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**ON A THEOREM OF M. IZUMI AND S. IZUMI**

SHIVA NARAIN LAL

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**This paper establishes a theorem on the absolute Nörlund summability of Fourier series which generalizes and unifies generalizations by the author and by M. and S. Izumi of an earlier result by McFadden.**

Let  $\sum a_n$  be a series with partial sums  $S_n$  and let  $p_n$  be a sequence of real constants with

$$P_n = \sum_{v=0}^n p_v, \quad p_0 > 0, \quad P_{-1} = p_{-1} = 0.$$

The series  $\sum a_n$  is said to be summable  $|N, p_n|$  if

$$\sum_{n=1}^{\infty} |t_n - t_{n-1}| < \infty,$$

where

$$(1.1) \quad t_n = \frac{1}{P_n} \sum_{v=0}^n p_{n-v} S_v.$$

We write  $P(t) = P_{[t]}$  and in the sequel we assume that  $p_n$  is nonnegative, nonincreasing and  $\lim_{n \rightarrow \infty} p_n = 0$ .

2. Let  $f(t)$  be a periodic function with period  $2\pi$  and integrable ( $L$ ) in  $(-\pi, \pi)$ . The Fourier series of  $f(t)$  is

$$\frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt) \equiv \sum_{n=0}^{\infty} A_n(t),$$

where  $a_n$  and  $b_n$  are given by the usual Euler-Fourier formulae. We write

$$\phi(t) = f(x+t) + f(x-t) - 2f(x),$$

$$\alpha(t) = \sum_{v=0}^{\infty} p_v \cos vt, \quad \beta(t) = \sum_{v=0}^{\infty} p_v \sin vt,$$

$$\alpha_n = \int_0^{\pi} \phi(t) \alpha(t) \cos nt \, dt, \quad \beta_n = \int_0^{\pi} \phi(t) \beta(t) \sin nt \, dt,$$

$$w(\delta) = \sup_{0 \leq |t| \leq \delta} |f(x+t) - f(x)|.$$

$p$  and  $q$  are mutually conjugate indices in the sense that  $1/p + 1/q = 1$ .

Recently M. Izumi and S. Izumi ([2, Th. 3]) proved the following

**THEOREM A.** *Let  $\{p_n\}$  be a positive decreasing and convex sequence tending to zero and satisfying the condition*

$$\sum_{n=1}^{\infty} p_n^p n^{p-2} < \infty, \quad (1 < p \leq 2).$$

*If the modulus of continuity  $\omega(\delta)$  of  $f$  satisfies the conditions*

$$\sum_{n=1}^{\infty} \frac{\omega\left(\frac{1}{n}\right)}{n^{1/q} P_n} < \infty,$$

*and*

$$(2.1) \quad \sum_{m=n}^{\infty} \frac{1}{m^p \left(\omega\left(\frac{1}{m}\right)\right)^{p-1}} \leq \frac{C}{\left(n\omega\left(\frac{1}{n}\right)\right)^{p-1}}$$

*then the Fourier series of  $f$  is  $|N, p_n|$  summable.*

In this note we prove that the condition (2.1) of the above theorem is *redundant* in that the assertion of the theorem holds without the condition (2.1) as well. The final result is then embodied in the following

**THEOREM.** *Let  $\{p_n - p_{n+1}\}$  be a nonincreasing sequence and*

$$(2.2) \quad \sum_{n=1}^{\infty} p_n^p n^{p-2} < C, \quad (1 < p \leq 2).$$

*If the modulus of continuity of the continuous function  $f(x)$  satisfies the condition*

$$(2.3) \quad \sum_{n=1}^{\infty} \omega(n^{-1}) P_n^{-1} n^{-1/q} < C,$$

*then the Fourier series of  $f$  is  $|N, p_n|$  summable.*

It is known that (see [4, Chapter XII, proof of Lemma 6.6]) the condition (2.2) of the theorem implies that

$$\sum_{n=1}^{\infty} P_n^p n^{-2} < C.$$

Also it is easy to show that the above condition implies the condition (2.2) of the theorem. Since  $p_n$  is nonnegative and nonincreasing

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<sup>1</sup> Throughout the paper  $C$  denotes a positive constant, not necessarily the same at each occurrence.

we have  $np_n \leq P_n$  and therefore

$$\sum_{n=1}^{\infty} p_n^p n^{p-2} \leq \sum_{n=1}^{\infty} P_n^p n^{-2} .$$

Thus the conditions (2.2) and  $\sum_{n=1}^{\infty} P_n^p n^{-2} < C$  are equivalent. In view of this equivalence it follows that the theorem established here generalises an earlier result of the author [3] as well.

3. The following lemmas are required for the proof of the theorem.

LEMMA 1. *Under the condition (2.2) of the theorem*

$$\int_0^{1/n} \omega(t)P(t^{-1})dt \leq C\omega(n^{-1})n^{-1/q} .$$

*Proof.* Remembering that the condition (2.2) of the theorem implies that

$$\sum_{n=1}^{\infty} P_n^p n^{-2} < C ,$$

we have

$$\begin{aligned} \int_0^{1/n} \omega(t)P(t^{-1})dt &\leq \sum_{v=n}^{\infty} \omega(v^{-1})P_v v^{-2} \\ &\leq \left[ \sum_{v=n}^{\infty} \{\omega(v^{-1})v^{-2+2/p}\}^q \right]^{1/q} \left[ \sum_{v=n}^{\infty} P_v^p v^{-2} \right]^{1/p} \\ &\leq C\omega(n^{-1})n^{1/p-1} , \end{aligned}$$

which is equivalent to the assertion of the lemma.

LEMMA 2. ([4, Chapter XII, Lemma 6.6]). *For the function  $\alpha(t)$  to belong to the class  $L^p(p > 1)$  it is necessary and sufficient that the condition (2.2) of the theorem is satisfied.*

LEMMA 3. ([1, Lemmas 5.11, 5.14 and 5.32]). *If  $p_n$  is non-negative and nonincreasing, then for  $0 \leq a < b \leq \infty$ ,  $0 < t \leq \pi$  and any  $n$*

$$(3.1) \quad \left| \sum_{v=a}^b p_v e^{i(n-v)t} \right| \leq CP(t^{-1}) ,$$

$$(3.2) \quad \sum_{v=n}^{\infty} \frac{v(p_v - p_{v+1})}{P_v P_{v-1}} \leq CP_{n-1}^{-1} ,$$

and

$$(3.3) \quad P(2^\lambda) \leq CP(2^{\lambda-1}),$$

as  $\lambda \rightarrow \infty$ .

LEMMA 4. ([3, Lemma 5.20]). *If  $p_n$  is nonnegative and non-increasing and if we take*

$$\gamma(t) = \sum_{v=0}^{\infty} p_v e^{ivt}$$

then for  $t$  in  $(h, \pi)$

$$|\gamma(t + 2h) - \gamma(t)| \leq Cht^{-1}P(h^{-1}).$$

LEMMA 5. ([1, see proof of Lemma 5.16]). *If  $p_n$  is nonnegative and nonincreasing,  $\lim_{n \rightarrow \infty} p_n = 0$  and  $\{p_n - p_{n+1}\}$  is nonincreasing, then*

$$\begin{aligned} & \left| \frac{1}{P_{n-1}} \int_{1/n}^{\pi} \phi(t) \left\{ \sum_{v=n}^{\infty} p_v \cos(n-v)t + \sum_{v=0}^{n-1} \frac{p_n P_v}{P_n} \cos(n-v)t \right\} dt \right| \\ & \leq C \frac{p_n}{P_n P_{n-1}} + C \left[ \frac{n(p_n - p_{n-1})}{P_n P_{n-1}} + \frac{p_n}{P_n P_{n-1}} \right] \int_{1/n}^{\pi} |\phi(t)| P(t^{-1}) t^{-1} dt. \end{aligned}$$

LEMMA 6. ([2]). *Under the conditions (2.2) and (2.3) of the theorem*

$$\sum_{n=1}^{\infty} \frac{\omega\left(\frac{1}{n}\right)}{n} \leq C.$$

4. *Proof of the theorem.* For the Fourier series

$$S_v(x) - f(x) = \frac{1}{\pi} \int_0^{\pi} \phi(t) \left( \frac{1}{2} + \sum_{k=1}^v \cos kt \right) dt,$$

so that from (1.1) and Abel's transformation we have

$$\begin{aligned} & \pi |t_n - t_{n-1}| \\ & = \left| \int_0^{\pi} \phi(t) \left\{ \sum_{v=0}^{n-1} \left( \frac{P_{n-v-1}}{P_n} - \frac{P_{n-v-2}}{P_{n-1}} \right) \cos(v+1)t \right\} dt \right| \\ & = \frac{1}{P_n P_{n-1}} \left| \int_0^{\pi} \phi(t) \sum_{v=0}^{n-1} (p_v P_n - p_n P_v) \cos(n-v)t dt \right| \\ & = \left| \frac{1}{P_{n-1}} \int_0^{\pi} \phi(t) \left( \sum_{v=0}^{\infty} p_v \cos(n-v)t \right) \right. \\ & \quad \left. - \frac{1}{P_n P_{n-1}} \int_0^{\pi} \phi(t) \left( \sum_{v=n}^{\infty} p_v P_n \cos(n-v)t + \sum_{v=0}^{n-1} p_n P_v \cos(n-v)t \right) dt \right| \end{aligned} \tag{4.1}$$

$$\begin{aligned} &\leq \frac{1}{P_{n-1}} \left| \int_0^\pi \phi(t)\alpha(t) \cos nt \, dt \right| + \frac{1}{P_{n-1}} \left| \int_0^\pi \phi(t)\beta(t) \sin nt \, dt \right| \\ &\quad + \frac{1}{P_{n-1}} \left| \int_0^{1/n} \phi(t) \sum_{v=n}^\infty p_v \cos(n-v)t \, dt \right| \\ &\quad + \frac{p_n}{P_n P_{n-1}} \left| \int_0^{1/n} \phi(t) \sum_{v=0}^{n-1} P_v \cos(n-v)t \, dt \right| \\ &\quad + \frac{1}{P_{n-1}} \left| \int_{1/n}^\pi \phi(t) \left\{ \sum_{v=n}^\infty p_v \cos(n-v)t + \sum_{v=0}^{n-1} \frac{p_n}{P_n} P_v \cos(n-v)t \right\} dt \right|. \\ &= \sum_{r=1}^5 |x_n^{(r)}|, \quad \text{say.} \end{aligned}$$

From (4.1) and the definition of the absolute Nörlund summability it is clear that for establishing the theorem we have to prove that

$$(4.2) \quad \sum_{n=2}^\infty |x_n^{(r)}| < \infty, \quad (r = 1, 2, \dots, 5).$$

Now

$$\begin{aligned} (4.3) \quad \sum_{n=2}^\infty |x_n^{(1)}| &= \sum_{\lambda=1}^\infty \sum_{n=2^{\lambda-1}+1}^{2^\lambda} |\alpha_n| P_{n-1}^{-1} \\ &\leq \sum_{\lambda=1}^\infty \left( \sum_{n=2^{\lambda-1}+1}^{2^\lambda} |\alpha_n|^q \right)^{1/q} \left( \sum_{n=2^{\lambda-1}+1}^{2^\lambda} P_{n-1}^{-p} \right)^{1/p} \\ &\leq C \sum_{\lambda=1}^\infty 2^{\lambda/p} P^{-1}(2^\lambda) \left( \sum_{n=1}^\infty |\alpha_n \sin \frac{n\pi}{2^{\lambda+1}}|^q \right)^{1/q} \end{aligned}$$

making use of (3.3) of Lemma 3.

Since the function  $\phi(t)$  is bounded in  $[0, \pi]$  and by Lemma 2, under the condition (2.2) of the theorem,  $\alpha(t) \in L^p$ , it follows that  $\phi(t)\alpha(t) \in L^p$ . Also, it is known [1] that the Fourier series of  $\phi(t+h)\alpha(t+h) - \phi(t-h)\alpha(t-h)$  is  $-4/\pi \sum \alpha_n \sin nt \sin nh$ , and therefore by Hausdorff-Young inequality we get

$$\begin{aligned} &\left( \sum_{n=1}^\infty |\alpha_n \sin nh|^q \right)^{p/q} \\ &\leq C \int_0^\pi |\phi(t+h)\alpha(t+h) - \phi(t-h)\alpha(t-h)|^p dt \\ &\leq C \int_0^\pi |\phi(t+h) - \phi(t-h)|^p |\alpha(t+h)|^p dt \\ &\quad + C \int_0^\pi |\alpha(t+h) - \alpha(t-h)|^p |\phi(t-h)|^p dt \\ (4.4) \quad &\leq C\omega^p(h) \int_0^\pi |\alpha(t+h)|^p dt + C \int_{-h}^{\pi-h} |\alpha(t+2h) - \alpha(t)|^p |\phi(t)|^p dt \\ &\leq C\omega^p(h) + C \int_{-h}^h \omega^p(|t|) |\alpha(t+2h)|^p dt \end{aligned}$$

$$\begin{aligned}
 &+ C \int_{-h}^h \omega^p(|t|) |\alpha(t)|^p dt + C \int_h^\pi |\alpha(t+2h) - \alpha(t)|^p \omega^p(t) dt \\
 &\leq C \omega^p(h) + Ch^p P^p(h^{-1}) \int_h^\pi \omega^p(t) t^{-p} dt
 \end{aligned}$$

using Lemma 4 and remembering that by virtue of Lemma 2,  $\alpha(t) \in L^p$ .

Taking  $h = \pi/2^{\lambda+1}$  in the estimate (4.4) and then substituting it in (4.3) we have

$$\begin{aligned}
 &\sum_{n=2}^\infty |x_n^{(1)}| \\
 &\leq C \sum_{\lambda=1}^\infty 2^{\lambda/p} P^{-1}(2^\lambda) \left[ \omega^p\left(\frac{\pi}{2^{\lambda+1}}\right) + 2^{-\lambda p} P^p(2^\lambda) \int_{\pi/2^{\lambda+1}}^\pi \omega^p(t) t^{-p} dt \right]^{1/p} \\
 &\leq C \sum_{\lambda=1}^\infty 2^{\lambda/p} P^{-1}(2^\lambda) \omega\left(\frac{\pi}{2^{\lambda+1}}\right) + C \sum_{\lambda=1}^\infty 2^{\lambda((1/p)-1)} \left( \int_{\pi/2^{\lambda+1}}^\pi \frac{\omega^p(t)}{t^p} dt \right)^{1/p} \\
 (4.5) \quad &\leq C \sum_{n=1}^\infty \omega\left(\frac{\pi}{n}\right) P_n^{-1} n^{-1/q} + C \sum_{\lambda=1}^\infty 2^{\lambda((1/p)-1)} \left\{ \left( \int_{1/\pi}^1 + \int_1^{2^\lambda} \right) \frac{\omega^p(t^{-1})}{t^{2-p}} dt \right\}^{1/p} \\
 &\leq C + C \sum_{\lambda=1}^\infty 2^{\lambda((1/p)-1)} \sum_{m=1}^\lambda \left( \sum_{n=2^{m-1}+1}^{2^m} \omega^p(n^{-1}) n^{p-2} \right)^{1/p} \\
 &\leq C + C \sum_{m=1}^\infty 2^{m(1-(1/p))} \omega(2^{-m}) \sum_{\lambda=m}^\infty 2^{\lambda((1/p)-1)} \\
 &\leq C + C \sum_{n=1}^\infty \omega(n^{-1}) n^{-1} \leq C,
 \end{aligned}$$

by virtue of the condition (2.3) of the theorem and Lemma 6.

Similarly, we can prove that

$$(4.6) \quad \sum_{n=2}^\infty |x_n^{(2)}| < \infty.$$

Also,

$$\begin{aligned}
 (4.7) \quad \sum_{n=2}^\infty |x_n^{(3)}| &\leq C \sum_{n=2}^\infty P_{n-1}^{-1} \int_0^{1/n} \omega(t) P(t^{-1}) dt \\
 &\leq C \sum_{n=1}^\infty \omega(n^{-1}) P_n^{-1} n^{-1/q} \leq C,
 \end{aligned}$$

by the application of (3.1) of Lemma 3, Lemma 1 and the condition (2.3) of the theorem.

For the proof of

$$(4.8) \quad \sum_{n=2}^\infty |x_n^{(4)}| < C,$$

see the proof of  $\sum_{n=1}^\infty K_n < \infty$  in [2]. Finally, by Lemma 5 we have

$$\begin{aligned}
& \sum_{n=2}^{\infty} |x_n^{(5)}| \\
\leq & C \sum_{n=1}^{\infty} \frac{p_n}{P_n P_{n-1}} + C \sum_{n=1}^{\infty} \left[ \frac{n(p_n - p_{n+1})}{P_n P_{n-1}} + \frac{p_n}{P_n P_{n-1}} \right] \int_{1/n}^{\pi} \omega(t) P(t^{-1}) t^{-1} dt \\
\leq & C \sum_{n=1}^{\infty} \frac{p_n}{P_n P_{n-1}} + C \sum_{n=1}^{\infty} \frac{n(p_n - p_{n+1})}{P_n P_{n-1}} \\
& + C \sum_{n=1}^{\infty} \frac{p_n}{P_n P_{n-1}} \sum_{v=1}^n \omega(v^{-1}) P_v v^{-1} \\
(4.9) \quad & + C \sum_{n=1}^{\infty} \frac{n(p_n - p_{n+1})}{P_n P_{n-1}} \sum_{v=1}^n \omega(v - 1) P_v v^{-1} \\
\leq & C + C \sum_{v=1}^{\infty} \omega(v^{-1}) P_v v^{-1} \sum_{n=v}^{\infty} \frac{p_n}{P_n P_{n-1}} \\
& + C \sum_{v=1}^{\infty} \omega(v^{-1}) P_v v^{-1} \sum_{n=v}^{\infty} \frac{n(p_n - p_{n+1})}{P_n P_{n-1}} \\
\leq & C + C \sum_{v=1}^{\infty} \omega(v^{-1}) v^{-1} \leq C,
\end{aligned}$$

by the application of the estimate (3.2) of Lemma 3 and Lemma 6.

Combining the estimates in (4.5) – (4.9) we find that (4.2) is established. This completes the proof of the theorem.

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