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ON IDEALS IN Ω_*^u

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The objective of these notes is to study the relations between and the structure of various ideals that occur in the study of the complex bordism homology functor and actions of abelian p-groups on closed weakly complex manifolds.

Let us fix a prime integer p and denote by $K(Z_p, n)$ an Eilenberg-MacLane space of type (Z_p, n). Let

$$f: S^n \longrightarrow K(Z_p, n)$$

represent a generator of $\pi_n(K(Z_p, n)) = Z_p$ and set

$$\sigma_n = [S^n, f] \in \Omega_n^U(K(Z_p, n)) .$$

We propose to study the annihilator ideal, $A(\sigma_n) \subset \Omega_*^U$, of the U-bordism class σ_n. To this end let us denote by K the ideal which is the kernel of the natural map

$$\Phi_{(p)}: \Omega_*^U \longrightarrow H_*(BU; Z_p) .$$

Thus the elements of K are those U-manifolds all of whose Chern numbers are congruent to zero mod p. The structure of the ideal K is known as a consequence of the work of Milnor [8]. Namely, for each nonnegative integer i, there exists a "Milnor" manifold V^{2p^i-2}, of dimension $2p^i - 2$, such that

$$K = ([V^0], [V^{2p-2}], \cdots) .$$

The first elementary fact concerning the ideal $A(\sigma_n)$ is the inclusion $A(\sigma_n) \subset K$. Thus it makes sense to inquire into which of the Milnor manifolds $[V^0], [V^{2p-2}], \cdots$, actually lie in $A(\sigma_n)$. One of our objectives in this note is to establish the following two results.

THEOREM A: $A(\sigma_n) \ni p, [V^{2p-2}], \cdots, [V^{2p^{n-1}-2}]$.

THEOREM B: $A(\sigma_n) \not\ni [V^{2p^n-2}], \cdots, [V^{2p^{n+m}-2}], \cdots$.

Thus we determine completely which of the Milnor manifolds annihilate the class σ_n. While this does not determine the structure of $A(\sigma_n)$ it is a step towards that goal. The study of annihilator ideals of spherical bordism classes has been a recurrent theme throughout the investigations [5], [6], [10], [11] of the complex bordism of finite complexes. The ideal $A(\sigma_n)$ is in an appropriate sense a universal example of such an ideal.

To connect up the study of the ideals $A(\sigma_n)$ with the study of free actions of abelian p-groups on closed weakly complex manifolds we introduce the infinite dimensional lens space $\boldsymbol{L}(\boldsymbol{Z}_p, \infty) = \boldsymbol{K}(\boldsymbol{Z}_p, 1) = B\boldsymbol{Z}_p$ that classifies principal $\boldsymbol{Z}_p$ bundles. (For $p = 2$ this may be taken to be the infinite dimensional real projective space.) We let $\gamma \in \widetilde{\Omega}_1^U(B\boldsymbol{Z}_p)$ denote the class represented by the canonical map $S^1 \to B\boldsymbol{Z}_p$. The exterior product

$$\underbrace{S^1 \times \cdots \times S^1}_{n\text{-fois}} \longrightarrow \underbrace{B\boldsymbol{Z}_p \times \cdots \times B\boldsymbol{Z}_p}_{n\text{-fois}}$$

represents an n-dimensional bordism class that we denote by $\bigotimes^n \gamma \in \widetilde{\Omega}_n^U(B\boldsymbol{Z}_p^n)$. With these notations fixed we introduce a trio of ideals:

$$A_n = A(\bigotimes^n \gamma) = \{[M] \in \Omega_*^U \mid [M] \bigotimes^n \gamma = 0 \in \widetilde{\Omega}_*^U(B\boldsymbol{Z}_p^n)\}$$

I_n: defined inductively by

$$I_0 = (0)$$

$$I_n = \{[M] \in \Omega_*^U \mid [M]\gamma \in I_{n-1}\widetilde{\Omega}_*^U(B\boldsymbol{Z}_p)\}$$

$$J_n = \left\{\begin{array}{l}[M] \in \Omega_*^U \mid \text{there exists a representative} \\ M \text{ of } [M] \text{ upon which } \boldsymbol{Z}_p \underset{(n\text{-fois})}{\times \cdots \times} \boldsymbol{Z}_p \text{ acts} \\ \text{without stationary points}\,.\end{array}\right\}$$

(Note that each of these ideals depends on the prime p but that we have supressed the dependence from our notation.) The ideals J_n were introduced in [2] where the study of their structure was first undertaken. In [6] E. E. Floyd computes both the ideals I_n and J_n and shows that they coincide.

Among the elementary facts that one has is that each of the above families of ideals forms an expanding sequence, viz.

$$(0) = A_0 \subset A_1 \subset A_2 \subset \cdots$$
$$(0) = S_0 \subset S_1 \subset S_2 \subset \cdots$$
$$(0) = I_0 \subset I_1 \subset I_2 \subset \cdots$$
$$(0) = J_0 \subset J_1 \subset J_2 \subset \cdots$$

where we have written S_n for $A(\sigma_n)$. This suggests that we introduce the stable ideals

$$A = \bigcup_{n=0}^{\infty} A_n,\ S = \bigcup_{n=0}^{\infty} S_n$$
$$I = \bigcup_{n=0}^{\infty} I_n,\ \ J = \bigcup_{n=0}^{\infty} J_n\,.$$

The results relating all these ideals that we shall establish are

THEOREM C: $I_n = J_n \subset A_n \subset S_n$.

THEOREM D: $I = J = A = S = K$.

It is to be emphasized that the structure theorem of Floyd [6] for the ideals I_n and J_n essentially contains all the information that we require to establish Theorems A, C, and D. Our main contribution is Theorem B. It follows from Floyd's work that

$$I_n = ([V^0], [V^{2p-2}], \cdots, [V^{2p^{n-1}-2}]) = J_n$$

and hence Theorem A is an instant consequence of Theorem C. Thus (i.e. from [6]) we see that

$$I = ([V^0], [V^{2p-2}], \cdots, [V^{2p^m-2}], \cdots) = J .$$

As noted previously the work of Milnor [8] shows

$$K = ([V^0], [V^{2p-2}], \cdots, [V^{2p^m-2}], \cdots) .$$

and thus Theorem D follows from the equality $S = K$ (or the observation that $S_n \subset K$ for all $n \geqq 0$) and Theorem C. The remainder of these notes is devoted to the proof of Theorem B and the verification of the necessary inclusion relations between the various ideals required to prove Theorem C and complete the proof of Theorem D.

1. Preliminaries. The proof of Theorem B requires that we review in some detail certain results of [5]. To this end let us denote by M a closed weakly complex manifold. We continue to denote by p a prime and we let

$$s_{p^i-1}(c) \in H^{2p^i-2}(M; \boldsymbol{Z}_p)$$

denote the usual s_ω-symmetric function of the Chern classes of M [3] [12]. Our purposes require that we have available a procedure for evaluating the number

$$\langle a \cup s_{p^i-1}(c), [M] \rangle \in \boldsymbol{Z}_p;\ a \in H^*(M, \boldsymbol{Z}_p) ,$$

in terms of "other" invariants of M. The required procedure consists of a *Wu* type formula established in [5]. To describe this formula we must collect some elementary facts concerning the Steenrod algebra [7].

Recollections and Notations. Let $\mathscr{A}^*(p)$ denote the mod p Steenrod algebra and $\mathscr{P}^*(p)$ the algebra of reduced power operations, i.e.,

$$\mathscr{P}^*(p) = \mathscr{A}^*(p)/(\beta)$$

where (β) denotes the two sided ideal generated by β. (Recall that $\beta = Sq^1$ when $p = 2$.)

According to Milnor [7] the dual Hopf algebra $\mathscr{P}_*(p)$ is given by

$$\mathscr{P}_*(p) = \boldsymbol{Z}_p[\{\mu_i\}]$$

where

$$\deg \mu_i = 2(p^i - 1)$$

and

$$\nabla^* \mu_k = \sum_{i=0}^{k} \mu_{k-i}^{p^i} \otimes \mu_i$$

with the convention that $\mu_0 = 1$.

The duals of the classes $\{\mu_i\}$ are primitive elements of $\mathscr{P}^*(p)$ which may be defined inductively by the formulas

$$S_i = \begin{cases} P_p^1;\ i = 1\ , \\ [S_{i-1},\ P_p^{p^{i-1}}];\ i > 1\ . \end{cases}$$

These formulas determine corresponding unique elements $S_i \in \mathscr{A}^*(p)$ of degree $2(p^i - 1)$ which are primitive provided that $p \neq 2$. (Recall that for $p = 2$, $P_2^1 = Sq^2$.)

The following result may be found in [5; 1.2].

THEOREM 1.1. *Let M^m be a closed weakly complex manifold of dimension m and $a \in H^{m-2(p^i-1)}(M; \boldsymbol{Z}_p)$. Then with the preceeding notations we have*

$$\langle a \cup s_{p^i-1}(c),\ [M]\rangle = \langle S_i a,\ [M]\rangle \in \boldsymbol{Z}_p$$

at least up to a unit in $\boldsymbol{Z}_p$. **

As an indication of how we intend to apply this theorem let us suppose that we are given a closed U-manifold M^m and a map

$$f\colon M^m \longrightarrow S^k\colon m - k = 2(p^i - 1)\ ,$$

and a cohomology class $a \in H^k(M; \boldsymbol{Z})$ such that

$$f^*(\iota) = pa$$

where $\iota \in H^k(S^k; \boldsymbol{Z})$ is the fundamental class. Then according to Milnor [8] and an elementary transverse regularity argument

$$[M, f] = [V^{2p^i-2}]\sigma \in \widetilde{\Omega}_*^U(S^k)$$

(where $\sigma \in \widetilde{\Omega}_k^U(S^k)$ is the fundamental bordism class) if and only if

$$\langle pa \cup s_{p^i-1}(c),\ [M]\rangle \equiv p \bmod p^2\ .$$

Now clearly this latter condition can hold if and only if

$$\langle a \cup s_{p^i-1}(c), [M]\rangle \neq 0 \bmod p .$$

In view of Poincare duality and Theorem 1.1 we thus obtain:

THEOREM 1.2. *Suppose given a closed weakly complex manifold M^m and a map*

$$f\colon M^m \longrightarrow S^k\colon m - k = 2p^i - 2 ,$$

together with a cohomology class $a \in H^k(M; \boldsymbol{Z})$ such that

$$f^*(\iota) = pa .$$

Then

$$[M, f] = [V^{2p^i-2}]\sigma \in \widetilde{\Omega}_*^U(S^k)$$

if and only if

$$\langle S_i a, [M]\rangle \neq 0 \in \boldsymbol{Z}_p$$

i.e., if and only if

$$S_i(a) \neq 0 \in H^m(M^m; \boldsymbol{Z}_p)$$

where we have written a for its own mod p reduction. □

2. The proof of Theorem B. We continue to denote by

$$f\colon S^n \longrightarrow \boldsymbol{K}(\boldsymbol{Z}_p, n)$$

a map representing a generator of $\pi_n(\boldsymbol{K}(\boldsymbol{Z}_n, n)) \cong \boldsymbol{Z}_n$. Let us introduce the cofibration

$$\boldsymbol{K}(\boldsymbol{Z}_p, n) \xrightarrow[h]{} \boldsymbol{K}(\boldsymbol{Z}_p, h) \bigcup_f e^{n+1} \xrightarrow[g]{} S^{n+1} .$$

We then have the following elementary result:

PROPOSITION 2.1. *With the notations as above we have*

$$[M] \in A(\sigma_n)$$

if and only if

$$[M]i_{n+1} \in \operatorname{Im}\{g_*\colon \widetilde{\Omega}_*^U(\boldsymbol{K}(\boldsymbol{Z}_p, , n) \bigcup_f e^{n+1}) \longrightarrow \widetilde{\Omega}_*^U(S^{n+1})\} ,$$

where

$$i_{n+1} = [S^{n+1}, id] \in \widetilde{\Omega}_{n+1}^U(S^{n+1})$$

is the canonical class.

Proof. This results from the exact triangle

$$\begin{array}{ccc} \widetilde{\Omega}_*^U(K(Z_p, n)) & \xrightarrow[h_*]{} & \widetilde{\Omega}_*^U(K(Z_p, n) \cup_f e^{n+1}) \\ & \partial_* \nwarrow \quad \swarrow g_* & \\ & \widetilde{\Omega}_*^U(S^{n+1}) & \end{array}$$

upon recalling that $\partial_* i_{n+1} = [S^n, f] = \sigma_n$. □

Notice the elementary fact that the sequence

$$\begin{aligned} 0 = H^{n+2}(S^{n+1}; Z) \longleftarrow & \, H^{n+1}(K(Z_p, n); Z) \\ \longleftarrow & \, H^{n+1}(K(Z_p, n) \cup_f e^{n+1}; Z) \longleftarrow H^{n+1}(S^{n+1}; Z) \\ \longleftarrow & \, H^n(K(Z_p, n); Z) = 0 \end{aligned}$$

is exact, not split, and has

$$H^{n+1}(S^{n+1}; Z) \cong Z \cong H^{n+1}(K(Z_p, n) \cup_f e^{n+1}; Z)$$
$$H^{n+1}(K(Z_p, n); Z) \cong Z_p .$$

Therefore for an appropriate choice of generator

$$e_{n+1} \in H^{n+1}(K(Z_p, n) \cup_f e^{n+1}; Z)$$

we have

$$g^*(\iota) = pe_{n+1} .$$

With these notations fixed we are now prepared to take up the proof of Theorem B.

Proof of Theorem B. Let us suppose that

$$[V^{2p^i-2}] \in A(\sigma_n) .$$

Then according to, and in the notation of, Proposition 2.1 we must have

$$[V^{2p^i-2}]i_{n+1} \in \mathrm{Im}\{g_*: \widetilde{\Omega}_*^U(K(Z_p, n) \cup_f e^{n+1}) \longrightarrow \widetilde{\Omega}_*^U(S^{n+1})\} .$$

Let $[M, \varphi] \in \widetilde{\Omega}_*^U(K(Z_p, n) \cup_f e^{n+1})$ be chosen with

$$g_*[M, \varphi] = [V^{2p^i-2}]i_{n+1} .$$

Then of course

$$[M, g\varphi] = [V^{2p^i-2}]i_{n+1} .$$

Recall that

$$g^*(\iota) = pe_{n+1} .$$

Thus by setting

$$a = \varphi^*(e_{n+1}) \in H^*(M; Z)$$

we find that with this cohomology class and the map

$$g\varphi: M \longrightarrow S^{n+1}$$

we may apply Theorem 1.2 to conclude that

$$\langle S_i a, [M] \rangle \neq 0 \bmod p.$$

Of course by naturality

$$\langle S_i a, [M] \rangle = \langle \varphi^* S_i e_{n+1}, [M] \rangle$$

(where we have written e_{n+1} for its own mod p reduction). Now recall that the map

$$h^*: H^*(K(\boldsymbol{Z}_p, n) \bigcup_f e^{n+1}; \boldsymbol{Z}_p) \longrightarrow H^*(K(\boldsymbol{Z}_p, n), \boldsymbol{Z}_p)$$

is an isomorphism for $* > n + 1$. As we have already observed that $p = [V^0] \in A(\sigma_n)$ we may as well assume that $i > 0$. It is now time to note that $h^*(e_{n+1}) = \beta\theta_n$, where $\theta_n \in H^n(K(\boldsymbol{Z}_p, n); \boldsymbol{Z}_p)$ is the fundamental class. Thus

$$h^* S_i e_{n+1} = S_i \beta\theta_n .$$

Suppose now that $i > n$. Examination of the results of Cartan [1] and Serre [9] reveal that for i $> n$

$$S_i \beta\theta_n = \rho v$$

where ρ is reduction mod p and $v \in H^*(K(\boldsymbol{Z}_p, n); \boldsymbol{Z})$ an integral class. Let $u \in H(K(\boldsymbol{Z}_p, n) \bigcup_f e^{n+1}; \boldsymbol{Z})$ be an integral class with $h^* u = v$. Thus we have found

$$[V^{2p^i-2}] \in A(\sigma_n) \Longrightarrow \langle \varphi^* S_i e_{n+1}, [M] \rangle \neq 0 .$$

Since $i > n$ the preceeding discussion shows

$$\begin{aligned} \langle \varphi^* S_i e_{n+1}, [M] \rangle &= \langle \varphi^* \rho u, [M] \rangle \\ &= \langle \rho \varphi^* u, [M] \rangle \end{aligned}$$

and since $[M]$ is an integral class

$$\begin{aligned} \langle \varphi^* S_i e_{n+1}, [M] \rangle &= \rho \langle \varphi^* u, [M] \rangle \\ &= \rho \langle u, \varphi^* [M] \rangle . \end{aligned}$$

However, since $H_*(K(\boldsymbol{Z}_p, n) \bigcup_f e^{n+1}; \boldsymbol{Z})$ is all torsion for $* > n + 1$ it follows that the Kronecker index over the integers $\boldsymbol{Z}$ for $K(\boldsymbol{Z}_p, n) \bigcup_f e^{n+1}$ is always zero for $* > n + 1$. Therefore

$$\langle u, \varphi_* [M] \rangle = 0$$

and hence

$$\langle \varphi^* S_i e_{n+1}, [M] \rangle = 0$$

contrary to our previous calculation. This contradiction shows that

$[V^{2p^i-2}]$ cannot belong to the ideal $A(\sigma_n)$ for any $i > n$ which is the desired conclusion. □

3. Various inclusions. We collect here in this section the various inclusion relations required to establish Theorem C and to complete the proof of Theorem D.

The Inclusion $A_n \subset A_{n+1}$. Consider the exterior cross product

$$\tilde{\Omega}_*^U(BZ_p^n) \otimes_{\Omega_*^U} \tilde{\Omega}_*^U(BZ_p) \longrightarrow \tilde{\Omega}_*^U(BZ_p^{n+1}) .$$

Let us suppose that $[M] \in A_n$, then by naturality of the cross-product we find

$$[M] \otimes^{n+1} \gamma = [M] \otimes^n \gamma \otimes \gamma = 0 \otimes \gamma = 0$$

and hence $[M] \in A_{n+1}$ which is the desired conclusion.

The Inclusion $S_n \subset S_{n+1}$. Introduce the map

$$u_n \colon \Sigma K(Z_p, n) \longrightarrow K(Z_p, n+1)$$

that classifies the cohomology class

$$\Sigma\theta_n \in H^{n+1}(K(Z_p, n); Z_p) .$$

It is easily checked that

$$(u_n)_*(\Sigma\sigma_n) = \sigma_{n+1} .$$

Thus if $[M] \in A(\sigma_n) = S_n$ then by naturality and stability under suspension we find

$$[M]\sigma_{n+1} = (u_n)_*(\Sigma[M]\sigma_n) = 0$$

and hence $[M] \in S_{n+1}$ as required.

The Equality $S = K$. Consider the morphism of spectra

$$v \colon S \longrightarrow K(Z_p)$$

that defines the unit of the latter. Its component morphisms are the maps

$$f \colon S^n \longrightarrow K(Z_p, n) \colon n = 0, 1, 2, \cdots$$

and as

$$S_n = \ker\{f_* \colon \tilde{\Omega}_*^U(S^n) \longrightarrow \tilde{\Omega}_*^U(K(Z_p, n))\}$$

we find

$$S = \cup S_n = \ker\{v_* \colon \tilde{\Omega}_*^U(S) \longrightarrow \tilde{\Omega}_*^U(K(Z_p))\}$$

since both $\boldsymbol{S}$ and $\boldsymbol{K}(\boldsymbol{Z}_p)$ are convergent spectra.

Now $\ker v_*$ may be given another interpretation as follows. Recall that

$$\tilde{\Omega}_*^U(\boldsymbol{K}(\boldsymbol{Z}_p)) = \pi_*^s(\boldsymbol{MU} \wedge \boldsymbol{K}(\boldsymbol{Z}_p)) = \tilde{H}_*(\boldsymbol{MU}; \boldsymbol{Z}_p)$$
$$\tilde{\Omega}_*^U(\boldsymbol{S}) = \Omega_*^U ,$$

and under the identifications above we may view v_* as the natural map

$$\phi_{(p)} = v_*: \Omega_*^U \longrightarrow \tilde{H}_*(\boldsymbol{MU}; \boldsymbol{Z}_p)$$

which by the Thom isomorphism may be viewed as

$$\Phi_{(p)} = v_*: \Omega_*^U \longrightarrow \tilde{H}_*(BU; \boldsymbol{Z}_p)$$

where

$$\langle v_*[M], c_\omega \rangle = c_\omega[M]$$

for any monomial $c_\omega \in H^*(BU; \boldsymbol{Z}_p)$ in the mod p Chern classes. As

$$\langle\, , \rangle: H_*(BU; \boldsymbol{Z}_p) \otimes H^*(BU: \boldsymbol{Z}_p) \longrightarrow \boldsymbol{Z}_p$$

is a dual pairing we find

$$\ker v_* = \{[M] \in \Omega_*^U \,|\, c_\omega[M] \equiv 0 \bmod p: \text{ all } \omega\} = K$$

and the desired equality follows.

The Inclusion $I_n \subset A_n$. Let us consider first the case $n = 1$. We find

$$I_1 = \{[M] \in \Omega_*^U \,|\, [M]\gamma = 0 \in \Omega_*^U(BZ_p)\} = A_1 .$$

Thus $I_1 \subseteqq A_1$. We may therefore proceed inductively and assume that $I_m \subseteqq A_m$ for all $m < n$. We wish to conclude $I_n \subseteqq A_n$. We may as well assume that $n > 1$. Accordingly we consider the exterior product

$$\Omega_*^U(BZ_p^{n-1}) \otimes_{\Omega_*^U} \Omega_*^U(BZ_p) \longrightarrow \Omega_*^U(BZ_p^n) .$$

One readily sees that with the obvious abuse of notation that

$$\bigotimes^{n-1} \gamma \otimes \gamma = \bigotimes^n \gamma$$

in $\Omega_*^U(BZ_p^n)$. Suppose that $[M] \in I_n$. Then we have

$$[M]\gamma \in I_{n-1}\Omega_*^U(BZ_p)$$

and thus

$$[M] = \Sigma[N_i]\lambda_i$$

where

$$[N_i] \in I_{n-1},\ \lambda_i \in \tilde{\Omega}_*^U(BZ_p) .$$

Thus we find

$$[M] \bigotimes^n \gamma = \bigotimes^{n-1} \gamma \otimes [M]\gamma = \bigotimes^{n-1} \gamma \otimes \Sigma[N_i]\lambda_i$$
$$= \Sigma[N_i] \bigotimes^{n-1} \gamma \otimes \lambda_i$$

by naturality. By our inductive assumption $I_{n-1} \subseteqq A_{n-1}$ and hence

$$[N_i] \bigotimes^{n-1} \gamma = 0 \in \Omega_*^U(BZ_p^{n-1}) .$$

Therefore we find

$$[M] \bigotimes^n \gamma = \Sigma[N_i] \bigotimes^{n-1} \gamma \otimes \lambda_i = \Sigma 0 \otimes \lambda_i = 0$$

and hence $[M] \in A_n$ as was to be shown. Thus the inclusion $I_n \subset A_n$ is established inductively for all $n \geqq 0$.

The Inclusion $A_n \subset S_n$. Consider the standard map

$$q_n: BZ_p^n \longrightarrow K(Z_p, n)$$

that classifies the cohomology class

$$\theta_1 \otimes \theta_1 \otimes \cdots \otimes \theta_1 \in H^n(BZ_p^n; Z_p) .$$

It is easily checked that

$$(q_n)_*(\bigotimes^n \gamma) = \sigma_n .$$

Suppose now that $[M] \in A_n$. Then by naturality we find

$$[M]\sigma_n = (q_n)_*[M] \bigotimes^n \gamma = 0$$

and therefore $[M] \in S_n$ as desired. □

Closing Remarks. For $n = 1$ and 2 it may be shown by brute force computational techniques that

$$I_n = A_n = S_n: n = 1, 2 .$$

In view of the work of Floyd [6] the structure of all these ideals are then known. We conjecture more generally that this equality holds for all $n \geqq 0$, and in particular that

$$A_n = (p, [V^{2p-2}], \cdots, [V^{2p^{n-1}-2}]) = S_n .$$

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