

Pacific Journal of Mathematics

SOME TRIPLE INTEGRAL EQUATIONS

JOHN S. LOWNDES

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In this paper we solve the triple integral equations

$$(1) \quad \mathfrak{M}^{-1} \left\{ \frac{\Gamma(\xi + s/\delta)}{\Gamma(\xi + \beta + s/\delta)} \Phi(s); x \right\} = 0, \quad 0 \leq x < a, \quad b < x < \infty,$$

$$(2) \quad \mathfrak{M}^{-1} \left\{ \frac{\Gamma(1 + \eta - s/\sigma)}{\Gamma(1 + \eta + \alpha - s/\sigma)} \Phi(s); x \right\} = f_2(x), \quad a < x < b,$$

where $\alpha, \beta, \xi, \eta, \delta > 0, \sigma > 0$, are real parameters, $f_2(x)$ is a known function, $\Phi(s)$ is to be determined and

$$(3) \quad \mathfrak{M}\{h(x); s\} = H(s), \quad \mathfrak{M}^{-1}\{H(s); x\} = h(x),$$

denote the Mellin transform of $h(x)$ and its inversion formula respectively.

The above equations are an extension of the dual integral equations solved in a recent paper by Erdélyi [2] by means of a systematic application of the Erdélyi-Kober operators of fractional integration [4].

Using the properties of some slightly extended forms of the Erdélyi-Kober operators we show, in a purely formal manner, that the solution of the triple integral equations can be expressed in terms of the solution of a Fredholm integral equation of the second kind. Srivastav and Parihar [5] have solved a very special case of the equations by a completely different method from that used in this paper. The method of solution employed here will be seen to follow closely that used by Cooke [1] to obtain the solution to some triple integral equations involving Bessel functions; indeed Cooke's equations may be regarded as a special case of equations (1) and (2) and it is shown that a solution of his equations can be readily obtained from that presented in this paper.

2. The integral operators. We shall use the integral operators defined by

$$(4) \quad I_{\gamma, \alpha}(a, x; \sigma)f(x) = \frac{\sigma x^{-\sigma(\alpha+\gamma)}}{\Gamma(\alpha)} \int_a^x (x^\sigma - t^\sigma)^{\alpha-1} t^{\sigma(\gamma+1)-1} f(t) dt, \quad \alpha > 0,$$

$$(5) \quad = \frac{x^{1-\sigma(\alpha+\gamma+1)}}{\Gamma(1+\alpha)} \frac{d}{dx} \int_a^x (x^\sigma - t^\sigma)^\alpha t^{\sigma(\gamma+1)-1} f(t) dt, \\ -1 < \alpha < 0,$$

$$(6) \quad K_{\gamma, \alpha}(x, b; \sigma)f(x) = \frac{\sigma x^{\sigma\gamma}}{\Gamma(\alpha)} \int_x^b (t^\sigma - x^\sigma)^{\alpha-1} t^{\sigma(1-\alpha-\gamma)-1} f(t) dt, \quad \alpha > 0,$$

$$(7) \quad = -\frac{x^{\sigma(\gamma-1)+1}}{\Gamma(1+\alpha)} \frac{d}{dx} \int_x^b (t^\sigma - x^\sigma)^\alpha t^{\sigma(1-\alpha-\gamma)-1} f(t) dt, \\ -1 < \alpha < 0,$$

where $a < x < b, \sigma > 0$.

When $a = 0, b = \infty$, these become the extended form of the Erdélyi-Kober operators used in [2] and when $\sigma = 2$ they are the same as the operators defined by Cooke [1].

From the theory of Abel integral equations it follows that the inverse operators are given by

$$(8) \quad I_{\gamma,\alpha}^{-1}(a, x; \sigma)f(x) = I_{\gamma+\alpha,-\alpha}(a, x; \sigma)f(x),$$

$$(9) \quad K_{\gamma,\alpha}^{-1}(x, b; \sigma)f(x) = K_{\gamma+\alpha,-\alpha}(x, b; \sigma)f(x).$$

We shall also find it convenient to have expressions for integral operators of the type

$$(10) \quad L_{\gamma,\alpha}(0, x; \sigma)f(x) = I_{\gamma,\alpha}^{-1}(a, x; \sigma) I_{\gamma,\alpha}(0, a; \sigma)f(x), \quad 0 < a < x,$$

$$(11) \quad M_{\gamma,\alpha}(x, b; \sigma)f(x) = K_{\gamma,\alpha}^{-1}(x, a; \sigma) K_{\gamma,\alpha}(a, b; \sigma)f(x), \quad x < a < b.$$

When $0 < \alpha < 1$, we see on using the results (4), (5) and (8) that

$$L_{\gamma,\alpha}(0, x; \sigma)f(x) = \frac{\sigma x^{1-\sigma(\gamma+1)}}{\Gamma(\alpha)\Gamma(1-\alpha)} \frac{d}{dx} \int_a^x (x^\sigma - t^\sigma)^{-\alpha} t^{\sigma-1} dt \\ \int_0^a (t^\sigma - u^\sigma)^{\alpha-1} u^{\sigma(\gamma+1)-1} f(u) du.$$

Inverting the order of integration and using the result

$$\frac{d}{dx} \int_a^x \frac{t^{\sigma-1} dt}{(x^\sigma - t^\sigma)^\alpha (t^\sigma - u^\sigma)^{1-\alpha}} = \frac{x^{\sigma-1} (a^\sigma - u^\sigma)^\alpha}{(x^\sigma - u^\sigma) (x^\sigma - a^\sigma)^\alpha}, \\ u < a < x, 0 < \alpha < 1,$$

we find

$$(12) \quad L_{\gamma,\alpha}(0, x; \sigma)f(x) = \frac{\sigma \sin(\alpha\pi)}{\pi} \frac{x^{-\sigma\gamma}}{(x^\sigma - a^\sigma)^\alpha} \\ \int_0^a \frac{u^{\sigma(\gamma+1)-1} (a^\sigma - u^\sigma)^\alpha}{x^\sigma - u^\sigma} f(u) du.$$

Similarly we can show that

$$(13) \quad M_{\gamma,\alpha}(x, b; \sigma)f(x) = \frac{\sigma \sin(\alpha\pi)}{\pi} \frac{x^{\sigma(\alpha+\gamma)}}{(a^\sigma - x^\sigma)^\alpha} \\ \int_a^b \frac{u^{\sigma(1-\alpha-\gamma)-1} (u^\sigma - a^\sigma)^\alpha}{u^\sigma - x^\sigma} f(u) du,$$

where $0 < \alpha < 1$.

When $-1 < \alpha < 0$, the formulae for $I_{\eta,\alpha}$ and $M_{\eta,\alpha}$ are exactly the same as those given by the above equations.

We also have the expressions

$$\begin{aligned}
 & I_{\eta+\alpha,-\alpha}(0, a; \sigma) I_{\eta,\alpha}(0, x; \sigma) f(x) \\
 (14) \quad & = [I_{\eta,\alpha}^{-1}(0, x; \sigma) - I_{\eta,\alpha}^{-1}(a, x; \sigma)] I_{\eta,\alpha}(0, x; \sigma) f(x) \\
 & = f(x) - I_{\eta,\alpha}^{-1}(a, x; \sigma) [I_{\eta,\alpha}(0, a; \sigma) + I_{\eta,\alpha}(a, x; \sigma)] f(x) \\
 & = -I_{\eta,\alpha}^{-1}(a, x; \sigma) I_{\eta,\alpha}(0, a; \sigma) f(x) = -L_{\eta,\alpha}(0, x; \sigma) f(x),
 \end{aligned}$$

$$(15) \quad K_{\eta+\alpha,-\alpha}(a, b; \sigma) K_{\eta,\alpha}(x, b; \sigma) f(x) = -M_{\eta,\alpha}(x, b; \sigma) f(x).$$

Two well known results [2] which play an important part in our solution are

$$(16) \quad \mathfrak{M}\{I_{\eta,\alpha}(0, x; \sigma) f(x); s\} = \frac{\Gamma(1 + \eta - s/\sigma)}{\Gamma(1 + \eta + \alpha - s/\sigma)} \mathfrak{M}\{f(x); s\},$$

$$(17) \quad \mathfrak{M}\{K_{\eta,\alpha}(x, \infty; \sigma) f(x); s\} = \frac{\Gamma(\eta + s/\sigma)}{\Gamma(\eta + \alpha + s/\sigma)} \mathfrak{M}\{f(x); s\}.$$

In what follows we are concerned with three ranges of the variable x , namely

$$(18) \quad I_1 = \{x: 0 \leq x < a\}, I_2 = \{x: a < x < b\}, I_3 = \{x: b < x < \infty\},$$

and we shall write any function $f(x)$, $x \geq 0$, in the form

$$(19) \quad f(x) = \sum_{i=1}^3 f_i(x),$$

where

$$(20) \quad f_i(x) = \begin{cases} f(x), & x \in I_i, \\ 0, & \text{otherwise,} \end{cases} \quad i = 1, 2, 3.$$

With these definitions it is easily seen that if we evaluate the equations

$$(21) \quad g(x) = I_{\eta,\alpha}(0, x; \sigma) f(x), h(x) = K_{\eta,\alpha}(x, \infty; \sigma) f(x),$$

on the intervals I_1 , I_2 and I_3 respectively, we get

$$(22) \quad \begin{aligned} g_1(x) &= I_{\eta,\alpha}(0, x; \sigma) f_1(x), \\ h_1(x) &= K_{\eta,\alpha}(x, a; \sigma) f_1(x) + K_{\eta,\alpha}(a, b; \sigma) f_2(x) + K_{\eta,\alpha}(b, \infty; \sigma) f_3(x), \end{aligned}$$

$$(23) \quad \begin{aligned} g_2(x) &= I_{\eta,\alpha}(0, a; \sigma) f_1(x) + I_{\eta,\alpha}(a, x; \sigma) f_2(x), \\ h_2(x) &= K_{\eta,\alpha}(x, b; \sigma) f_2(x) + K_{\eta,\alpha}(b, \infty; \sigma) f_3(x), \end{aligned}$$

$$(24) \quad \begin{aligned} g_3(x) &= I_{\eta,\alpha}(0, a; \sigma) f_1(x) + I_{\eta,\alpha}(a, b; \sigma) f_2(x) + I_{\eta,\alpha}(b, x; \sigma) f_3(x), \\ h_3(x) &= K_{\eta,\alpha}(x, \infty; \sigma) f_3(x). \end{aligned}$$

3. Solution of the integral equations. Using the notation of equations (19) and (20) we can write the triple integral equations (1) and (2) as

$$(25) \quad \mathfrak{M}^{-1} \left\{ \frac{\Gamma(\xi + s/\delta)}{\Gamma(\xi + \beta + s/\delta)} \Phi(s); x \right\} = g(x) ,$$

$$(26) \quad \mathfrak{M}^{-1} \left\{ \frac{\Gamma(1 + \eta - s/\sigma)}{\Gamma(1 + \eta + \alpha - s/\sigma)} \Phi(s); x \right\} = f(x) ,$$

where $g_1 = g_3 = 0$, f_2 is given and g_2, f_1 and f_3 are unknown functions.

If we write

$$(27) \quad \Phi(s) = \mathfrak{M}\{\phi(x); s\} ,$$

and use the formulae (16) and (17) we find that equations (25) and (26) assume the operational form

$$(28) \quad I_{\gamma, \alpha}(0, x: \sigma) \phi(x) = f(x) ,$$

$$(29) \quad K_{\xi, \beta}(x, \infty: \delta) \phi(x) = g(x) .$$

Using the formulae (8) and (9) and solving the above equations for $\phi(x)$ we obtain

$$(30) \quad \phi(x) = I_{\gamma+\alpha, -\alpha}(0, x: \sigma) f(x)$$

$$(31) \quad = K_{\xi+\beta, -\beta}(x, \infty: \delta) g(x) .$$

Now remembering that $g_1 = g_3 = 0$, and using the relations (22), (23) and (24) to evaluate equation (28) on the interval I_1 , equation (30) on I_2 , equation (31) on I_3 , equation (29) on I_2 and equation (31) on I_1 , we arrive at the following results

$$(32) \quad f_1(x) = I_{\gamma, \alpha}(0, x: \sigma) \phi_1(x) ,$$

$$(33) \quad \phi_2(x) = I_{\gamma+\alpha, -\alpha}(0, a: \sigma) f_1(x) + I_{\gamma, \alpha}^{-1}(a, x: \sigma) f_2(x) ,$$

$$(34) \quad \phi_3(x) = K_{\xi, \beta}^{-1}(x, \infty: \delta) g_3(x) = 0 ,$$

$$(35) \quad g_2(x) = K_{\xi, \beta}(x, b: \delta) \phi_2(x) ,$$

$$(36) \quad \phi_1(x) = K_{\xi+\beta, -\beta}(a, b: \delta) g_2(x) .$$

After eliminating $f_1(x)$ between equations (32) and (33), and eliminating $g_2(x)$ between equations (35) and (36), we find that the functions $\phi_1(x)$ and $\phi_2(x)$ satisfy the pair of simultaneous integral equations

$$(37) \quad \phi_2(x) = -L_{\gamma, \alpha}(0, x: \sigma) \phi_1(x) + I_{\gamma, \alpha}^{-1}(a, x: \sigma) f_2(x) ,$$

$$(38) \quad \phi_1(x) = -M_{\xi, \beta}(x, b: \delta) \phi_2(x) ,$$

where we have used the formulae (14) and (15).

From these results it is easily seen that $\phi_2(x)$ can be determined from the Fredholm integral equation of the second kind

$$(39) \quad \phi_2(x) = L_{\gamma,\alpha}(0, x; \sigma) M_{\xi,\beta}(x, b; \delta) \phi_2(x) + I_{\gamma,\alpha}^{-1}(a, x; \sigma) f_2(x) .$$

The solution to the triple integral equations can then be obtained from equations (27), (34), (38) and (39).

As an example we consider the case when $0 < \alpha < 1$, and $-1 < \beta < 0$, or $0 < \beta < 1$; in this instance equation (39) when written out in detail is

$$(40) \quad \begin{aligned} \phi_2(x) - \int_a^b \phi_2(u) S(x, u) du \\ = \frac{x^{1-\sigma(\gamma+1)}}{\Gamma(1-\alpha)} \frac{d}{dx} \int_a^x \frac{t^{\sigma(\alpha+\gamma+1)-1}}{(x^\sigma - t^\sigma)^\alpha} f_2(t) dt , \end{aligned}$$

where

$$(41) \quad \begin{aligned} S(x, u) = \frac{\sigma\delta}{\pi^2} \sin(\alpha\pi) \sin(\beta\pi) \frac{x^{-\sigma\gamma} u^{\delta(1-\beta-\xi)-1}}{(x^\sigma - a^\sigma)^\alpha (u^\delta - a^\delta)^{-\beta}} \\ \int_0^a \frac{t^{\sigma(\gamma+1)+\delta(\beta+\xi)-1} (a^\sigma - t^\sigma)^\alpha}{(x^\sigma - t^\sigma) (u^\delta - t^\delta) (a^\delta - t^\delta)^\beta} dt . \end{aligned}$$

4. An application. Certain mixed boundary value problems [4] may be reduced to the solution of triple integral equations of the type

$$(42) \quad \int_0^\infty \psi(u) J_{2p}(ux) du = 0 , \quad 0 \leq x < a , \quad b < x < \infty ,$$

$$(43) \quad \int_0^\infty u^{-2n} \psi(u) J_{2q}(ux) du = F(x) , \quad a < x < b ,$$

where $J_{2p}(ux)$ is the Bessel function of the first kind of order $2p$, $F(x)$ is a prescribed function and $\psi(u)$ is to be determined. When $p = q$ these are the equations investigated by Cooke [1]. We now show, in a fairly straightforward manner, that the above equations can be transformed into equations of the type (1) and (2).

Denoting the Mellin transform of $\psi(u)$ by

$$(44) \quad \mathfrak{M}\{\psi(u); s\} = \Psi(s) ,$$

and using the result [3]

$$(45) \quad \mathfrak{M}\{\xi^{-2n} J_{2q}(\xi); s\} = 2^{s-1-2n} \frac{\Gamma(q - n + s/2)}{\Gamma(1 + n + q - s/2)} ,$$

we have, on applying the Faltung theorem for Mellin transforms [3],

that the integral equations (42) and (43) can be written in the form

$$(46) \quad \mathfrak{M}^{-1}\left\{\frac{\Gamma(p + s/2)}{\Gamma(q - n + s/2)} \Phi(s); x\right\} = 0, \quad 0 \leq x < a, \quad b < x < \infty,$$

$$(47) \quad \mathfrak{M}^{-1}\left\{\frac{\Gamma(1 + p - s/2)}{\Gamma(1 + n + q - s/2)} \Phi(s); x\right\} = 2^{1+2n}x^{-2n}F(x), \quad a < x < b,$$

where

$$(48) \quad \Phi(s) = 2^s \frac{\Gamma(q - n + s/2)}{\Gamma(1 + p - s/2)} \Psi(1 - s).$$

These are the same as equations (1) and (2) with

$$(49) \quad \begin{aligned} \sigma &= \delta = 2, \quad \xi = \eta = p, \quad \alpha = q - p + n, \quad \beta = q - p - n, \\ f_2(x) &= 2^{1+2n}x^{-2n}F(x). \end{aligned}$$

Using the results of the previous section we have therefore that the solution of equations (46) and (47) can be found in terms of a function $\phi(x)$ by

$$(50) \quad \Phi(s) = \mathfrak{M}\{\phi(x); s\},$$

where $\phi_3(x) = 0$ and the functions $\phi_1(x)$ and $\phi_2(x)$ are obtained from equations (38) and (39) with the parameters ξ, η , etc. given by equations (49).

Finally, in order to find the solution of the integral equations (42) and (43) in terms of $\phi(x)$, we proceed in the following way.

From equation (44) we have that the solution is

$$(51) \quad \begin{aligned} \psi(u) &= \mathfrak{M}^{-1}\{\Psi(s); u\} \\ &= \mathfrak{M}^{-1}\left\{2^{s-1} \frac{\Gamma(1/2 + p + s/2)}{\Gamma(1/2 + q - n - s/2)} \mathfrak{M}\{\phi(x); 1 - s\}; u\right\}, \end{aligned}$$

on using equations (48) and (50). Inverting the order of integration in the last equation we get

$$(52) \quad \begin{aligned} \psi(u) &= \int_0^\infty \phi(x) \mathfrak{M}^{-1}\left\{2^{s-1} \frac{\Gamma(1/2 + p + s/2)}{\Gamma(1/2 + q - n - s/2)}; ux\right\} dx \\ &= \int_0^\infty \left(\frac{ux}{2}\right)^{1+n+p-q} \phi(x) J_{p+q-n}(ux) dx, \end{aligned}$$

after applying the result (45). When $p = q$ this solution is exactly the same as that found by Cooke [1, pp. 61-62].

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