

Pacific Journal of Mathematics

PLANAR SURFACES IN KNOT MANIFOLDS

HOWARD WILSON LAMBERT

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Let K be a knot manifold, that is the 3-sphere S^3 minus an open regular neighborhood of a polygonal simple closed curve in S^3 . Whether K can be embedded in S^3 differently or in a homotopy 3-sphere different from S^3 (if such really exist) leads in a natural way to the question of which planar surfaces can be embedded in K . Geometric conditions are imposed on the embedded planar surfaces which are sufficient to imply that K is not knotted, that is K is homeomorphic to a disk cross S^1 .

1. Introduction and definitions. In this paper we consider some geometric problems motivated by the so called "Property P " [3] of a knot manifold K . In particular, we will investigate whether there is a continuous map f of a planar surface S (compact, submanifold of E^2) into K such that $f(\text{Int } S) \subset \text{Int } K$, $f|_{BdS} \subset BdK$ and f is 1-1 on each component $\Delta_1, \dots, \Delta_n$ of BdS (each Δ_i is a simple closed curve (scc)). We are interested in the cases of either *I.* f is 1-1 and no $f(\Delta_i)$ is contained in a disk on BdK or *II.* S is connected, $f(\Delta_i)$ is parallel to K 's longitude and each $f(\Delta_i)$, $2 \leq i \leq n$, is parallel to a fixed exotic homotopy killer of K (definitions below). For example, if $\pi_1(K) \neq Z$, Case II holds and the homotopy killer of K is exotic, then we would have a counter-example to "Property P ". Conversely, if we had a K violating "Property P ", then there exists $f: S \rightarrow K$ as in Case II and each $f(\Delta_i)$, $2 \leq i \leq n$, is parallel to an exotic homotopy killer of K . In Theorem 1 we develop a geometric condition which is sufficient to imply K is unknotted and in Theorem 2 we develop a related geometric condition which is sufficient to imply K has "Property P ".

Everything here is taken to be polyhedral. Definitions for such terminology as "properly embedded" and "boundary-irreducible" may be found in [17]. A knot manifold K is a submanifold of S^3 such that $Cl(S^3 - K)$ is a solid torus $T = S^1 \times D^2$. On BdK , but not separating BdK , there exists a unique (up to isotopy on BdK) scc homologous to zero (Mod Z) in K , called K 's longitude. A meridian of K is $x \times BdD^2$, $x \in S^1$, and we call it K 's ordinary homotopy killer. Any other scc on BdK which kills $\pi_1(K)$ (by attaching a 3-cell along this scc) will be called an exotic homotopy killer. An exotic homotopy killer is of the form $m(l)^n$, where m is the meridian of K , l is the longitude of K and $n \neq 0$. If K has no exotic homotopy killer, then K is said to have "Property P ". Some results on "Property P " have been obtained by R. Bing and J. Martin [3], A. C. Connor [4], F.

Gonzales [9], J. Hempel [12] and J. Simon [15]. Results about the existence of surfaces (singular or not) in 3-manifolds have been obtained by W. R. Alford [2], C. Feustel [5], C. Feustel and N. Max [6], W. Heil [11], J. Hempel and W. Jaco [13], H. Lambert [14], J. Simon [16], and F. Waldhausen [18] among others.

2. **Results for Case I.** Suppose $f: S \rightarrow K$ as in Case I (since f is a homeomorphism, identify S with $f(S)$) and that each A_i is not parallel to K 's ordinary homotopy killer. Let X_n be the 3-manifold obtained by adding $T (= \text{Cl}(S^3 - K))$ to a regular neighborhood, $S \times [0, 1]$, of S in K (see Figure 1 for a picture of an X_3 with S connected).

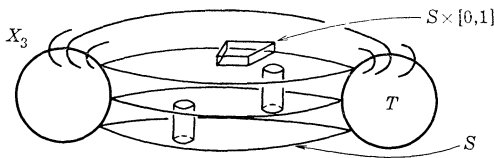


FIGURE 1

Recall from the first paragraph that n is the number of boundary components of S and picture X_n as being obtained by attaching $BdS \times [0, 1]$ to n disjoint annuli A_1, \dots, A_n on BdT .

LEMMA 1. X_n is boundary-irreducible.

Proof. Assume S is connected, as the proof is similar if not. Suppose BdX_n is compressible, i.e., there exists a properly embedded disk D in X_n ($BdD \subset BdX_n$ and $\text{Int } D \subset \text{Int } X_n$) such that BdD does not bound a disk in BdX_n . Put D in general position relative to $\bigcup_{i=1}^n A_i$. After removing simple closed curves of $D \cap \bigcup_{i=1}^n A_i$ which bound disks in $\bigcup_{i=1}^n A_i$, it follows that there exists a subdisk D' of D such that either 1. $D' = D$ and $D' \cap (\bigcup_{i=1}^n A_i) = \emptyset$, 2. $BdD' \subset A_i$ and $\text{Int } D' \cap (\bigcup_{i=1}^n A_i) = \emptyset$ or 3. BdD' consists of two arcs, one in BdX_n and the other in A_i , and $\text{Int } D' \cap (\bigcup_{i=1}^n A_i) = \emptyset$. In Case 1, if $D \subset S \times [0, 1]$, then it follows by Proposition 3.1 of [17] that BdD bounds a disk in BdX_n , contradiction. If $D \subset T$, then either each $f(A_i)$ is parallel to K 's ordinary homotopy killer, contradiction, or BdD bounds a disk in BdX_n , contradiction. Case 2 cannot occur since the center line of each A_i is not homologous to zero in either $S \times [0, 1]$ or T . In Case 3 if $D' \subset S \times [0, 1]$, the arc $BdD' \cap A_i$ intersects one boundary component of A_i and, by using Proposition 3.1 of [17], the number of components of $D \cap (\bigcup_{i=1}^n A_i)$ can be reduced. Similarly, in Case 3 for $D' \subset T$ it follows that the number of components of $D \cap (\bigcup_{i=1}^n A_i)$ can be reduced (assume $n > 1$, since X_1 is a 3-cell). All three cases now imply D could not have existed and

therefore X_n is boundary-irreducible.

Suppose M is a 3-manifold. If D is a disk properly embedded in M such that BdD does not bound a disk on BdM , then we say M has a handle D . More generally, if S is a connected planar 2-manifold properly embedded in M such that 1. n , the number of boundary components $\Delta_1, \dots, \Delta_n$ of S , is odd and 2. there exists an annulus $A = S^1 \times [1, n]$ on BdM such that each $\Delta_i = S^1 \times i, 1 \leq i \leq n$, then call A handle-like in M .

LEMMA 2. *Suppose M is a 3-manifold with a handle D and a handle-like annulus A . Then M has a handle D_0 such that $D_0 \cap A = \emptyset$ and A is handle-like in $M - D_0$.*

Proof. The case $n = 1$ is easy. Suppose then that $n \geq 3$ (and n odd) but that $BdD \cap A = \emptyset$ (we may need to pull BdD off A by an isotopy in BdM to achieve this). If S is in general position relative to D , we may choose a subdisk D' of D such that $BdD' \subset S$ and $\text{Int } D' \cap S = \emptyset$. Now cut S at BdD' and fill in the resulting two holes by disks close to but on opposite sides of D' to obtain two planar surfaces, at least one of which, S' , has an odd number of boundary components ($BdS' \subset BdS$) and $S' \cap D$ has fewer components than $S \cap D$. Repeating this argument a finite number of times yields $D_0 (= D)$ in this special case.

Now suppose $BdD \cap A (\neq \emptyset)$ consists of arcs, each connecting one boundary component of A to its other, and that $D \cap S$ consists of arcs only (simple closed curves may be removed as in the special case). Note that each arc of $D \cap S$ starts and ends in $BdD \cap A$ and that n such arcs start at each arc of $BdD \cap A$. If an arc of $D \cap S$ starts and ends on the same arc of $BdD \cap A$, then there exists a subdisk D' of D such that $D' \cap A$ is an arc on BdD' , the complementary arc of BdD' is contained in $D \cap S$ and $\text{Int } D' \cap S = \emptyset$. Now cut S at $BdD' \cap S$ and attach two disks close to but on opposite sides of D' . The resulting S' then contains one boundary component which bounds a disk in A . Fill in this boundary component to obtain S'' such that S'' is planar, $BdS'' \subset BdS$ and S'' has $n-2$ boundary components.

If no arc of $D \cap S$ has both its end points in the same arc of $BdD \cap A$, then, in D , there are two adjacent arcs Q_1, Q_2 of $BdD \cap A$ (relative to BdD) such that $Q_1 \times (n + 1)/2 (= Q_1 \cap \Delta(n + 1)/2)$ is connected to $Q_2 \times (n + 1)/2$ by an arc γ_0 of $D \cap S$. Since S is orientable and γ_0 has both ends in the same boundary component of S , namely $\Delta(n + 1)/2$, γ_0 does not separate $Q_1 \times 1$ from $Q_2 \times 1$ in D . Hence there is an arc of $D \cap S$ with both ends in Δ_1 (or Δ_n). Since all arcs of $D \cap S$ with one end point in $\Delta_1 \cup \Delta_n$ have both end points in $\Delta_1 \cup \Delta_n$, we may ignore all these arcs and repeat the above argument $((n + 1)/2) - 2$

times more to conclude that for each boundary component A_i of S there exists an arc of $S \cap D$ with both endpoints in A_i . Since S is planar, one of these arcs together with an arc on BdS bounds a disk D' in S such that $\text{Int } D' \cap D = \emptyset$. Now cut D at $BdD' \cap D$ and attach two disks close to but on opposite sides of D' to obtain two disks properly embedded in M and at least one of them is a handle of M which intersects A in fewer arcs than D does. Applying the various cases above a finite number of times yields the desired handle D_0 .

It follows as a corollary to Lemma 2 that if M is a cube with handles, then $n = 1$, i.e., the center line of A bounds a disk in M .

THEOREM 1. *Suppose $f: S \rightarrow K$ as in Case I, that $f(S)$ has at least two components S_1, S_2 such that each has an odd number of boundary components and that there exists an annulus on BdK whose boundary separates BdS_1 , from BdS_2 in $BdK (= S^1 \times S^1)$. Then K is unknotted (homeomorphic to $T = \text{Cl}(S^3 - K)$).*

Proof. Since S_1 and S_2 have an odd number of boundary components and no boundary component of $f(S)$ is contained in a disk on BdK , it follows that each boundary component of $f(S)$ is parallel to K 's longitude. Let A_1, A_2 be disjoint annuli in BdK , parallel to K 's longitude, such that $BdS_1 \subset A_1$ and $BdS_2 \subset A_2$. Let U_1, U_2 be disjoint regular neighborhoods of $S_1 \cup A_1, S_2 \cup A_2$ in K , respectively. Then U_1 is homeomorphic to an X_n of Lemma 1; hence it is boundary-irreducible. Similarly U_2 is boundary-irreducible and by [7] it follows that there is a properly embedded disk D in $\text{Cl}(S^3 - U_1 \cup U_2)$ such that BdD does not bound a disk in $Bd(\text{Cl}(S^3 - U_1 \cup U_2)) = BdU_1 \cup BdU_2$. Suppose, without loss of generality, that $BdD \subset BdU_1$. Since $D \cap U_2 = \emptyset$, it follows that we may cut D and fill in on the two annuli components of $\text{Cl}(BdK - U_1 \cup U_2)$ so as to assume $D \cap T = \emptyset$ (note that obtaining $D \cap T = \emptyset$ involves assuming K is knotted). Now add to U_1 a regular neighborhood of D in $\text{Cl}(S^3 - U_1 \cup U_2)$ to obtain a new 3-manifold U'_1 (if BdD separates BdU_1 , also add the component of $\text{Cl}(S - U_1) - D$ not containing U_2 to U'_1). Note that the genus of BdU'_1 is less than the genus of BdU_1 . Repeat these steps on U'_1, U_2 . But now it is possible that U'_1 is not boundary-irreducible. If $D \subset U'_1$, Lemma 2 says we may assume $D \cap S_1 = \emptyset$ and cut out an open regular neighborhood of D in U'_1 to obtain the new U''_1 . Again the genus of BdU''_1 is less than the genus of BdU'_1 . Continuing, we eventually conclude that there is a 3-cell B in K such that $B \cap BdK$ is either A_1 or A_2 and hence K is unknotted.

3. Results for Case II. Suppose $f: S \rightarrow K$ as in Case II and, in addition, assume each $f(A_i)$, $2 \leq i \leq n$, is parallel to a fixed exotic homotopy killer of K . We may also assume that f is in general posi-

tion, that is the singularities of f on S consist of pairwise disjoint arcs with endpoints in BdS and f sews these arcs together in pairs, each pair forming a single arc in the image (see W. Haken's [10] to see how to eliminate branch points and triple points at the expense of increasing n). There are two types of such arcs of singularities, Type α where the arc runs from A_1 to some $A_i, i \neq 1$, and Type β where the arc has both endpoints in A_1 and its associated arc runs from A_i to $A_j, i, j > 1$ and $i \neq j$. In [10], Haken shows that we can always make every arc of Type α . Unfortunately, from the point of view of studying "Property P ", Type α arcs seem to be particularly intractible. If all arcs are of Type β , then K corresponds to being like a ribbon knot [8, p. 172] relative to its exotic homotopy killer. It is a very particular case of Type β arcs we wish to look at. Suppose S contains a pair of arcs β_1, β_2 of Type β sewed together by f where $Bd\beta_1 \subset A_1$ and one of the two components of $S - \beta_1$ contains no other arc of singularity but β_2 . Denote the closure of this component of $S - \beta_1$ by Γ (Γ is a disk with 2 holes, see Figure 2 for a picture of $f(\Gamma) \cup T$).

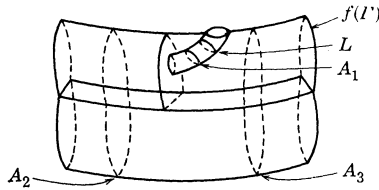


FIGURE 2

THEOREM 2. *Suppose 1. $f: S \rightarrow K$ as in Case II, 2. S contains two (disjoint) Γ 's, Γ_1 and Γ_2 , and 3. n , the number of boundary components of S , is minimal with respect to property 1. Then K is unknotted.*

Proof. First assume $n > 1$, since $n = 1$ implies, by Dehn's Lemma, that K is unknotted. Let A_1, A_2 be disjoint annuli on BdK such that $f(\Gamma_1) \cap BdK \subset A_1$ and $f(\Gamma_2) \cap BdK \subset A_2$. Let U_1, U_2 be disjoint regular neighborhoods of $A_1 \cup f(\Gamma_1), A_2 \cup f(\Gamma_2)$ in K , respectively. We claim both U_1 and U_2 are homeomorphic to an X_3 of Lemma 1. (To see this we have indicated in Figure 2 where the three annuli A_1, A_2 and A_3 of Lemma 1 would be located in U_1 .) By Lemma 1, U_1, U_2 are boundary irreducible and we follow the technique used in the proof of Theorem 1 to conclude that there is a disk D properly embedded in $Cl(S^3 - U_1 \cup U_2)$ such that $BdD \subset U_1$ (or U_2) and $D \cap T = \emptyset$. As in the proof of Theorem 1, we add a regular neighborhood of D to U_1 to obtain U'_1 . Now BdU'_1 is a torus, $S^1 \times S^1$. By [1], the closure of one complementary domain of $S^1 \times S^1$ in S^3 is a solid torus T' . If $f(\Gamma_1) \subset T'$, then the sec

L of Figure 2 can be shrunk to a point in T' since homology and homotopy are the same in T' . (To see that L is homologous to zero Mod Z , note that L bounds an orientable surface in $f(\Gamma_1) \cup A_1$.) Suppose $f(\Gamma_2) \subset T'$. Then $T' - (\text{Int } T \cup \text{Int } A_1)$ is a solid torus, $L \sim 0$ Mod Z in $f(\Gamma_2) \cup A_2 \subset T' - (\text{Int } T \cup \text{Int } A_1)$ and hence the L of $f(\Gamma_2)$ can be shrunk to a point in $T' - \text{Int } T$. In either case, by using the singular disk that L bounds, it follows that there is an $f': S' \rightarrow K$ as in Case II with $n' < n$, contradicting property 3 of the hypothesis. Then $n = 1$ and K is unknotted.

4. A question. Suppose $f: S \rightarrow K$ as in Case II, each $f(\Delta_i)$, $2 \leq i \leq n$, is parallel to a fixed exotic homotopy killer of K and each are of singularity in S is of Type β . We can say in general that there exist disjoint Γ_1, Γ_2 in S as before but now Γ_1, Γ_2 contain holes whose boundaries go parallel to the exotic homotopy killer under f . It does not seem likely that K is knotted if Γ_1, Γ_2 exist, but the author could not show this. We conclude then with the following

Question. If K does not have "Property P " and all singularities of the resulting $f: S \rightarrow K$ are of Type β , then is K unknotted?

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