Pacific Journal of Mathematics

ON IDEALS IN Ω^u_*

LARRY SMITH

Vol. 37, No. 2

February 1971

ON IDEALS IN $\Omega^{\scriptscriptstyle U}_*$

Larry Smith

The objective of these notes is to study the relations between and the structure of various ideals that occur in the study of the complex bordism homology functor and actions of abelian *p*-groups on closed weakly complex manifolds.

Let us fix a prime integer p and denote by $K(Z_p, n)$ an Eilenberg-MacLane space of type (Z_p, n) . Let

$$f: S^n \longrightarrow \mathbf{K}(\mathbf{Z}_p, n)$$

represent a generator of $\pi_n(K(Z_p, n)) = Z_p$ and set

$$\sigma_n = [S^n, f] \in \Omega^{\scriptscriptstyle U}_n(\pmb{K}(\pmb{Z}_p, n))$$
.

We propose to study the annihilator ideal, $A(\sigma_n) \subset \Omega_*^{U}$, of the U-bordism class σ_n . To this end let us denote by K the ideal which is the kernel of the natural map

$$\Phi_{(p)}: \Omega^{\scriptscriptstyle U}_* \longrightarrow H_*(BU; \boldsymbol{Z}_p)$$
 .

Thus the elements of K are those U-manifolds all of whose Chern numbers are congruent to zero mod p. The structure of the ideal Kis known as a consequence of the work of Milnor [8]. Namely, for each nonnegative integer i, there exists a "Milnor" manifold V^{2p^i-2} , of dimension $2p^i - 2$, such that

$$K = ([V^{\circ}], [V^{2p-2}], \cdots)$$
 .

The first elementary fact concerning the ideal $A(\sigma_n)$ is the inclusion $A(\sigma_n) \subset K$. Thus it makes sense to inquire into which of the Milnor manifolds $[V^0], [V^{2p-2}], \cdots$, actually lie in $A(\sigma_n)$. One of our objectives in this note is to establish the following two results.

THEOREM A:
$$A(\sigma_n) \ni p$$
, $[V^{2p^{-2}}]$, ..., $[V^{2p^{n-1}-2}]$.
THEOREM B: $A(\sigma_n) \ni [V^{2p^{n-2}}]$, ..., $[V^{2p^{n+m-2}}]$, ...,

Thus we determine completely which of the Milnor manifolds annihilate the class σ_n . While this does not determine the structure of $A(\sigma_n)$ it is a step towards that goal. The study of annihilator ideals of spherical bordism classes has been a recurrent theme throughout the investigations [5], [6], [10], [11] of the complex bordism of finite complexes. The ideal $A(\sigma_n)$ is in an appropriate sense a universal example of such an ideal. To connect up the study of the ideals $A(\sigma_n)$ with the study of free actions of abelian *p*-groups on closed weakly complex manifolds we introduce the infinite dimensional lens space $L(Z_p, \infty) = K(Z_p, 1) =$ BZ_p that classifies principal Z_p bundles. (For p = 2 this may be taken to be the infinite dimensional real projective space.) We let $\gamma \in \widetilde{\mathcal{Q}}_1^U(BZ_p)$ denote the class represented by the canonical map $S^1 \to$ BZ_p . The exterior product

$$\underbrace{S^{1} \times \cdots \times S^{1}}_{n-\text{fois}} \longrightarrow B \underbrace{Z_{p} \times \cdots \times B}_{n-\text{fois}} B Z_{p}$$

represents an *n*-dimensional bordism class that we denote by $\bigotimes^n \gamma \in \widetilde{Q}_n^U(BZ_p^n)$. With these notations fixed we introduce a trio of ideals:

$$egin{aligned} &A_n = A(igodots^n \gamma) = \{[M] \in \mathcal{Q}_*^{\scriptscriptstyle U} | [M] igodots^n \gamma = 0 \in \mathcal{Q}_*^{\scriptscriptstyle U}(Boldsymbol{Z}_p^{\scriptscriptstyle n})\} \ &I_n : ext{ defined inductively by} \ &I_0 = (0) \ &I_n = \{[M] \in \mathcal{Q}_*^{\scriptscriptstyle U} | [M] \gamma \in I_{n-1} \widetilde{\mathcal{Q}}_*^{\scriptscriptstyle U}(Boldsymbol{Z}_p)\} \ &J_n = \left\{ \begin{matrix} [M] \in \mathcal{Q}_*^{\scriptscriptstyle U} | [M] \gamma \in I_{n-1} \widetilde{\mathcal{Q}}_*^{\scriptscriptstyle U}(Boldsymbol{Z}_p)
brace \\ &M & of & [M] \circ \mathcal{Q}_*^{\scriptscriptstyle U} | ext{ there exists a representative} \\ &M & of & [M] & upon & \text{which} & oldsymbol{Z}_p \times \cdots \times oldsymbol{Z}_p & ext{acts} \\ & & \text{ (n-fois)} \end{array}
ight\} \end{aligned}$$

(Note that each of these ideals depends on the prime p but that we have supressed the dependence from our notation.) The ideals J_n were introduced in [2] where the study of their structure was first undertaken. In [6] E. E. Floyd computes both the ideals I_n and J_n and shows that they coincide.

Among the elementary facts that one has is that each of the above families of ideals forms an expanding sequence, viz.

$$(0) = A_0 \subset A_1 \subset A_2 \subset \cdots$$
$$(0) = S_0 \subset S_1 \subset S_2 \subset \cdots$$
$$(0) = I_0 \subset I_1 \subset I_2 \subset \cdots$$
$$(0) = J_0 \subset J_1 \subset J_2 \subset \cdots$$

where we have written S_n for $A(\sigma_n)$. This suggests that we introduce the stable ideals

$$egin{aligned} A &= egin{smallmatrix} \overset{igodots}{\mathop{}_{n=0}} A_n, \, S &= egin{smallmatrix} \overset{igodots}{\mathop{}_{n=0}} S_n \ I &= egin{smallmatrix} \overset{igodots}{\mathop{}_{n=0}} I_n, \ J &= egin{smallmatrix} \overset{igodots}{\mathop{}_{n=0}} J_n \ . \end{aligned}$$

The results relating all these ideals that we shall establish are

THEOREM C: $I_n = J_n \subset A_n \subset S_n$.

THEOREM D: I = J = A = S = K.

It is to be emphasized that the structure theorem of Floyd [6] for the ideals I_n and J_n essentially contains all the information that we require to establish Theorems A, C, and D. Our main contribution is Theorem B. It follows from Floyd's work that

$$I_n = ([V^0], [V^{2p-2}], \dots, [V^{2p^{n-1}-2}]) = J_n$$

and hence Theorem A is an instant consequence of Theorem C. Thus (i.e. from [6]) we see that

$$I = ([V^0], [V^{2p-2}], \cdots, [V^{2p^m-2}], \cdots) = J$$
 .

As noted previously the work of Milnor [8] shows

$$K = ([V^0], [V^{2p-2}], \cdots, [V^{2p^m-2}], \cdots)$$
.

and thus Theorem D follows from the equality S = K (or the observation that $S_n \subset K$ for all $n \ge 0$) and Theorem C. The remainder of these notes is devoted to the proof of Theorem B and the verification of the necessary inclusion relations between the various ideals required to prove Theorem C and complete the proof of Theorem D.

1. Preliminaries. The proof of Theorem B requires that we review in some detail certain results of [5]. To this end let us denote by M a closed weakly complex manifold. We continue to denote by p a prime and we let

$$s_{p^{i}-1}(c) \in H^{2p^{i}-2}(M; \mathbb{Z}_p)$$

denote the usual s_{ω} -symmetric function of the Chern classes of M [3] [12]. Our purposes require that we have available a procedure for evaluating the number

$$\langle a \cup s_{p^i-1}(c), [M] \rangle \in Z_p; a \in H^*(M, Z_p)$$
,

in terms of "other" invariants of M. The required procedure consists of a Wu type formula established in [5]. To describe this formula we must collect some elementary facts concerning the Steenrod algebra [7].

Recollections and Notations. Let $\mathscr{N}^*(p)$ denote the mod p Steenrod algebra and $\mathscr{P}^*(p)$ the algebra of reduced power operations, i.e.,

$$\mathscr{P}^{*}(p) = \mathscr{M}^{*}(p)/(\beta)$$

where (β) denotes the two sided ideal generated by β . (Recall that $\beta = Sq^1$ when p = 2.)

According to Milnor [7] the dual Hopf algebra $\mathscr{P}_*(p)$ is given by

 $\mathscr{P}_*(p) = \mathbf{Z}_p[\{\mu_i\}]$

where

$$\deg \mu_i = 2(p^i - 1)$$

and

$$abla^*\mu_k = \sum\limits_{i=0}^k \mu_{k-i}^{p^i} \otimes \mu_i$$

with the convention that $\mu_0 = 1$.

The duals of the classes $\{\mu_i\}$ are primitive elements of $\mathscr{P}^*(p)$ which may be defined inductively by the formulas

$$S_i = egin{cases} P_p^{_1};\, i=1 \ , \ [S_{i-1},\, p_p^{p^{i-1}}];\, i>1 \ . \end{cases}$$

These formulas determine corresponding unique elements $S_i \in \mathscr{H}^*(p)$ of degree $2(p^i - 1)$ which are primitive provided that $p \neq 2$. (Recall that for p = 2, $P_2^1 = Sq^2$.)

The following result may be found in [5; 1.2].

THEOREM 1.1. Let M^m be a closed weakly complex manifold of dimension m and $a \in H^{m-2(p^{i}-1)}(M; \mathbb{Z}_p)$. Then with the preceeding notations we have

$$\langle a \cup s_p i_{-1}(c), [M]
angle = \langle S_i a, [M]
angle \in Z_p$$

at least up to a unit in Z_p . *

As an indication of how we intend to apply this theorem let us suppose that we are given a closed U-manifold M^m and a map

$$f: M^m \longrightarrow S^k: m - k = 2(p^i - 1)$$
 ,

and a cohomology class $a \in H^k(M; \mathbb{Z})$ such that

$$f^*(\iota) = pa$$

where $\iota \in H^k(S^k; \mathbb{Z})$ is the fundamental class. Then according to Milnor [8] and an elementary transverse regularity argument

$$\llbracket M, f
bracket = \llbracket V^{2p^i-2}
bracket \sigma \in \widetilde{\mathcal{Q}}^{\scriptscriptstyle U}_*(S^k)$$

(where $\sigma \in \widetilde{\Omega}_k^U(S^k)$ is the fundamental bordism class) if and only if

$$ig\langle pa\cup s_{{}_{p^i-1}}\!(c),\,[M]ig
angle\equiv p mod p^2$$
 .

Now clearly this latter condition can hold if and only if

$$\langle a \cup s_{p^{i}-1}(c), [M]
angle
eq 0 mod p \ .$$

In view of Poincare duality and Theorem 1.1 we thus obtain:

THEOREM 1.2. Suppose given a closed weakly complex manifold M^m and a map

$$f: M^{m} \longrightarrow S^{k}: m - k = 2p^{i} - 2$$
 ,

together with a cohomology class $a \in H^k(M; \mathbb{Z})$ such that

$$f^*(\iota) = pa$$
.

Then

$$\llbracket M, f \rrbracket = \llbracket V^{2p^i-2} \rrbracket \sigma \in \widetilde{\mathcal{Q}}^{\scriptscriptstyle U}_*(S^k)$$

if and only if

$$\langle S_i a, [M] \rangle \neq 0 \in \mathbb{Z}_p$$

i.e., if and only if

$$S_i(a)
eq 0 \in H^m(M^m; \boldsymbol{Z}_p)$$

where we have written a for its own mod p reduction. \Box

2. The proof of Theorem B. We continue to denote by

 $f: S^n \longrightarrow K(\mathbb{Z}_p, n)$

a map representing a generator of $\pi_n(K(Z_n, n)) \cong Z_n$. Let us introduce the cofibration

$$K(\mathbf{Z}_p, n) \xrightarrow{h} K(\mathbf{Z}_p, h) \bigcup_f e^{n+1} \xrightarrow{g} S^{n+1}$$
.

We then have the following elementary result:

PROPOSITION 2.1. With the notations as above we have

$$[M] \in A(\sigma_n)$$

if and only if

$$[M]i_{n+1} \in \operatorname{Im} \{g_* \colon \widetilde{\mathcal{Q}}^U_*(K(Z_p, , n) \bigcup_f e^{n+1}) \longrightarrow \widetilde{\mathcal{Q}}^U_*(S^{n+1})\},$$

where

$$i_{n+1} = [S^{n+1}, \, id] \in \widetilde{arPi}^{_U}_{n+1}(S^{n+1})$$

is the canonical class.

Proof. This results from the exact triangle

$$\widetilde{\mathcal{Q}}_{*}^{U}(\boldsymbol{K}(\boldsymbol{Z}_{p}, n)) \xrightarrow{h_{*}} \widetilde{\mathcal{Q}}_{*}^{U}(\boldsymbol{K}(\boldsymbol{Z}_{p}, n) \bigcup_{f} e^{n+1})$$

$$\overbrace{\mathcal{Q}}_{*} \swarrow g_{*}$$

$$\widetilde{\mathcal{Q}}_{*}^{U}(S^{n+1})$$

upon recalling that $\partial_* i_{n+1} = [S^n, f] = \sigma_n$. \square

Notice the elementary fact that the sequence

$$0 = H^{n+2}(S^{n+1}; \mathbb{Z}) \longleftarrow H^{n+1}(\mathbb{K}(\mathbb{Z}_p, n); \mathbb{Z})$$

$$\longleftarrow H^{n+1}(\mathbb{K}(\mathbb{Z}_p, n) \bigcup_f e^{n+1}; \mathbb{Z}) \longleftarrow H^{n+1}(S^{n+1}; \mathbb{Z})$$

$$\longleftarrow H^n(\mathbb{K}(\mathbb{Z}_p, n); \mathbb{Z}) = 0$$

is exact, not split, and has

$$egin{aligned} H^{n+1}(S^{n+1};oldsymbol{Z})&\congoldsymbol{Z}&\cong H^{n+1}(oldsymbol{K}(oldsymbol{Z}_p,n)igcup_f e^{n+1};oldsymbol{Z})\ &H^{n+1}(oldsymbol{K}(oldsymbol{Z}_p,n);oldsymbol{Z})&\congoldsymbol{Z}_p\;. \end{aligned}$$

Therefore for an appropriate choice of generator

$$e_{n+1} \in H^{n+1}(K(Z_p, n) \bigcup_{f} e^{n+1}; Z)$$

we have

$$g^*(\iota) = p e_{n+1}$$
 .

With these notations fixed we are now prepared to take up the proof of Theorem B.

Proof of Theorem B. Let us suppose that

 $[V^{2p^{i}-2}] \in A(\sigma_n)$.

Then according to, and in the notation of, Proposition 2.1 we must have

$$[V^{2p^{i}-2}]i_{n+1} \in \operatorname{Im} \{g_* \colon \widetilde{\mathcal{Q}}^U_*(K(Z_p, n) \bigcup_f e^{n+1}) \longrightarrow \widetilde{\mathcal{Q}}^U_*(S^{n+1})\}$$

Let $[M, \varphi] \in \widetilde{\Omega}^{U}_{*}(K(\mathbb{Z}_{p}, n) \bigcup_{f} e^{n+1})$ be chosen with

$$g_*[M, \varphi] = [V^{2p^i-2}]i_{n+1}$$
 .

Then of course

$$[M, g\varphi] = [V^{2p^i-2}]i_{n+1}$$

Recall that

$$g^*(\iota) = p e_{n+1}$$
 .

Thus by setting

$$a = arphi^*(e_{n+1}) \in H^*(M; oldsymbol{Z})$$

we find that with this cohomology class and the map

 $g\varphi \colon M \longrightarrow S^{n+1}$

we may apply Theorem 1.2 to conclude that

 $\langle S_i a, [M] \rangle \neq 0 \mod p.$

Of course by naturality

$$ig \langle S_i a, \, [M] ig
angle = ig arphi^* S_i e_{n+1}, \, [M] ig
angle$$

(where we have written e_{n+1} for its own mod p reduction). Now recall that the map

$$h^*: H^*(K(\mathbb{Z}_p, n) \bigcup_f e^{n+1}; \mathbb{Z}_p) \longrightarrow H^*(K(\mathbb{Z}_p, n), \mathbb{Z}_p)$$

is an isomorphism for * > n + 1. As we have already observed that $p = [V^{\circ}] \in A(\sigma_n)$ we may as well assume that i > 0. It is now time to note that $h^*(e_{n+1}) = \beta \theta_n$, where $\theta_n \in H^n(K(\mathbf{Z}_p, n); \mathbf{Z}_p)$ is the fundamental class. Thus

$$h^*S_ie_{n+1}=S_ieta heta_n$$
 .

Suppose now that i > n. Examination of the results of Cartan [1] and Serre [9] reveal that for i > n

$$S_ieta heta_n=
ho v$$

where ρ is reduction mod p and $v \in H^*(K(\mathbb{Z}_p, n); \mathbb{Z})$ an integral class. Let $u \in H(K(\mathbb{Z}_p, n) \bigcup_f e^{n+1}; \mathbb{Z})$ be an integral class with $h^*u = v$. Thus we have found

$$[V^{{}_{2p^i-2}}] \in A(\sigma_{\scriptscriptstyle n}) \Longrightarrow ig< \varphi^*S_i e_{{}_{n+1}}, [M] ig>
eq 0$$
 .

Since i > n the preceeding discussion shows

$$ig< arphi^*S_i e_{n+1}, \ [M] ig> = ig< arphi^*
ho u, \ [M] ig> \ = ig< arphi^* u, \ [M] ig>$$

and since [M] is an integral class

However, since $H_*(K(\mathbb{Z}_p, n) \bigcup_f e^{n+1}; \mathbb{Z})$ is all torsion for * > n + 1 it follows that the Kronecker index over the integers \mathbb{Z} for $K(\mathbb{Z}_p, n) \bigcup_f e^{n+1}$ is always zero for * > n + 1. Therefore

$$\langle u, \varphi_*[M] \rangle = 0$$

and hence

$$\langle \varphi^* S_i e_{n+1}, [M] \rangle = 0$$

contrary to our previous calculation. This contradiction shows that

 $[V^{2p^{i}-2}]$ cannot belong to the ideal $A(\sigma_n)$ for any i>n which is the desired conclusion. \square

3. Various inclusions. We collect here in this section the various inclusion relations required to establish Theorem C and to complete the proof of Theorem D.

The Inclusion $A_n \subset A_{n+1}$. Consider the exterior cross product

$$\widetilde{\mathcal{Q}}^{\scriptscriptstyle U}_*(BZ_p^{\,n}) \bigotimes_{{}^{\scriptscriptstyle U}} \widetilde{\mathcal{Q}}^{\scriptscriptstyle U}_*(BZ_p) \longrightarrow \widetilde{\mathcal{Q}}^{\scriptscriptstyle U}_*(BZ_p^{\,n+1}) \ .$$

Let us suppose that $[M] \in A_n$, then by naturality of the cross-product we find

$$[M] \bigotimes^{n+1} \gamma = [M] \bigotimes^n \gamma \otimes \gamma = 0 \otimes \gamma = 0$$

and hence $[M] \in A_{n+1}$ which is the desired conclusion.

The Inclusion $S_n \subset S_{n+1}$. Introduce the map

 $u_n: \Sigma K(\mathbf{Z}_p, n) \longrightarrow K(\mathbf{Z}_p, n+1)$

that classifies the cohomology class

$$\Sigma \theta_n \in H^{n+1}(K(Z_p, n); Z_p)$$
.

It is easily checked that

$$(u_n)_*(\Sigma\sigma_n) = \sigma_{n+1}$$
.

Thus if $[M] \in A(\sigma_n) = S_n$ then by naturality and stability under suspension we find

$$[M]\sigma_{n+1} = (u_n)_*(\Sigma[M]\sigma_n) = 0$$

and hence $[M] \in S_{n+1}$ as required.

The Equality S = K. Consider the morphism of spectra

$$v: S \longrightarrow K(Z_p)$$

that defines the unit of the latter. Its component morphisms are the maps

$$f: S^n \longrightarrow K(Z_p, n): n = 0, 1, 2, \cdots$$

and as

$$S_n = \ker \{ f_* \colon \widetilde{\mathcal{Q}}^{\scriptscriptstyle U}_*(S^n) \longrightarrow \widetilde{\mathcal{Q}}^{\scriptscriptstyle U}_*(\pmb{K}(\pmb{Z}_p, n)) \}$$

we find

$$S = \bigcup S_n = \ker \{ v_* \colon \widetilde{\mathcal{Q}}^{\scriptscriptstyle U}_*(S) \longrightarrow \widetilde{\mathcal{Q}}^{\scriptscriptstyle U}_*(K(Z_p)) \}$$

since both S and $K(Z_p)$ are convergent spectra.

Now ker v_* may be given another interpretation as follows. Recall that

$$\widetilde{arPsi}^{\scriptscriptstyle U}_*(\pmb{K}(\pmb{Z}_p)) = \pi^s_*(\pmb{M} oldsymbol{U} \wedge \pmb{K}(\pmb{Z}_p)) = \widetilde{H}_*(\pmb{M} oldsymbol{U}; \pmb{Z}_p) \ \widetilde{arDsigma}^{\scriptscriptstyle U}_*(\pmb{S}) = arDsigma^{\scriptscriptstyle U}_* \;,$$

and under the identifications above we may view v_* as the natural map

 $\phi_{\scriptscriptstyle (p)} = v_* \colon \Omega^{\scriptscriptstyle V}_* \longrightarrow \widetilde{H}_*(MU;Z_p)$

which by the Thom isomorphism may be viewed as

$${ { \varPhi}_{\scriptscriptstyle (p)} } = v_* { : } { arGamma}^{\scriptscriptstyle U}_* { \longrightarrow } { \widetilde{H}}_* (BU; { old Z}_p)$$

where

$$\langle v_*[M], \, c_\omega
angle = c_\omega[M]$$

for any monomial $c_{\omega} \in H^*(BU; \mathbb{Z}_p)$ in the mod p Chern classes. As

 \langle , \rangle : $H_*(BU; \mathbf{Z}_p) \otimes H^*(BU: \mathbf{Z}_p) \longrightarrow \mathbf{Z}_p$

is a dual pairing we find

$$\ker v_* = \{[M] \in \Omega^{\scriptscriptstyle U}_* | c_{\scriptscriptstyle \omega}[M] \equiv 0 \bmod p: \text{ all } \omega\} = K$$

and the desired equality follows.

The Inclusion $I_n \subset A_n$. Let us consider first the case n = 1. We find

$$I_1 = \{ [M] \in \mathcal{Q}^{\scriptscriptstyle U}_* | \, [M] \gamma = 0 \in \mathcal{Q}^{\scriptscriptstyle U}_* (B \boldsymbol{Z}_p) \} = A_1$$
 .

Thus $I_1 \subseteq A_1$. We may therefore proceed inductively and assume that $I_m \subseteq A_m$ for all m < n. We wish to conclude $I_n \subseteq A_n$. We may as well assume that n > 1. Accordingly we consider the exterior product

$$\Omega^{\scriptscriptstyle U}_*(B{\boldsymbol Z}_p^{n-1})\bigotimes_{{\scriptscriptstyle {\mathcal Q}}_*^{\scriptscriptstyle U}} {\mathscr Q}^{\scriptscriptstyle U}_*(B{\boldsymbol Z}_p) \longrightarrow {\mathscr Q}^{\scriptscriptstyle U}_*(B{\boldsymbol Z}_p^{n})$$
.

One readily sees that with the obvious abuse of notation that

$$\bigotimes^{n-1} \gamma \bigotimes \gamma = \bigotimes^n \gamma$$

in $\Omega^{\mathcal{Y}}_*(BZ_p^n)$. Suppose that $[M] \in I_n$. Then we have

 $[M]\gamma \in I_{n-1} \Omega^{\scriptscriptstyle U}_*(BZ_p)$

and thus

$$[M] = \Sigma[N_i]\lambda_i$$

where

 $[N_i] \in I_{n-1}, \, \lambda_i \in \widetilde{\mathcal{Q}}^{\scriptscriptstyle U}_*(BZ_p)$.

Thus we find

$$\begin{split} \llbracket M \rrbracket \bigotimes^{n} \gamma &= \bigotimes^{n-1} \gamma \bigotimes \llbracket M \rrbracket \gamma &= \bigotimes^{n-1} \gamma \bigotimes \varSigma \llbracket N_i \rrbracket \lambda_i \\ &= \varSigma \llbracket N_i \rrbracket \bigotimes^{n-1} \gamma \bigotimes \lambda_i \end{split}$$

by naturality. By our inductive assumption $I_{n-1} \subseteq A_{n-1}$ and hence

$$[N_i]igotimes^{n-1}\gamma = 0 \in arOmega_*^{\scriptscriptstyle U}(BZ_p^{\scriptscriptstyle n-1})$$
 .

Therefore we find

$$[M] \bigotimes^n \gamma = \varSigma[N_i] \bigotimes^{n-1} \gamma \otimes \lambda_i = \varSigma 0 \otimes \lambda_i = 0$$

and hence $[M] \in A_n$ as was to be shown. Thus the inclusion $I_n \subset A_n$ is established inductively for all $n \ge 0$.

The Inclusion $A_n \subset S_n$. Consider the standard map

 $q_n: BZ_p^n \longrightarrow K(Z_p, n)$

that classifies the cohomology class

$$heta_1 \otimes heta_1 \otimes \cdots \otimes heta_1 \in H^n(BZ_p^n; Z_p)$$
 .

It is easily checked that

$$(q_n)_*(\bigotimes^n \gamma) = \sigma_n$$

Suppose now that $[M] \in A_n$. Then by naturality we find

$$[M]\sigma_n = (q_n)_*[M] \bigotimes^n \gamma = 0$$

and therefore $[M] \in S_n$ as desired. \square

Closing Remarks. For n = 1 and 2 it may be shown by brute force computational techniques that

$$I_n=A_n=S_n$$
: $n=1,\,2$.

In view of the work of Floyd [6] the structure of all these ideals are then known. We conjecture more generally that this equality holds for all $n \ge 0$, and in particular that

$$A_n = (p, [V^{2p-2}], \cdots, [V^{2p^{n-1}-2}]) = S_n$$
.

References

1. H. Cartan, Algebres d'Eilenberg-MacLane et Homotopie, Seminar H. Cartan 1954/55, E.N.S. Paris.

2. P. E. Conner and E. E. Floyd, *Differentiable Periodic Maps*, Springer-Verlag, New York, 1964.

3. _____, The Relation of Cobordism Theories to K-Theories, Springer Lecture Notes in Math. No. 28, 1966.

4. P. E. Conner and L. Smith, On the Complex Bordism of Finite Complexes, I.H.E.S. Journal de Math., No. 37, (1970), 117-221.

_____, On the Complex Bordism of Finite Complexes II, U.VA. Preprint 1970.
 E. E. Floyd, Actions of Z^k_p Without Stationary Points, (to appear).

7. J. W. Milnor, On the Steenrod Algebra and its Dual, Annals of Math., 67 (1958), 150-171.

8. _____, On the Cobordism Ring Ω^* and a Complex Analog, Amer. J. Math., 82 (1960), 505-521.

9. J-P. Serre, Cohomologie modulo deux des complexes d'Eilenberg-MacLane, Comm. Math. Helv., **27** (1953), 198-231.

10. L. Smith, An Application of Complex bordism to the stable homotopy groups of spheres, Bull. Amer. Math. Soc., **76** (1970), 601-604.

11. ____, On Realizing Complex Bordism Modules, Amer. J. Math., **92** (1970), 793-856.

12. R. E. Stong, Notes on Cobordism Theory, Princeton University Press, 1968.

Received May 4, 1970.

Institute des Hautes Études Scientifiques, 91-Bures-sur-Yvette, France and

THE UNIVERSITY OF VIRGINIA

PACIFIC JOURNAL OF MATHEMATICS

EDITORS

H. SAMELSON Stanford University Stanford, California 94305 J. DUGUNDJI Department of Mathematics University of Southern California Los Angeles, California 90007

RICHARD ARENS

University of California Los Angeles, California 90024

ASSOCIATE EDITORS

E. F. BECKENBACH

B. H. NEUMANN

F. WOLE

K. YOSHIDA

SUPPORTING INSTITUTIONS

UNIVERSITY OF BRITISH COLUMBIA CALIFORNIA INSTITUTE OF TECHNOLOGY UNIVERSITY OF CALIFORNIA MONTANA STATE UNIVERSITY UNIVERSITY OF NEVADA NEW MEXICO STATE UNIVERSITY OREGON STATE UNIVERSITY UNIVERSITY OF OREGON OSAKA UNIVERSITY UNIVERSITY OF SOUTHERN CALIFORNIA STANFORD UNIVERSITY UNIVERSITY OF TOKYO UNIVERSITY OF UTAH WASHINGTON STATE UNIVERSITY UNIVERSITY OF WASHINGTON

AMERICAN MATHEMATICAL SOCIETY CHEVRON RESEARCH CORPORATION NAVAL WEAPONS CENTER

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies.

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be in typed form or offset-reproduced, (not dittoed), double spaced with large margins. Underline Greek letters in red, German in green, and script in blue. The first paragraph or two must be capable of being used separately as a synopsis of the entire paper. The editorial "we" must not be used in the synopsis, and items of the bibliography should not be cited there unless absolutely necessary, in which case they must be identified by author and Journal, rather than by item number. Manuscripts, in duplicate if possible, may be sent to any one of the four editors. Please classify according to the scheme of Math. Rev. Index to Vol. **39**. All other communications to the editors should be addressed to the managing editor, Richard Arens, University of California, Los Angeles, California, 90024.

50 reprints are provided free for each article; additional copies may be obtained at cost in multiples of 50.

The Pacific Journal of Mathematics is published monthly. Effective with Volume 16 the price per volume (3 numbers) is \$8.00; single issues, \$3.00. Special price for current issues to individual faculty members of supporting institutions and to individual members of the American Mathematical Society: \$4.00 per volume; single issues \$1.50. Back numbers are available.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley, California, 94708.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), 7-17, Fujimi 2-chome, Chiyoda-ku, Tokyo, Japan.

C. R. HOBBY University of Washington Seattle, Washington 98105

Pacific Journal of Mathematics Vol. 37, No. 2 February, 1971

Charles Compton Alexander, Semi-developable spaces and quotient images of	
metric spaces	277
Ram Prakash Bambah and Alan C. Woods, <i>On a problem of Danzer</i>	295
John A. Beekman and Ralph A. Kallman, <i>Gaussian Markov expectations and</i> related integral equations	303
Frank Michael Cholewinski and Deborah Tepper Haimo, <i>Inversion of the Hankel</i>	
potential transform	319
John H. E. Cohn. <i>The diophantine equation</i>	
$Y(Y+1)(Y+2)(Y+3) = 2X(X+1)(X+2)(X+3) \dots$	331
Philip C. Curtis. Ir. and Henrik Stetkaer. A factorization theorem for analytic	001
functions operating in a Banach algebra	337
Doyle Otis Cutler and Paul F. Dubois. <i>Generalized final rank for arbitrary limit</i>	
ordinals	345
Keith A Ekblaw. The functions of hounded index as a subspace of a space of	515
entire functions	353
Dennis Michael Girord. The asymptotic behavior of norms of powers of	555
absolutely convergent Fourier series	357
Lohn Crossony An approximation theory for alliptic and ratio forms on Hilbert	557
John Gregory, An approximation theory for elliptic quadratic forms on Hilbert	
spaces. Application to the eigenvalue problem for compact quadratic	383
Doub C. Kainon, Universal as efficient the server for a second line the server	585
raul C. Kainen, Universal coefficient theorems for generalized homology and	207
Alla Lange Lange et Lange Danald Datherford, N. January C. J. J.	397
Aldo Joram Lazar and James Ronald Retnerford, <i>Nuclear spaces, Schauder</i>	400
bases, and Choquer simplexes	409
David Lowell Lovelady, Algebraic structure for a set of nonlinear integral	401
operations	421
John McDonald, <i>Compact convex sets with the equal support</i> property	429
Forrest Miller, <i>Quasivector topologies</i>	445
Marion Edward Moore and Arthur Steger, <i>Some results on completability in</i>	
commutative rings	453
A. P. Morse, <i>Taylor's theorem</i>	461
Richard E. Phillips, Derek J. S. Robinson and James Edward Roseblade,	
Maximal subgroups and chief factors of certain generalized soluble	
groups	475
Doron Ravdin, On extensions of homeomorphisms to homeomorphisms	481
John William Rosenthal, <i>Relations not determining the structure of</i> L	497
Prem Lal Sharma, <i>Proximity bases and subbases</i>	515
Larry Smith, <i>On ideals in</i> Ω^{μ}_{*}	527
Warren R. Wogen, von Neumann algebras generated by operators similar to	
normal operators	539
R. Grant Woods, <i>Co-absolutes of remainders of Stone-Čech</i>	
compactifications	545
compactifications	545