

# Pacific Journal of Mathematics

**ON COMPLETELY HAUSDORFF-COMPLETION OF A  
COMPLETELY HAUSDORFF SPACE**

ASIT BARAN RAHA

## ON COMPLETELY HAUSDORFF-COMPLETION OF A COMPLETELY HAUSDORFF SPACE

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**R. M. Stephenson, Jr. (Trans. Amer. Math. Soc. 133 (1968), 537-546) has established the existence of a completely Hausdorff-closed extension  $X'$  of an arbitrary completely Hausdorff space  $X$ . Stephenson demonstrates that  $X'$  enjoys many interesting properties of the Stone-Ćech compactification. This paper shows that, by a modification of the topology,  $X'$  is made also to possess a property which is in the line of the celebrated property of the Stone-Ćech compactification of a completely regular Hausdorff space that it is the largest amongst all Hausdorff compactifications.**

1. Introduction. A topological space  $X$  is called completely Hausdorff if for every pair  $x, y$  of distinct points of  $X$  there exists a continuous real valued function  $f$  on  $X$  such that  $f(x) \neq f(y)$ . A completely Hausdorff space is called completely Hausdorff-closed if every homeomorphic image of it in any completely Hausdorff space is closed. A space  $Y$  is termed a completely Hausdorff-closed extension of a completely Hausdorff space  $X$  if  $X$  is dense in  $Y$  and  $Y$  is completely Hausdorff-closed. R. M. Stephenson, Jr. in [4] has established the existence of a completely Hausdorff-closed extension (referred to as the completely Hausdorff-completion)  $X'$  of an arbitrary completely Hausdorff space  $X$ . If  $X$  is completely regular (which, of course, assumes Hausdorff property and is necessarily completely Hausdorff) then  $X'$  is the Stone-Ćech compactification of  $X$ . Stephenson shows {[4], Theorem 4} that, even if  $X$  is completely Hausdorff but not necessarily completely regular,  $X'$  continues to enjoy many interesting properties of the Stone-Ćech compactification. By enlarging the topology of  $X'$  we shall, in fact, strengthen Theorem 4 of [4] in the sense that property (vii) therein will be replaced by the following:

$X'$  is a projective maximum in the class of completely Hausdorff-closed extensions  $Y$  of  $X$  with the property that any element in  $F(X)$ , the set of all continuous functions on  $X$  into  $[0, 1]$ , admits an extension to  $F(Y)$ .

The above property is, obviously, akin to the well-known fact that the Stone-Ćech compactification is largest among the Hausdorff compactifications of a completely regular Hausdorff space.

2. Notations and definitions. We shall try to follow the notations and definitions of [4] as far as possible.

$C(X)$  will stand for the set of all bounded continuous functions on  $X$ . If  $Z$  is any topological space, we shall denote by  $C(X, Z)$  the set of all continuous mappings of  $X$  into  $Z$ .

A topological space  $Y$  is an *extension space* of another space  $X$  if  $X$  is dense in  $Y$ . If  $T$  is an extension space of a topological space  $S$ , the *tracefilters* of  $T$  are the filters  $\mathcal{F}(t)$ ,  $t \in T - S$ , where  $\mathcal{F}(t)$  is the filter on  $S$  given by  $\{U \cap S: U \text{ a neighbourhood of } t \text{ in } T\}$ .

Banaschewski [1] introduced the notion of a projective maximum in a set  $E$  of extensions of  $X$ ; an extension  $Y$  in  $E$  is a projective maximum in  $E$  if for each  $Z$  in  $E$  there is a continuous function from  $Y$  onto  $Z$  which leaves  $X$  pointwise fixed.

A filter  $\mathcal{F}$  on a space  $X$  is called completely regular provided that it has a base  $\mathcal{B}$  of open sets such that for each  $B \in \mathcal{B}$ , there is a set  $B' \subset B$  in  $\mathcal{B}$  and a function  $f \in F(X)$  such that  $f(B') = 0$  and  $f = 1$  on  $X - B$ .

**3. Main result.** Let  $X$  be a completely Hausdorff space, and let  $\mathcal{M}$  be the set of all maximal completely regular filters on  $X$  which have empty adherences. (If  $\mathcal{F}$  is a completely regular filter,

$$\cap \{F: F \in \mathcal{F}\} = \cap \{\bar{F}: F \in \mathcal{F}\} = \text{adherence of } \mathcal{F},$$

where  $\bar{F}$  = closure of  $F$  in  $X$ . If  $\cap \{F: F \in \mathcal{F}\} = \emptyset$ ,  $\mathcal{F}$  is called *free*, otherwise it is called *fixed*.) Put  $X' = X \cup \mathcal{M}$ . We shall endow  $X'$  with a topology as follows:

Any set, open in  $X$ , is also open in  $X'$ . If  $\mathcal{F} \in \mathcal{M}$ , basic neighbourhoods of  $\mathcal{F}$  are of the form  $G \cup \{\mathcal{F}\}$  where  $G \in \mathcal{F}$ . With this topology (will, henceforth, be called the Katětov topology)  $X'$  becomes a completely Hausdorff-closed space and will be called the completely Hausdorff-completion of  $X$ . The trace filters of  $X'$  are the filters  $\{\mathcal{F}(\mathcal{F}): \mathcal{F} \in \mathcal{M}\}$  and for each  $\mathcal{F} \in \mathcal{M}$ ,  $\mathcal{F}(\mathcal{F}) = \{U \cap X: U \subset X' \text{ and } U \text{ a neighbourhood of } \mathcal{F}\} = \{G: G \in \mathcal{F}\} = \mathcal{F}$ . Thus the trace filters of  $X'$  are the maximal completely regular filters  $\mathcal{F}$  on  $X$  such that

$$\cap \{G: G \in \mathcal{F}\} = \emptyset.$$

Now we are in a position to state our main theorem which is identical with Theorem 4 of [4] with the exception of property (vii).

**THEOREM 1.** *Let  $X$  be a completely Hausdorff space. The completely Hausdorff-completion  $X'$  of  $X$  has the following properties:*

(i) *If  $Z$  is a compact Hausdorff space, then each function in  $C(X, Z)$  has a unique extension in  $C(X', Z)$ .*

(ii) *The Stone-Weierstrass theorem holds for  $X'$ .*

(iii)  $X'$  is locally connected if and only if  $X$  is locally connected and each trace filter of  $X'$  has a base consisting of connected open sets.

(iv)  $X'$  is locally connected only if  $X$  is locally connected and pseudocompact.

(v)  $X'$  is connected if and only if  $X$  is connected.

(vi)  $C(X')$  and  $C(X)$  are isomorphic, and if  $R$  is the real line,  $C(X')$  and  $C(X, R)$  are isomorphic only if  $X$  is pseudocompact.

(vii) Suppose  $Y$  is a completely Hausdorff-closed space containing  $X$  as a dense subset and each element of  $F(X)$  has an extension to  $F(Y)$ . Then there exists a one-to-one function  $g \in C(X', Y)$  such that  $g(X') = Y$  and  $g$  is identity on  $X$ . In short,  $X'$  is a projective maximum in the class of completely Hausdorff-closed extensions  $Y$  of  $X$  with the property that any element in  $F(X)$  admits an extension to  $F(Y)$ .

*Proof.* Proofs of (i) – (vi) are omitted as they are same as those given in Theorem 4 of [4] (page 540). We shall only give a proof for (vii). Let  $Y$  be a completely Hausdorff-closed topological space containing  $X$  as a dense subset and such that every function in  $F(X)$  admits an (unique) extension to  $F(Y)$ . If  $\mathcal{F}$  is a nonconvergent maximal completely regular filter on  $X$  (i.e.,  $\mathcal{F} \in \mathcal{M}$ ) define  $Z = \{f \in F(X) : \text{for some } G', G \in \mathcal{F} \text{ with } G' \subset G, \text{ one has } f(G') = 0 \text{ and } f(X - G) = 1\}$ .  $Z$  is nonvoid as  $\mathcal{F}$  is completely regular. For  $f \in F(X)$  let  $f'$  denote its extension in  $F(Y)$ . Put  $Z' = \{f' : f \in Z\}$ . Take  $\mathcal{S} = \{V(f', t) = f'^{-1} [0, t) : f' \in Z', 0 < t \leq 1\}$ . The empty set does not belong to  $\mathcal{S}$ . Consider,  $V(f'_i, t_i) \in \mathcal{S}, i = 1, 2, \dots, n$  and choose, for each  $i, 0 < s_i < t_i$ . By using the normality of  $[0, 1]$  we can get  $g_i \in F(Y)$  such that  $g_i(V(f'_i, s_i)) = 0$  and  $g_i [Y - V(f'_i, t_i)] = 1$  for  $i = 1, 2, \dots, n$ . Put  $g = \max_{1 \leq i \leq n} g_i$ . Then  $g \in F(Y)$  and  $g[\bigcap_{i=1}^n V(f'_i, s_i)] = 0$  and

$$g[Y - \bigcap_{j=1}^n V(f'_j, t_j)] = 1.$$

Note also that  $\bigcap_{j=1}^n V(f'_j, s_j) \subset \bigcap_{j=1}^n V(f'_j, t_j)$ . Thus, we have shown that finite intersections of sets of  $\mathcal{S}$  form a completely regular filter base on  $Y$ . Let  $\mathcal{G}$  be the completely regular filter on  $Y$  generated by  $\mathcal{S}$  and let  $\mathcal{U}$  denote a maximal completely regular filter on  $Y$  such that  $\mathcal{G} \subset \mathcal{U}$ . Since  $Y$  is completely Hausdorff-closed every completely regular filter on  $Y$  has nonempty adherence (See [4] Theorem 1, and [2]). Consequently adherence of  $\mathcal{U}$  ( $= ad(\mathcal{U})$ ) is nonempty and maximality of  $\mathcal{U}$  will make  $\mathcal{U}$  converge to each point in  $ad(\mathcal{U})$ . But  $Y$  is Hausdorff, so  $ad(\mathcal{U})$  must contain exactly one point, i.e.,  $\bigcap U = \bigcap \{U : U \in \mathcal{U}\}$  is a singleton. We now claim that  $\mathcal{F} = \{U \cap X : U \in \mathcal{U}\}$ .

*Proof of the claim.* Since  $\mathcal{U}$  is a maximal completely regular

open filter it has a completely regular filter base  $\mathcal{V}$  consisting of open sets. As  $X$  is dense in  $Y$ , it is easy to see that  $\mathcal{U} \cap X = \{U \cap X : U \in \mathcal{U}\}$  is an open filter on  $X$  with an open base given by  $\mathcal{V} \cap X = \{V \cap X : V \in \mathcal{V}\}$ . Let  $V \cap X \in \mathcal{V} \cap X$ . Since  $V \in \mathcal{V}$  there exist  $V' \in \mathcal{V}$  with  $V' \subset V$  and  $h \in F(Y)$  such that  $h(V') = 0$  and  $h(Y - V) = 1$ . Obviously,  $h(V' \cap X) = 0$  and  $h(X - V \cap X) = 1$ . Let  $f$  denote the restriction of  $h$  to  $X$ . Then  $f \in F(X)$  and  $f(V' \cap X) = 0$  and  $f(X - V \cap X) = 1$  i.e.,  $\mathcal{V} \cap X$  is a completely regular filter base on  $X$  for  $\mathcal{U} \cap X$ . Therefore  $\mathcal{U} \cap X$  is a completely regular filter on  $X$ . Again  $\mathcal{F}$  is a completely regular filter on  $X$ , so  $F \in \mathcal{F}$  implies that there exist  $F' \in \mathcal{F}$  with  $F' \subset F$  and  $f \in F(X)$  such that  $f(F') = 0$  and  $f(X - F) = 1$ . This gives  $F' \subset f^{-1}[0, 1) \subset F$ . Hence  $f \in Z$  and  $F' \subset f^{-1}[0, 1) \cap X \subset F$  where  $f' \in Z'$ . Now,  $f'^{-1}[0, 1) \in \mathcal{D} \subset \mathcal{U}$ . Thus  $X \cap f'^{-1}[0, 1) \in \mathcal{U} \cap X$  and  $F \supset X \cap f'^{-1}[0, 1)$  implies  $F \in \mathcal{U} \cap X$  (since it is a filter). We get  $\mathcal{F} \subset \mathcal{U} \cap X$  and maximality of  $\mathcal{F}$  forces  $\mathcal{F} = \mathcal{U} \cap X$ . Immediately we have from the above fact,  $(\cap U) \cap X = \cap (U \cap X) = \cap \{F : F \in \mathcal{F}\} = \emptyset$  as  $\mathcal{F}$  is a free maximal completely regular filter. So the single point contained in  $\cap U$  is actually in  $Y - X$ . Let the point be denoted by  $y(\mathcal{F})$ . Next we show that if  $\mathcal{F}_1$  and  $\mathcal{F}_2$  are two distinct points in  $\mathcal{M}$ , the points  $y(\mathcal{F}_1)$  and  $y(\mathcal{F}_2)$  are distinct points of  $Y - X$ . Since  $\mathcal{F}_1$  and  $\mathcal{F}_2$  are two distinct free maximal completely regular filters there must exist  $G_1 \in \mathcal{F}_1$  and  $G_2 \in \mathcal{F}_2$  such that  $G_1$  and  $G_2$  are open in  $X$  and  $G_1 \cap G_2 = \emptyset$ . As shown earlier, we can associate two maximal completely regular filters  $\mathcal{U}_1$  and  $\mathcal{U}_2$  on  $Y$  with  $\mathcal{F}_1$  and  $\mathcal{F}_2$  respectively. By definition  $\{y(\mathcal{F}_i)\} = \cap \{U : U \in \mathcal{U}_i\}$ ,  $i = 1, 2$  and we also know that  $\mathcal{F}_i = \mathcal{U}_i \cap X$ . Consequently there exists  $U_i \in \mathcal{U}_i$  such that  $U_i \cap X = G_i$  and  $U_i$  is open ( $i = 1, 2$ ). Since  $G_1 \cap G_2 = \emptyset$  and  $X$  is dense in  $Y$  we have  $U_1 \cap U_2 = \emptyset$ . Since  $y(\mathcal{F}_i) \in U_i$  for  $i = 1, 2$  we get  $y(\mathcal{F}_1) \neq y(\mathcal{F}_2)$ . So far we have shown that  $\mathcal{F} \mapsto y(\mathcal{F})$  is a one-to-one map of  $\mathcal{M}$  into  $Y - X$ . Let  $i$  denote the identity map on  $X$  into  $Y$ .

Define  $\bar{i} : X' \rightarrow Y$  as follows:

$$\begin{aligned} \bar{i}(x) &= i(x) = x && \text{if } x \in X, \text{ and} \\ \bar{i}(\mathcal{F}) &= y(\mathcal{F}) && \text{if } \mathcal{F} \in \mathcal{M} = X' - X. \end{aligned}$$

Claim:  $\bar{i}$  is continuous.

We shall establish the continuity by showing the continuity at each point.

(i) Suppose  $x \in X$ . Then  $\bar{i}(x) = x$ . Let  $W$  be an open neighbourhood of  $x$  in  $Y$ , then  $\bar{i}^{-1}(W) \cap X = i^{-1}(W) = G$ , an open neighbourhood of  $x$  in  $X$  and hence open in  $X'$  and also  $\bar{i}^{-1}(G) \subset W$ .

(ii) For  $\mathcal{F} \in \mathcal{M}$ , we have  $\bar{i}(\mathcal{F}) = y(\mathcal{F})$ . By construction of

$y(\mathcal{F})$  we know that it is the point of convergence of a maximal completely regular filter  $\mathcal{U}$  on  $Y$  such that  $\mathcal{F} = \mathcal{U} \cap X$ .

If  $W$  is an open neighbourhood of  $y(\mathcal{F})$  in  $Y$  then  $W \in \mathcal{U}$  i.e.,  $W \cap X \in \mathcal{F}$ . But  $W \cap X$  is open in  $X$  and hence  $(W \cap X) \cup \{\mathcal{F}\}$  is an open neighbourhood of  $\mathcal{F}$  in  $X'$  such that

$$\begin{aligned} \bar{i}[(W \cap X) \cup \{\mathcal{F}\}] &= \bar{i}(W \cap X) \cup \bar{i}(\mathcal{F}) = i(W \cap X) \cup \{y(\mathcal{F})\} \\ &= (W \cap X) \cup \{y(\mathcal{F})\} \subset W. \end{aligned}$$

Thus the continuity of  $\bar{i}$  has been proved. But  $X'$  is, in particular, completely Hausdorff-closed and  $\bar{i}$  is a continuous function on  $X'$  into a completely Hausdorff space  $Y$  in which  $X$  is dense. Consequently, from the following fact it will follow that  $\bar{i}$  is onto  $Y$ .

*Fact.* Let  $X$  be a completely Hausdorff-closed space and let  $Y$  be a completely Hausdorff space such that there is a continuous function  $f: X \rightarrow Y$ . Then  $f(X)$  is a completely Hausdorff closed subspace of  $Y$ .

Let us put  $g = \bar{i}$ . Then  $g \in C(X', Y)$  with  $g(X') = Y$  and  $g$  restricted to  $X$  equals  $i$ , the identity map on  $X$ .

**COROLLARY 1.** *Suppose  $Y$  is completely Hausdorff-closed space satisfying the conditions stated in Theorem 1(vii) and  $f$  is a homeomorphism of  $X$  onto  $X$ , then there exists a one-to-one function  $g \in C(X', Y)$  such that  $g(X') = Y$  and  $g$  restricted to  $X$  equals  $f$ .*

*Proof.* We first note that if  $\mathcal{F}$  is a nonconvergent maximal completely regular filter on  $X$ ,  $f(\mathcal{F})$  is a nonconvergent maximal completely regular filter on  $X$ . Then the proof follows by a reasoning similar to one presented in the proof of Theorem 1(vii) where  $i$  is replaced by  $f$ .

**4. REMARKS.** The completely Hausdorff-completion  $X'$  of  $X$  in Theorem 1 is essentially unique, i.e., if  $T$  is any completely Hausdorff closed extension of  $X$  and  $T$  satisfies the properties of Theorem 1 then  $X'$  and  $T$  are homeomorphic. For there exists  $g \in C(X', T)$  such that  $g(X') = T$  and  $g$  is identity on  $X$ . Also, there exists  $h \in C(T, X')$  such that  $h(T) = X'$  and  $h$  is identity on  $X$ . Therefore by the following result {[3], page 5} we can assert that  $X'$  and  $T$  are homeomorphic.

*Result.* Let  $X$  be dense in each of the Hausdorff spaces  $S$  and  $T$ . If the identity mapping on  $X$  has continuous extensions  $s$  from

$S$  into  $T$ , and  $t$  from  $T$  into  $S$ , then  $s$  is a homeomorphism onto, and  $s^{-1} = t$ .

One can raise the following two questions regarding Theorem 1: (a) Is a  $Y$  satisfying the condition (vii) of Theorem 1 homeomorphic to  $X'$ ? (b) Is  $X'$  a one-to-one continuous image of such  $Y$ ? We shall answer both the questions in the negative. Let  $N$  denote the set of natural numbers with discrete topology. On  $N$  any free maximal completely regular filter is nothing but a free ultrafilter. Thus  $\beta N = NU\mathcal{M}$  where  $\mathcal{M}$  is the set of all free ultrafilters on  $N$ . The topology by which  $\beta N$  is the Stone-Čech compactification of  $N$  will be called Stone-Čech topology ( $S - \check{C}$  topology) for  $\beta N$ . Its open sets are generated by  $\{V' : V \text{ open in } N\}$  where  $V' = V \cup \{\mathcal{F} \in \mathcal{M} : V \in \mathcal{F}\}$ . But, according to our definition,  $\beta N$  endowed with the Katětov topology is the completely Hausdorff-completion of  $N$  and in this topology  $\mathcal{M} = \beta N - N$  is a closed, discrete infinite subspace of  $\beta N$  and, thus, cannot be compact. While in the  $S - \check{C}$  topology of  $\beta N$ ,  $\mathcal{M}$  is closed, no doubt, and hence compact. Clearly, the  $S - \check{C}$  topology is strictly weaker than the Katětov topology. As  $S - \check{C}$  topology of  $\beta N$  is compact, *no continuous map from  $\beta N$  with  $S - \check{C}$  topology onto  $\beta N$  with Katětov topology can exist*. So homeomorphism is ruled out. But the Stone-Čech compactification  $\beta N$  satisfies all the conditions enjoyed by a  $Y$  in Theorem 1(vii).

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