

# Pacific Journal of Mathematics

**INTERPOLATION IN  $C(\Omega)$**

BENJAMIN BAXTER WELLS, JR.

## INTERPOLATION IN $C(\Omega)$

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It is known from the work of Bade and Curtis that if  $\mathfrak{A}$  is a Banach subalgebra of  $C(\Omega)$ ,  $\Omega$  a compact Hausdorff space, and if  $\Omega$  is an  $F$ -space in the sense of Gillman and Henriksen then  $\mathfrak{A} = C(\Omega)$ . This paper is concerned with the extension of this and similar results to the setting of Grothendieck spaces ( $G$ -spaces for short). An important feature of the extension is that emphasis is shifted from the underlying topological structure of  $\Omega$  to the linear topological character of  $C(\Omega)$ .

As a corollary we show that if  $\Omega_1$  and  $\Omega_2$  are infinite compact Hausdorff spaces, then  $\Omega_1 \times \Omega_2$  is not a  $G$ -space. Consequently if  $\Omega$  is a  $G$ -space then  $C(\Omega)$  is not linearly isomorphic to  $C(\Omega \times \Omega)$ .

If  $A$  is a commutative Banach algebra whose spectrum is a totally disconnected  $G$ -space, a second corollary of our extension is that the Gelfand homomorphism is onto. This establishes for  $G$ -spaces a result due to Seever for  $N$ -spaces.

Two definitions of  $G$ -space are to be found in the literature.

(A) A Banach space  $X$  is a  $G$ -space if every weak-\* convergent sequence in  $X^*$ , the dual of  $X$ , is weakly convergent.

(B) A compact Hausdorff space  $\Omega$  is a  $G$ -space if  $C(\Omega)$  is a  $G$ -space in the sense of (A).

Unless otherwise noted we shall accept (B) as our definition.

It is known from the work of Seever [7] that if  $\Omega$  is an  $F$ -space, i.e., if disjoint open  $F_\sigma$  subsets of  $\Omega$  have disjoint closures, then  $\Omega$  is a  $G$ -space. A result due to Rudin [3] states that if  $\Omega_1$  and  $\Omega_2$  are infinite compact Hausdorff spaces then  $\Omega_1 \times \Omega_2$  is not an  $F$ -space. Corollary 2.6 is an extension of this to  $G$ -spaces. Although an example of a  $G$ -space which is not an  $F$ -space is given in [7], no necessary and sufficient topological characterization of the  $G$  property is known.

1. Preliminaries. Let  $M(\Omega)$  be the space of regular Borel measures on  $\Omega$  equipped with the total variation norm. A sequence  $\{\mu_n\}$  in  $M(\Omega)$  converges for the weak-\* topology if for each  $f$  in  $C(\Omega)$ , the space of continuous complex valued functions on  $\Omega$ , the sequence  $\{\mu_n(f)\}$  is convergent. Weak convergence of  $\{\mu_n\}$  means convergence of  $\{\gamma(\mu_n)\}$  for every  $\gamma$  in  $M^*(\Omega)$ , the dual of  $M(\Omega)$ . If  $\Omega$  is any set  $l_1(\Omega)$  will denote the Banach space of point mass measures on  $\Omega$  with the total variation norm.

A Banach subalgebra (subspace)  $\mathfrak{A}$  of  $C(\Omega)$  is a subalgebra (subspace)

of  $C(\Omega)$  under the pointwise operations and is a Banach algebra (space) such that the embedding  $\mathfrak{A} \rightarrow C(\Omega)$  is continuous.  $\mathfrak{A}$  is said to be normal if for each pair  $F_1, F_2$  of disjoint compact subsets of  $\Omega$  there is an  $f \in \mathfrak{A}$  such that  $f = 1$  on  $F_1$  and  $f = 0$  on  $F_2$ . Following [2] we call  $\mathfrak{A}$   $\varepsilon$ -normal if for each pair  $F_1, F_2$  of disjoint compact subsets of  $\Omega$  there exists an  $f \in \mathfrak{A}$  satisfying

$$(i) \quad |f(\omega) - 1| < \varepsilon, \omega \in F_1,$$

$$(ii) \quad |f(\omega)| < \varepsilon, \omega \in F_2.$$

If  $\Omega_1$  and  $\Omega_2$  are compact Hausdorff spaces the projective tensor product  $V = C(\Omega_1) \hat{\otimes} C(\Omega_2)$  is the set of all functions of the form

$$\sum_{i=1}^{\infty} f_i(x)g_i(y), \quad f_i(x) \in C(\Omega_1)$$

and  $g_i(y) \in C(\Omega_2)$  such that  $\sum_{i=1}^{\infty} \|f_i\|_{\infty} \|g_i\|_{\infty} < \infty$ . If  $h \in V$  then

$$\|h\|_V = \inf \left\{ \sum_{i=1}^{\infty} \|f_i\|_{\infty} \|g_i\|_{\infty} : h = \sum_{i=1}^{\infty} f_i g_i \right\}.$$

Two Banach spaces  $X_1$  and  $X_2$  are isomorphic if there is a one-to-one continuous linear map from  $X_1$  onto  $X_2$ . If  $X_2$  is a closed subspace of  $X_1$ , it is said to be complemented in  $X_1$  if there exists a closed subspace  $Y$  of  $X_1$  such that  $X_2 + Y = X_1$  and  $X_2 \cap Y = \{0\}$ . We write  $X_1 = X_2 \oplus Y$ .

If  $D$  is a discrete space,  $C(D)$  will denote the bounded continuous functions on  $D$ . It is well known that  $C(D)$  is isometrically isomorphic to  $C(\beta D)$  where  $\beta D$  is the Stone-Ćech compactification of  $D$ . A compact Hausdorff space is totally disconnected if there is a basis for the topology consisting of open and closed neighborhoods.

2. We shall need to recall here a criterion due to Grothendieck [5] for relative weak compactness in  $M(\Omega)$ . Namely, a bounded sequence  $\{\mu_n\}$  in  $M(\Omega)$  is relatively weakly compact if and only if for every sequence  $\{0_i\}$  of pairwise disjoint Borel sets  $\lim_{i \rightarrow \infty} \mu_n(0_i) = 0$  uniformly in  $n$ . By the Eberlein Smulian theorem this is equivalent to every subsequence of  $\{\mu_n\}$  having a weakly convergent subsequence.

**LEMMA 2.1.** *If  $\Omega$  is a  $G$ -space and  $K$  is a closed subspace of  $\Omega$ , then  $K$  is a  $G$ -space.*

*Proof.* Suppose  $\{\mu_n\}$  in  $M(K)$  is weak-\* convergent. One may regard  $\{\mu_n\}$  as a weak-\* convergent sequence in  $M(\Omega)$ . It is therefore weakly convergent as a sequence in  $M(\Omega)$ , and so by the Hahn-Banach Theorem it is a weakly convergent sequence in  $M(K)$ .

**LEMMA 2.2.** *Let  $\Omega$  be a  $G$ -space and  $X$  a dense Banach subspace such that  $X \neq C(\Omega)$ . Then for every  $M > 0$  there is a measure  $\mu$  with no atomic part such that  $\|\mu\| \geq M$  and  $\sup\{|\mu(f)| : f \in X, \|f\|_X \leq 1\} \leq 1$ .*

*Proof.* We shall write  $\mu_a$  for the atomic part of  $\mu$  and  $\mu_c$  for the continuous part. By a well known theorem of Banach there is a sequence  $\{\mu_n\}$  of measures such that  $\|\mu_n\| \geq n$  and  $\sup$

$$\{|\mu_n(f)| : f \in X, \|f\|_X \leq 1\}$$

for each  $n$ . Since  $X$  is dense in  $C(\Omega)$  setting  $\nu_n = \mu_n/\|\mu_n\|$  we have  $\lim_n \nu_n = 0$  weak- $*$  and hence  $\lim_n \nu_n = 0$  weakly since  $\Omega$  is a  $G$ -space. The natural projection  $p: M(\Omega) \rightarrow l_1(\Omega)$  given by  $p\mu = \mu_a$  is continuous and hence weakly continuous. Hence  $\lim_n \nu_{n,a} = 0$  weakly. Since in  $l_1(\Omega)$  weakly convergent sequences are norm convergent, it follows that  $\lim_n \|\nu_{n,a}\| = 0$ . Thus for an appropriate sequence of scalars  $\{c_n\}$  we have  $\lim_n \|c_n \nu_{n,c}\| = \infty$  and

$$\sup\{|c_n \nu_{n,c}(f)| : f \in X, \|f\|_X \leq 1\} \leq 1$$

for every  $n$ .

**THEOREM 2.3.** *Let  $\Omega$  be a  $G$ -space and let  $X$  be a dense Banach subspace of  $C(\Omega)$ . Then there exists a finite open covering  $U_1, \dots, U_n$  of  $\Omega$  such that  $X|_{\bar{U}_i} = C(\bar{U}_i)$ ,  $1 \leq i \leq n$ .*

*Proof.* From the compactness of  $\Omega$  it suffices to show that each point  $p$  of  $\Omega$  has a neighborhood  $U_p$  such that  $X|_{\bar{U}_p} = C(\bar{U}_p)$ . Suppose this fails for some  $p$ , and choose  $U_1$  a neighborhood of  $p$ . Let  $X_1$  denote the quotient space of  $X$  by all functions in  $X$  vanishing on  $\bar{U}_1$ . Applying Lemmas 2.1 and 2.2 it follows that there is a regular Borel measure  $\mu_1$  with no atomic part such that  $\|\mu_1\| \geq 1$ ,  $\text{supp } \mu_1 \subseteq \bar{U}_1$  and such that  $|\mu_1(f)| \leq \|f\|_{X_1} \leq \|f\|_X$  for every  $f \in X$ .

From the regularity of  $\mu_1$  we may choose open  $U_2 \subseteq U_1$ ,  $p \in U_2$  such that  $|\mu_1|(\bar{U}_1 - \bar{U}_2) > 1/2 \|\mu_1\|$ . Since  $X|_{\bar{U}_2} \neq C(\bar{U}_2)$  we may choose in the same way a  $\mu_2$  with no atomic part such that  $\text{supp } \mu_2 \subseteq \bar{U}_2$ ,  $\|\mu_2\| \geq 2$  and  $|\mu_2(f)| \leq \|f\|_X$  for all  $f \in X$ .

Continuing in this fashion, define inductively a sequence of measures  $\{\mu_n\}$  with no atomic parts such that  $\|\mu_n\| \geq n$ ,  $|\mu_n(f)| \leq \|f\|_X$  for every  $f \in X$ ,  $\text{supp } \mu_n \subseteq \bar{U}_n$  and  $|\mu_n|(\bar{U}_n - \bar{U}_{n+1}) > 1/2 \|\mu_n\|$ .

Setting  $\nu_n = \mu_n/\|\mu_n\|$  we see  $\lim_n \nu_n = 0$  weak- $*$  from the density of  $X$ . However, since  $|\nu_n|(\bar{U}_n - \bar{U}_{n+1}) > 1/2$  for each  $n$ ,  $\{\nu_n\}$  is not weakly convergent by the Grothendieck criterion. This contradiction establishes the theorem.

**REMARK.** Theorem 2.3 is the sharpest result in the sense that

for every compact Hausdorff space  $\Omega$  there is a dense Banach subspace  $X$  of  $C(\Omega)$  such that  $X \neq C(\Omega)$ . By a result of [8] (corollary 3.2 page 201) there are closed subspaces  $Y, W$  of  $C(\Omega)$  such that  $Y + W$  is dense in  $C(\Omega)$  but  $Y + W \neq C(\Omega)$ ; in the terminology of that paper every  $C(\Omega)$  contains a quasi-complemented uncomplemented subspace. Setting  $X = Y \oplus W$  we have the result.

Our next theorem is an extension to  $G$ -spaces of a result of [2]. The work is all done by the following:

LEMMA 2.4. [2] *Let  $\Omega$  be a compact Hausdorff space, and let  $\mathfrak{A}$  be a Banach subalgebra of  $C(\Omega)$  such that*

(i)  *$\mathfrak{A}$  is  $\varepsilon$ -normal for some  $\varepsilon < 1/2$ ,*

(ii) *There is an open covering  $U_1, \dots, U_n$  of  $\Omega$  such that  $\mathfrak{A}|_{\bar{U}_i} = C(\bar{U}_i)$ ,  $1 \leq i \leq n$ .*

*Then  $\mathfrak{A} = C(\Omega)$ .*

Combining this with Theorem 2.3 and the remark that density implies  $\varepsilon$ -normality we obtain:

THEOREM 2.5. *Let  $\Omega$  be a  $G$ -space, and let  $\mathfrak{A}$  be a dense Banach subalgebra of  $C(\Omega)$ . Then  $\mathfrak{A} = C(\Omega)$ .*

REMARK. As demonstrated in [2]  $\varepsilon$ -normality for some  $\varepsilon < 1/4$  and density of a Banach subspace of  $C(\Omega)$  are equivalent in case  $\Omega$  is an  $F$ -space. We do not know if "dense" may be replaced by " $\varepsilon$ -normal" in Theorem 2.5.

COROLLARY 2.6. *If  $\Omega_1$  and  $\Omega_2$  are infinite compact Hausdorff spaces then  $\Omega_1 \times \Omega_2$  is not a  $G$ -space.*

*Proof.* We need only take  $\mathfrak{A} = C(\Omega_1) \hat{\otimes} C(\Omega_2)$  and note that  $\mathfrak{A}$  is a dense Banach subalgebra of  $C(\Omega_1 \times \Omega_2)$ . ( $\mathfrak{A}$  happens to be normal as well.) But it is well known that  $\mathfrak{A} \neq C(\Omega_1 \times \Omega_2)$ .

Let  $X_1$  and  $X_2$  be Banach spaces such that  $X_2$  is a continuous linear image of  $X_1$ . It is an easy consequence of the Hahn Banach theorem that if  $X_1$  is a  $G$ -space in the sense of definition A, then so is  $X_2$ . Consequently if  $\Omega$  is a  $G$ -space then  $C(\Omega \times \Omega)$  is not even a continuous linear image of  $C(\Omega)$ . This is contrasted with a result of Milutin [6, p. 42] which states that if  $\Omega_1$  and  $\Omega_2$  are uncountable compact metric spaces then  $C(\Omega_1)$  is isomorphic to  $C(\Omega_2)$ . In particular for such  $\Omega$ ,  $C(\Omega)$  is isomorphic to  $C(\Omega \times \Omega)$ .

These notions may be of use in solving complementation problems. Suppose that  $X_2$  is a complemented subspace of  $X_1$ . Then if  $X_1$  is a  $G$ -space in the sense of definition A, so is  $X_2$ . For example, if  $D$

denotes an infinite discrete space,  $C(\beta D \times \beta D)$  may be viewed in a natural way as a closed subspace of  $C(D \times D)$ . Since  $\beta(D \times D)$  is a  $G$ -space, by the above remarks  $C(\beta D \times \beta D)$  has no complement in  $C(D \times D)$ .

**COROLLARY 2.7.** [cf. [7] corollary 2 p. 278]. *Let  $A$  be a commutative Banach algebra whose spectrum  $\Omega$  is a totally disconnected  $G$ -space. Then the Gelfand homomorphism is onto.*

*Proof.* By the Šilov idempotent theorem the image of  $A$  in  $C(\Omega)$  contains the characteristic functions of open closed sets. Hence  $A$  is a dense Banach subalgebra of  $C(\Omega)$  and the theorem applies.

**REMARK.** An interesting fact suggested by the proof of Theorem 2.3 is that if  $\Omega$  is a  $G$ -space then no normal subalgebra  $A$  of  $C(\Omega)$ , closed in the uniform norm, is such that  $C(\Omega)/A$  has countable (infinite) dimension. To see this suppose to the contrary that  $C(\Omega)/A$  has countable dimension. Recall that if  $A$  is a normal subalgebra of  $C(\Omega)$  such that every point  $p$  of  $\Omega$  has a neighborhood  $U_p$  such that  $A|_{\bar{U}_p} = C(\bar{U}_p)$  then  $A = C(\Omega)$ . Thus there is a point  $p \in \Omega$  such that for every neighborhood  $U_p$  of  $p$ ,  $A|_{\bar{U}_p} \neq C(\bar{U}_p)$ . Since  $A$  contains the constant functions, by a result of Glicksberg [4 p. 421] we may choose  $\mu_1 \in A^\perp$ ,  $\|\mu_1\| = 1$  such that  $|\mu_1|(\bar{U}_1^*) > \delta > 0$  where  $\bar{U}_1^*$  is a closed deleted neighborhood of  $p$ . By regularity of  $\mu_1$  we may choose a neighborhood  $U_2$  of  $p$  such that  $\bar{U}_2 \subseteq U_1$  and  $|\mu_1|(\bar{U}_2^*) < \delta/2$ . Again we may choose  $\mu_2 \in A^\perp$ ,  $\|\mu_2\| = 1$ , such that  $|\mu_2|(\bar{U}_2^*) > \delta > 0$ . Continuing in this fashion we get a sequence of measures  $\{\mu_n\} \in A^\perp$ ,  $\|\mu_n\| = 1$ , and a nested sequence of neighborhoods of  $p$ ,  $\{U_n\}$ ,  $\bar{U}_{n+1} \subseteq U_n$  such that  $|\mu_n|(\bar{U}_n - \bar{U}_{n+1}) > \delta/2$  for each  $n$ . By Grothendieck's criterion no subsequence of  $\{\mu_n\}$  is weakly convergent. Since  $C(\Omega)/A$  is separable, the unit ball in  $A^\perp$  is weak-\* sequentially compact. Thus a subsequence of  $\{\mu_n\}$  may be found which is weak-\* convergent and hence weakly convergent. This contradiction completes the proof.

In [7] the following theorem is proved.

**THEOREM 2.8.** *If  $\Omega$  is an  $F$ -space, and if  $X$  is a normal Banach subspace of  $C(\Omega)$ , then  $X = C(\Omega)$ .*

*Question [7].* In Theorem 2.8 can " $F$ " be replaced by " $G$ "? In the terminology of that paper is every  $G$ -space an  $N$ -space? The following may be of help in giving an answer.

**THEOREM 2.9.** *Let  $X$  be a  $G$ -space in the sense of definition A, and let  $Y$  be a closed subspace such that  $X/Y$  is separable. Then  $Y$*

is a  $G$ -space.

*Proof.* Let  $\{y_n^*\}$  denote a sequence in  $Y^*$ . It suffices to show that if  $\lim_n y_n^* = 0$  weak- $*$  then  $\{y_n^*\}$  has a subsequence  $\{y_{n_k}^*\}$  such that  $\lim_k y_{n_k}^* = 0$  weakly. Let  $x_n^*$  be any normpreserving extension of  $y_n^*$  to all of  $X$ . Since  $X/Y$  is separable, a sequence  $\{w_n\}$  in  $X$  may be found such that  $sp\{w_n\} + Y$  is dense in  $X$ . By a diagonal argument a subsequence  $\{x_{n_k}^*\}$  of  $\{x_n^*\}$  may be found such that  $\{x_{n_k}^*\}$  converges on each member of  $\{w_n\}$  and hence on  $sp\{w_n\} + Y$ . Since  $\{\|x_{n_k}^*\|\}$  is bounded,  $\{x_{n_k}^*\}$  is weak $^*$  convergent in  $X$  and hence weakly convergent. Thus  $\lim_k y_{n_k}^* = 0$  weakly.

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