N}) = \mathcal{L}_0(\text{ad } \mathcal{N})$, for if t is the semi-simple part of an element y of \mathcal{N} , $t \text{ ad } x = -\alpha(y)x$ so that $0 = t(\text{ad } x)^2 = \dots = t(\text{ad } x)^p = [t, x^p]$. Thus, x^p , hence x , is nilpotent. Thus, the $\mathcal{L}_\alpha(\text{ad } \mathcal{N})$ again consist of nilpotent transformations for all α . We now can apply [2; p. 33] to the weakly closed set $\cup \mathcal{L}_\alpha(\text{ad } \mathcal{N})$ of nilpotent transformations. This implies that the Lie algebra generated by $\cup \mathcal{L}_\alpha(\text{ad } \mathcal{N})$, namely \mathcal{L} itself, consists of nilpotent transformations.

Proof of theorem. We first show that $\alpha^*(\text{Cart } \mathcal{L}) \subset \text{Cart } \mathcal{L}'$. Thus, let, $\mathcal{H} \in \text{Cart } \mathcal{L}$. Then $\mathcal{H} \cap \mathcal{L}' = \mathcal{L}_0(\text{ad } \mathcal{H}) \cap \mathcal{L}' = (\mathcal{L}')_0(\text{ad } \mathcal{H})$. Now $\mathcal{N} = \text{ad } \mathcal{H}|_{\mathcal{L}'}$, is a nilpotent Lie algebra of derivations of \mathcal{L}' and $\mathcal{H} \cap \mathcal{L}'$ is trivially a Cartan subalgebra of $(\mathcal{L}')_0(\mathcal{N}) = \mathcal{H} \cap \mathcal{L}'$. Thus, Theorem 5 of [4] applies and shows that $(\mathcal{L}')_0(\text{ad } (\mathcal{H} \cap \mathcal{L}')) = \alpha^*(\mathcal{H})$ is a Cartan subalgebra of \mathcal{L}' .

Next suppose that $\mathcal{H}' \in \text{Cart } \mathcal{L}'$ and that $\mathcal{N} \in \text{Cart } \mathcal{L}_0(\text{ad } \mathcal{H}')$. Since $\mathcal{L}_0(\text{ad } \mathcal{H}')$ normalizes $\mathcal{L}_0(\text{ad } \mathcal{H}') \cap \mathcal{L}' = (\mathcal{L}')_0(\text{ad } \mathcal{H}') = \mathcal{H}'$, we have:

$$(1) \quad \mathcal{N} \text{ normalizes } \mathcal{H}' .$$

In view of (1), we have $\mathcal{H}' = \mathcal{H}'_0 \oplus \mathcal{H}'_*$ where $\mathcal{H}'_0 = (\mathcal{H}')_0(\text{ad } \mathcal{N})$

and $\mathcal{H}' = (\mathcal{H}')_*(\text{ad } \mathcal{N})$. Note that $\mathcal{H}'_0 = \mathcal{H}' \cap \mathcal{N}$ since $\mathcal{N} \in \text{Cart } \mathcal{L}'_0(\text{ad } \mathcal{H}')$ and $\mathcal{H}' \subset \mathcal{L}'_0(\text{ad } \mathcal{H}')$. Let $V = (\mathcal{L}')_0(\text{ad } \mathcal{H}'_0)$. Since $\mathcal{H}'_0 \subset \mathcal{H}'$ and $\mathcal{H}'_0 \subset \mathcal{N}$, V is stable under $\text{ad } \mathcal{H}'$ and $\text{ad } \mathcal{N}$ [cf. 2; p. 58]. Now we prepare the way for applying the above lemma to $(V, \text{ad } \mathcal{H}'|_V, \text{ad } \mathcal{N}|_V)$. Thus, note that $\text{ad } \mathcal{H}'|_V$ is a subalgebra of $\text{Hom}_F V$ normalized by the nilpotent subalgebra $\text{ad } \mathcal{N}|_V$ and that, if the characteristic of F is $p \neq 0$, $\text{ad } \mathcal{H}'|_V$ is closed under p -th powers. (In fact, $\text{ad}_{\mathcal{H}'} \mathcal{H}'$ is closed under p -th powers since $\text{ad}_{\mathcal{L}'} \mathcal{L}'$ is closed under p -th powers and since \mathcal{H}' is a Cartan subalgebra of \mathcal{L}' : for $x \in \mathcal{H}'$, $(\text{ad } x)^p = \text{ad } y$ for some $y \in \mathcal{L}'$, and $y \in \mathcal{H}'$, since $\mathcal{H} \supset \mathcal{H}'$ $(\text{ad } x)^p = [\mathcal{H}', y]$). Moreover

$$(\text{ad } \mathcal{H}'|_V)_0(\text{ad } (\text{ad } \mathcal{N}|_V)) = \text{ad } \mathcal{H}'_0|_V$$

and $\text{ad } \mathcal{H}'_0|_V$ consists of nilpotent transformations by the definition of V . Thus, by the lemma, $\text{ad } \mathcal{H}'|_V$ consists of nilpotent transformations. Thus, $(\mathcal{L}')_0(\text{ad } \mathcal{H}'_0) = ((\mathcal{L}')_0(\text{ad } \mathcal{H}')) = \mathcal{H}'$. We therefore have:

$$(2) \quad \mathcal{H}' = (\mathcal{L}')_0(\text{ad } (\mathcal{H}' \cap \mathcal{N})) .$$

We show that (2) implies $\mathcal{H}' = (\mathcal{L}')_0(\text{ad } (\mathcal{H} \cap \mathcal{L}'))$ for substable $\mathcal{H} \in \text{Cart } \mathcal{L}$. Thus, let $\mathcal{H} = \mathcal{L}'_0(\text{ad } \mathcal{N})$. Then $\mathcal{H} \in \text{Cart } \mathcal{L}$, by Theorem 5 of [4], since \mathcal{N} is a Cartan subalgebra of $\mathcal{L}'_0(\text{ad } \mathcal{H}')$. Since $\mathcal{H}' \cap \mathcal{N} \subset \mathcal{L}' \cap \mathcal{H}$, (2) implies that

$$\alpha^*(\mathcal{H}) = (\mathcal{L}')_0(\text{ad } (\mathcal{L}' \cap \mathcal{H})) \subset (\mathcal{L}')_0(\text{ad } (\mathcal{H}' \cap \mathcal{N})) = \mathcal{H}' .$$

Thus, since $\alpha^*(\mathcal{H}) \in \text{Cart } \mathcal{L}'$, by the preceding paragraph, $\alpha^*(\mathcal{H}) = \mathcal{H}'$, by the maximal nilpotency of Cartan subalgebras. Thus, we have:

$$(3) \quad \mathcal{L}'_0(\text{ad } \mathcal{N}) \in \text{Cart } \mathcal{L} \text{ and } \alpha^*(\mathcal{L}'_0(\text{ad } \mathcal{N})) = \mathcal{H}' .$$

We have $\alpha^*(\text{Cart } \mathcal{L}) \subset \text{Cart } \mathcal{H}'$, from the first paragraph. Thus, it follows from (3) that $\alpha^*(\text{Cart } \mathcal{L}) = \text{Cart } \mathcal{L}'$. Note, however, that the existence of $\mathcal{N} \in \text{Cart } \mathcal{L}'_0(\text{ad } \mathcal{H}')$ is used for this conclusion. But $\text{ad } \mathcal{L}'_0(\text{ad } \mathcal{H}')$ is a linear Lie p -algebra for $p \neq 0$, as the null component of the linear Lie p -algebra $\text{ad } \mathcal{L}$ with respect to the subalgebra $\text{ad } \mathcal{H}'$. Thus, $\text{ad } \mathcal{L}'_0(\text{ad } \mathcal{H}')$ has a Cartan subalgebra, by [3; p. 121], so that $\mathcal{L}'_0(\text{ad } \mathcal{H}')$ has a Cartan subalgebra.

Finally, we suppose that $\mathcal{H}' \in \text{Cart } \mathcal{L}'$. Let $\mathcal{H} \in \alpha^{*-1}(\mathcal{H}')$. Then $\mathcal{H} \subset \mathcal{L}'_0(\text{ad } (\mathcal{H} \cap \mathcal{L}'))$, so that \mathcal{H} normalizes

$$\mathcal{H}' = \mathcal{L}'_0(\text{ad } (\mathcal{H} \cap \mathcal{L}')) \cap \mathcal{L}' .$$

Thus, $\mathcal{H} \subset \mathcal{L}'_0(\text{ad } \mathcal{H}')$, so that $\mathcal{H} \in \text{Cart } \mathcal{L}'_0(\text{ad } \mathcal{H}')$. Suppose, conversely, that $\mathcal{N} \in \text{Cart } \mathcal{L}'_0(\text{ad } \mathcal{H}')$. By (3), $\mathcal{L}'_0(\text{ad } \mathcal{N}) \in \text{Cart } \mathcal{L}$

and $\alpha^*(\mathcal{L}_0(\text{ad } \mathcal{N})) = \mathcal{H}'$. Thus, by first part of this paragraph, $\mathcal{L}_0(\text{ad } \mathcal{N}) \subset \mathcal{L}_0(\text{ad } \mathcal{H}')$. But then

$$\mathcal{L}_0(\text{ad } \mathcal{N}) = \mathcal{L}_0(\text{ad } \mathcal{H}')_0(\text{ad } \mathcal{N}) = \mathcal{N},$$

since $\mathcal{N} \in \text{Cart } \mathcal{L}_0(\text{ad } \mathcal{H}')$ [cf. 2; p. 57–58]. Thus, \mathcal{N} is a Cartan subalgebra of \mathcal{L} , by [2; p. 57–58]. Now

$$\alpha^*(\mathcal{N}) = \alpha^*(\mathcal{L}_0(\text{ad } \mathcal{N})) = \mathcal{H}',$$

by (3), and $\mathcal{N} \in \alpha^{*-1}(\mathcal{H}')$. Thus, $\alpha^{*-1}(\mathcal{H}') = \text{Cart } \mathcal{L}_0(\text{ad } \mathcal{H}')$.

We now turn to two related results. The first is concerned with the fibres of α^* . The second is concerned with the relations between the sequences

$$\begin{array}{ccccc} \text{Cart } \mathcal{L}' & \xleftarrow{\alpha^*} & \text{Cart } \mathcal{L} & \xleftarrow{i} & \text{Cart } \mathcal{L}_0(\text{ad } \mathcal{H}') \\ \text{Cart } \beta^{-1}(\overline{\mathcal{H}}) & \xrightarrow{i} & \text{Cart } \mathcal{L} & \xrightarrow{\beta_*} & \text{Cart } \overline{\mathcal{L}} \end{array}$$

where i is inclusion, $\mathcal{H} \in \text{Cart } \mathcal{L}$, $\mathcal{H}' = \alpha^*(\mathcal{H})$, $\overline{\mathcal{H}} = \beta(\mathcal{H})$ and β_* is defined by $\beta_*(\mathcal{I}) = \beta(\mathcal{I})$ for $\mathcal{I} \in \text{Cart } \mathcal{L}$.

PROPOSITION 1. *Let the hypothesis be as in the theorem, and let $\mathcal{H} \in \text{Cart } \mathcal{L}$, $\mathcal{H}' \in \text{Cart } \mathcal{L}'$. Then the following conditions are equivalent.*

- (1) $\mathcal{H}' = \alpha^*(\mathcal{H})$;
- (2) \mathcal{H} normalizes \mathcal{H}' ;
- (3) $\mathcal{H} \cap \mathcal{L}' \subset \mathcal{H}'$.

Proof. If $\mathcal{H}' = \alpha^*(\mathcal{H})$, then $\mathcal{H} \subset \mathcal{L}_0(\text{ad } (\mathcal{L}' \cap \mathcal{H}))$ and \mathcal{H} normalizes $\mathcal{H}' = \mathcal{L}_0(\text{ad } (\mathcal{L}' \cap \mathcal{H})) \cap \mathcal{L}'$. Thus, (1) implies (2). If \mathcal{H} normalizes \mathcal{H}' , then $\mathcal{H} \subset \mathcal{L}_0(\text{ad } \mathcal{H}')$ and

$$\mathcal{H} \cap \mathcal{L}' \subset (\mathcal{L}')_0(\text{ad } \mathcal{H}') = \mathcal{H}'.$$

Thus, (2) implies (3). Suppose, finally, that $\mathcal{H} \cap \mathcal{L}' \subset \mathcal{H}'$. Then $\mathcal{H}' = (\mathcal{L}')_0(\text{ad } \mathcal{H}') \subset (\mathcal{L}')_0(\text{ad } (\mathcal{H} \cap \mathcal{L}')) = \alpha^*(\mathcal{H})$. But $\alpha^*(\mathcal{H}) \in \text{Cart } \mathcal{L}'$ and \mathcal{H}' is, a Cartan subalgebra of \mathcal{L}' , maximal nilpotent in \mathcal{L}' . Thus, $\mathcal{H}' = \mathcal{L}(\mathcal{H})^*(\mathcal{H})$. Thus (3) implies (1), and the conditions (1)–(3) are equivalent.

PROPOSITION 2. *Let the hypothesis be as in the theorem. Let $\mathcal{H} \in \text{Cart } \mathcal{L}$, $\mathcal{H}' = \alpha^*(\mathcal{H})$, $\overline{\mathcal{H}} = \beta(\mathcal{H})$. Then \mathcal{H} normalizes \mathcal{H}' , $\text{Cart } \mathcal{L}$ contains $\text{Cart } (\mathcal{H} + \mathcal{H}')$, $\text{Cart } \mathcal{L}_0(\text{ad } \mathcal{H}')$ and $\text{Cart } \beta^{-1}(\overline{\mathcal{H}})$, and $(\text{Cart } \mathcal{L}_0(\text{ad } \mathcal{H}')) \cap \text{Cart } \beta^{-1}(\overline{\mathcal{H}}) = \text{Cart } (\mathcal{H} + \mathcal{H}')$.*

Proof. \mathcal{H} normalizes \mathcal{H}' , by Proposition 1. Since

$$\beta^{-1}(\overline{\mathcal{H}}) = \mathcal{H} + \mathcal{H}', \mathcal{L}_0(\text{ad } \mathcal{H}') \cap \beta^{-1}(\overline{\mathcal{H}}) = \mathcal{H} + \mathcal{H}' .$$

Thus, it suffices to show that $\text{Cart } \mathcal{L}$ contains $\text{Cart } \mathcal{L}_0(\text{ad } \mathcal{H}')$, $\text{Cart } \beta^{-1}(\overline{\mathcal{H}})$ and $\text{Cart } (\mathcal{H} + \mathcal{H}')$. The first two sets are contained in $\text{Cart } \mathcal{L} - \text{Cart } \mathcal{L}_0(\text{ad } \mathcal{H}')$ by the theorem and $\text{Cart } \beta^{-1}(\overline{\mathcal{H}})$ by [1]. Thus, it remains only to show that

$$\text{Cart } (\mathcal{H} + \mathcal{H}') \subset \text{Cart } \mathcal{L}_0(\text{ad } \mathcal{H}') .$$

But $\mathcal{H}' = \mathcal{L}_0(\text{ad } \mathcal{H}') \cap \mathcal{L}'$ is an ideal of $\mathcal{L}_0(\text{ad } \mathcal{H}')$ and $\mathcal{H} \in \text{Cart } \mathcal{L}_0(\text{ad } \mathcal{H}')$. Thus, $\text{Cart } (\mathcal{H} + \mathcal{H}') \subset \text{Cart } \mathcal{H}_0(\text{ad } \mathcal{H}')$, by [1], since $\mathcal{H} + \mathcal{H}'$ is the preimage in $\mathcal{L}_0(\text{ad } \mathcal{H}')$ of the Cartan subalgebra $(\mathcal{H} + \mathcal{H}')/\mathcal{H}'$ of $\mathcal{L}_0(\text{ad } \mathcal{H}')/\mathcal{H}'$.

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