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For X a Hausdorff space let $H(X)$ be the group of homeomorphisms of X. We study here certain subgroups of $H(E)$ where E is an infinite-dimensional normed linear space.

The set of homeomorphisms from a topological space X onto itself forms a group $H(X)$ under composition. There are many topologies which can be given to $H(X)$, some of which may make $H(X)$ a topological group. It is natural to ask about the properties of $H(X)$, both algebraic and topological. Also, what relationships are there between X and $H(X)$? One way to attack these questions is to study various subgroups of $H(X)$. In this paper we shall investigate certain subgroups of $H(E)$, where E is a normed linear space.

1. Algebraic properties of $H(E)$. Let X be a Hausdorff space. If $A \subset X$, $S(A)$ will denote the set of elements of $H(X)$ which are supported on A. That is, $h \in S(A)$ if and only if $h|_{X-A}$ is the identity on $X-A$. Let $\mathscr{B}$ be a base for the topology on X. Define $B(X)$ to be the subgroup of $H(X)$ which is generated by those elements of $H(X)$ which are supported on elements of $\mathscr{B}$. Then $h \in B(X)$ if and only if $h = h_n \cdots h_1$, where for each $i \leqq n$, $h_i \in S(B_i)$ for some $B_i \in \mathscr{B}$. A homeomorphism $h \in H(X)$ is said to be stable if $h = h_n \cdots h_1$, where for each $i \leqq n$, $h_i \in S(X - U_i)$ for some nonempty open set U_i in X. The stable homeomorphisms of X, $SH(X)$, form a subgroup of $H(X)$.

We shall consider the following possible conditions on $\mathscr{B}$.

B 1. For every B_1, $B_2 \in \mathscr{B}$, there exists an $h \in H(X)$ such that $h(B_1) \subset B_2$.

B 1′. For every $B_1, B_2 \in \mathscr{B}$, there exists an $h \in B(X)$ such that $h(B_1) \subset B_2$.

B 2. For every $B \in \mathscr{B}$, there exists an $x \in B$ and a pairwise disjoint sequence $\{B_i \in \mathscr{B} \mid B_i \subset B, i = 1, 2, \cdots\}$ which converges to x (i.e., for every open set U containing x, there is some B_i contained in U), and there exists an $h \in S(B)$ such that $h(B_i) = B_{i+1}$ for every i.

B 3. For every $B \in \mathscr{B}$ and $h \in H(X)$, $h(B) \in \mathscr{B}$.

B 4. For every $B \in \mathscr{B}$, there exists $B' \in \mathscr{B}$ such that $B \cup B' = X$, and no $B \in \mathscr{B}$ is dense in X.

LEMMA 1.1. *If* $\mathscr{B}$ *satisfies* B 3, *then* $B(X)$ *is a normal subgroup of* $H(X)$.

Proof. Let $h \in B(X)$ and $f \in H(X)$. Then $h = h_n \cdots h_1$, where for each $i \leqq n$, $h_i \in S(B_i)$ for some $B_i \in \mathscr{B}$. Then

$$fhf^{-1} = (fh_nf^{-1}) \cdots (fh_1f^{-1}) .$$

Each $fh_if^{-1} \in S(f(B_i))$, so that $fhf^{-1} \in B(X)$.

The following two lemmas can be proved in a manner similar to the proof of Theorem 2 in [9]. Also see [1], [2], and [16].

LEMMA 1.2. *Let* $\mathscr{B}$ *satisfy* B 1 *and* B 2, *and let* $h \in H(X)$ *such that* h *is not the identity. If* $f \in B(X)$, *then* f *is a product of conjugates of* h *and* h^{-1} *by members of* $H(X)$.

LEMMA 1.3. *Let* $\mathscr{B}$ *satisfy* B 1′ *and* B 2, *and let* $h \in H(X)$ *such that* h *is not the identity. If* $f \in B(X)$, *then* f *is a product of conjugates of* h *and* h^{-1} *by members of* $B(X)$.

THEOREM 1.1. *If* $\mathscr{B}$ *satisfies* B 1′ *and* B 2, *then* $B(X)$ *is simple.*

Proof. Let N be a normal subgroup of $B(X)$ having more than one element. Let $f \in B(X)$. Choose $h \in N$ such that h is not the identity. Then by Lemma 1.3, f is a product of conjugates of h and h^{-1} by members of $B(X)$. But Since $h \in N$ and N is normal in $B(X)$, f is a product of elements of N. Therefore $f \in N$, so that $B(X) = N$.

THEOREM 1.2. *If* $\mathscr{B}$ *satisfies* B 1, B 2, *and* B 3, *then if* $B(X)$ *is nontrivial, it is the smallest nontrivial normal subgroup of* $H(X)$.

Proof. By Lemma 1.1, $B(X)$ is a normal subgroup of $H(X)$. Suppose that N is a normal subgroup of $H(X)$ having more than one element. Let $f \in B(X)$. Choose $h \in N$ such that h is not the identity. Then by Lemma 1.2, f is a porduct of conjugates of h and h^{-1} by members of $H(X)$. But since $h \in N$ and N is normal in $H(X)$, f is a product of elements of N. Therefore $f \in N$, so that $B(X) \subset N$.

LEMMA 1.4. *If* $\mathscr{B}$ *satisfies* B 4, *then* $B(X) = SH(X)$.

Proof. Clearly $B(X) \subset SH(X)$. Suppose that $h \in SH(X)$. Then $h = h_n \cdots h_1$, where for each $i \leqq n$, $h_i \in S(X - U_i)$ for some nonempty open set U_i in X. Since $\mathscr{B}$ is a base for the topology on X, for each $i \leqq n$, there is some $B_i \in \mathscr{B}$ such that $B_i \subset U_i$. By property B 4, for each $i \leqq n$, there exists $B_i' \in \mathscr{B}$ such that $B_i \cup B_i' = X$. Then each h_i is an element of $S(B_i')$. Thus $h \in B(X)$.

Theorem 1.1 and Lemma 1.4 then give conditions which imply that $H(X)$ is a simple group.

THEOREM 1.3. *If* $\mathscr{B}$ *satisfies* B 1′, B 2, *and* B 4, *and if every element of* $H(X)$ *is stable, then* $H(X)$ *is simple.*

Now let us consider the special case of the group of homeomorphisms on a normed linear space or a manifold modeled on a normed linear space. E will always denote a normed linear space, and M will be a connected manifold modeled on E. By that we mean a connected paracompact space such that every point in M is contained in an open subset of M which is homeomorphic to E. If E is finite-dimensional it will be permissible to allow M to have boundary.

For finite-dimensional E, Fisher defined in [9] a base for M which satisfies B 1, B 1′, B 2, and B 3. A similar base for M can be found when E is infinite-dimensional.

LEMMA 1.5. *If E is infinite-dimensional, M has a base* $\mathscr{B}$ *which satisfies* B 1, B 1′, B 2, *and* B 3.

Proof. Take $\mathscr{B}$ to consist of all collared open cells in M. By a collared open cell in M is meant the interior of a collared cell in M. C is a collared cell in M if there exists a homeomorphism from the triple $(B_2; B_1, S_2)$ in E onto the triple $(C'; C, BdC')$ in M, where C' is some subset of M, where $B_r = \{x \in E \mid \|x\| \leqq r\}$, and where $S_r = BdB_r$.

Property B 1 follows from B 1′, and B 3 follows from the definition of $\mathscr{B}$. We shall outline the proof that $\mathscr{B}$ satisfies B 1′ and B 2 by using a similar technique to that which was used in [9]. Let $Q_1, Q_2 \in \mathscr{B}$. Since M is connected, there are a finite number of elements of $\mathscr{B}$, say $Q^1, \cdots, Q^n$, such that $Q^1 = Q_1, Q^n = Q_2$, and $Q^i \cap Q^{i+1} \neq \varnothing$ for $i < n$. For each $i < n$, let f_i be a homeomorphism from $(B_2; B_1, S_2)$ onto $(C_i; ClQ^i, BdC_i)$, where C_i is some subset of M. Also for each $i < n$, we can define a $g_i \in S(B_{3/2})$ such that

$$g_i(B_1) \subset f_i^{-1}(Q^i \cap Q^{i+1}) .$$

Then define $h = f_{n-1}g_{n-1}f_{n-1}^{-1} \cdots f_1 g_1 f_1^{-1}$. Since for each $i < n$, $f_i(\text{Int } B_{3/2}) \in \mathscr{B}$, then $h \in B(M)$. Also $h(Q_1) \subset Q_2$.

To establish that $\mathscr{B}$ satisfies B 2, let $Q \in \mathscr{B}$. Let f be a homeomorphism from $(B_2; B_1, S_2)$ onto $(Cl; CQ, BdC)$ for some set C in M.

Define $g \in H(B_2)$ by $g(y) = \|y\| y$ for $y \in B_1$, and $g(y) = y$ for $y \in B_2 - B_1$. Let $x = f(0)$, and choose $z \in S_{3/8}$. For each positive integer i, set $Q_i = fg^i(\mathrm{Int}\, B_{1/9}(z))$. Then define $h \in S(Q)$ by $h(y) = fgf^{-1}(y)$ if $y \in C$, and $h(y) = y$ if $y \in M - C$. It can be verified that the sequence $\{Q_i\}$ is pairwise disjoint and converges to x, and that $h(Q_i) = Q_{i+1}$ for every i.

LEMMA 1.6. *If E is infinite-dimensional, it has a base $\mathscr{B}$ which satisfies* B 1, B 1′, B 2, B 3, *and* B 4.

Proof. As in Lemma 1.5, take $\mathscr{B}$ to consist of all collared open cells in E. Hence $\mathscr{B}$ satisfies B 1, B 1′, B 2, and B 3. Klee showed in [13] that if E is infinite-dimensional, there is a $\varphi \in H(E)$ such that $\varphi(B_1) = E - \mathrm{Int}\, B_1$. Therefore complements of collared cells are collared open cells. Then to see that $\mathscr{B}$ satisfies B 4, let $Q \in \mathscr{B}$. From Theorem 4.1 in [14] it is seen that Q is tame, so that there exists an $f \in H(E)$ such that $f(Q) = \mathrm{Int}\, B_1$. Let $Q' = E - f^{-1}(B_{1/2})$, which is thus in $\mathscr{B}$ because of Klee's result. Clearly $Q \cup Q' = E$.

The next two theorems then follow from Theorem 1.1, Theorem 1.2, Lemma 1.4, Lemma 1.5 and Lemma 1.6.

THEOREM 1.4. *M has a base $\mathscr{B}$ such that $B(M)$ is the smallest nontrivial normal subgroup of $H(M)$ and is simple.*

THEOREM 1.5. *If E is infinite-dimentional, then $SH(E)$ is the smallest nontrivial normal subgroup of $H(E)$ and is simple.*

It was shown in [8] that if E is homeomorphic to the countably infinite product of copies of itself (we shall abreviate this statement as $E \sim E^{\omega}$), then $SH(E) = H(E)$.

THEOREM 1.6. *If $E \sim E^{\omega}$, then $H(E)$ is simple.*

It should be noted that if E is an infinite-dimensional Hilbert space, then $E \sim E^{\omega}$ [5]. Also, all reflexive Banach spaces are homeomorphic to Hilbert spaces [6]. In fact, at this time there seems to be no known infinite-dimensional E which is not homeomorphic to E^{ω}.

2. Stable structure on E.

Whittaker defines the following terms in [18]. Let $\mathscr{K}(X)$ be the set of nonempty connected open subsets U of X such that for every $x, y \in U$, there exists an $f \in S(U)$ with $f(x) = y$. Set $K(X) = \cup \mathscr{K}(X)$, which is an open subset of X.

Finally, define $R(X)$ to be the set of $h \in H(X)$ such that for every $x \in K(X)$ and every connected open subset U of $K(X)$ containing x and $h(x)$, there is a neighborhood V of x and an $f \in S(U)$ satisfying $f|_V = h|_V$.

It was shown in [18] that if X is a Hausdorff space such that each open subset contains a member of $\mathscr{K}(X)$, and $K(X)$ cannot be separated by any two points, then $R(X)$ is a normal subgroup of $H(X)$.

As in the previous section, E will denote a normed linear space, and M will be a connected manifold modeled on E.

LEMMA 2.1. *$\mathscr{K}(M)$ is a base for the topology on M, and $K(M) = M$.*

Proof. If $z \in M$, then there exists a collared open cell Q in M containing z. Let g be a homeomorphism from $(B_2; B_1, S_2)$ onto $(C; ClQ, BdC)$, for some set C in M (see the proof of Lemma 1.5 for terminology). Let $x, y \in Q$, and set $a = g^{-1}(x)$ and $b = g^{-1}(y)$. Define $h \in H(B_1)$ as follows. First define $h(a) = b$. Next let $c \in B_1 - \{a\}$. Let $\{c'\} = \text{Ray}\,[a : c] \cap S_1$, where Ray $[a : c]$ is the infinite ray from a through c. Then $c = a + \alpha(c' - a)$ for some $0 < \alpha \leqq 1$. Define $h(c) = b + \alpha(c' - b)$. With h thus defined, define $f \in H(M)$ by $f(\omega) = ghg^{-1}(\omega)$ if $\omega \in Q$, and $f(\omega) = \omega$ if $\omega \in M - Q$. Then $f \in S(Q)$ and $f(x) = y$. Therefore $Q \in \mathscr{K}(M)$, which makes $\mathscr{K}(M)$ a base for the topology on M. Then obviously $K(M) = M$.

THEOREM 2.1. *If the dimension of E is greater than one, then $R(M)$ is a normal subgroup of $H(M)$.*

It was also shown in [18] that M has a stable structure if and only if $R(M)$ does not consist only of the identity on M. The concept of a stable structure was introduced and studied in [7]. M has a stable structure if $M = \cup \{U_\alpha \mid \alpha \in A\}$, where the U_α are the images of homeomorphisms h_α from B_1 in E into M which satisfy the condition that if $U_\alpha \cap U_\beta \neq \varnothing$ and $x \in h^{-1}(U_\alpha \cap U_\beta)$, then there is a neighborhood V of x and an $f \in S(B_1)$ such that $f|_V = h_\beta^{-1} h_\alpha|_V$. In the next theorem we shall see that for a large class of spaces E, $R(M)$ is all of $H(M)$.

THEOREM 2.2. *If $E \sim E^\omega$, then $R(M) = H(M)$.*

Proof. Let $h \in H(M)$. By Lemma 2.1, $K(M) = M$. So let $x \in M$, and let U be a connected open subset of M containing x and $h(x)$.

Since $E \sim E^{\omega}$, by a result of Henderson and Schori in [10], there exists a homeomorphism φ from M into E such that $\varphi(M)$ is open in E. Since $\varphi(U)$ is connected, there is a piecewise linear arc, α, joining $\varphi(x)$ and $\varphi h(x)$, such that $\alpha \subset \varphi(U)$. By taking an appropriate ε-neighborhood of α, a collared cell C can be found contained in $\varphi(U)$ and containing α in its interior. Choose $\delta > 0$ such that

$$B_\delta(\varphi h(x)) \subset \operatorname{Int} C .$$

Then choose $\varepsilon > 0$ such that $B_\varepsilon(\varphi(x)) \subset \varphi h^{-1}\varphi^{-1}(\operatorname{Int} B_\delta(\varphi h(x))) \cap \operatorname{Int} C$. In [8] it is shown that $SH(E) = H(E)$ if and only if the strong annulus conjecture for E is true. Then since $SH(E) = H(E)$ for E such that $E \sim E^{\omega}$, we may apply the strong annulus conjecture here. Thus there exists $g \in S(C)$ such that $g|_{B_\varepsilon(\varphi(x))} = \varphi h \varphi^{-1}|_{B_\varepsilon(\varphi(x))}$. Define $f \in S(U)$ by $f = \varphi^{-1} g \varphi$ and let $V = \varphi^{-1}(\operatorname{Int} B_\varepsilon(\varphi(x)))$. Then $f|_V = h|_V$ as desired, so that $h \in R(M)$.

COROLLARY. *If $E \sim E^{\omega}$, then M has a stable structure.*

3. Topological propeties of $H(E)$. Let X be a Hausdorff space, and let $\mathscr{C}$ be a collection of closed subsets of X. Define $H_{\mathscr{C}}(X)$ to be $H(X)$ along with the topology generated by the collection

$$\{[C, U] \mid C \in \mathscr{C} \text{ and } U \text{ is open in } X\} ,$$

where

$$[C, U] = \{h \in H(X) \mid h(C) \subset U\} .$$

X is (stably) $\mathscr{C}$-homogeneous if every homeomorphism between elements of $\mathscr{C}$ can be extended to a (stable) homeomorphism in $H(X)$.

For the remainder of this section, F will be a locally convex, linear topological space such that $F \sim F \times F$. If A is a closed subset of F, then A is F-deficient if there exists a homeomorphism h from F onto $F \times F$ such that $h(A) \subset F \times \{0\}$. It is a standard technique (see [12] and [4]) that F is stably $\mathscr{C}$-homogeneous if $\mathscr{C}$ has the property that for $C, D \in \mathscr{C}$, $C \cup D$ is F-deficient. Lemma 3.1 is a partial converse to this. In Lemma 3.1, Theorem 3.1, and Theorem 3.2, we shall take $\mathscr{C}$ to be closed under finite unions and under homeomorphisms (i.e., if $C, D \in \mathscr{C}$, then $C \cup D \in \mathscr{C}$; and if $C \in \mathscr{C}$, then $h(C) \in \mathscr{C}$ for every $h \in H(F)$).

LEMMA 3.1. *If F is $\mathscr{C}$-homogeneous, then every element of $\mathscr{C}$ is F-deficient.*

Proof. Let $C \in \mathscr{C}$, and let f be a homeomorphism from F onto

$F \times F$. Then the homeomorphism from C onto $f^{-1}(C \times \{0\})$ can be extended to some $g \in H(F)$. Let $h = fg$, so that

$$h(C) = fg(C) = ff^{-1}(C \times \{0\}) = C \times \{0\} \subset F \times \{0\} .$$

THEOREM 3.1. *If F is C-homogeneous, then it is stably $\mathscr{C}$-homogeneous.*

THEOREM 3.2. *Let F be $\mathscr{C}$-homogeneous. Then $SH(F) = H(F)$ if and only if $SH_{\mathscr{C}}(F)$ is open in $H_{\mathscr{C}}(F)$.*

Proof. Suppose $SH_{\mathscr{C}}(F)$ is open in $H_{\mathscr{C}}(F)$, and let $h \in H(F)$. Let $\bigcap_{i=1}^{n} [C_i, U_i]$ be a neighborhood of the identity on F which is contained in $SH(F)$, where $C_i \in \mathscr{C}$ and U_i is open for $i \leqq n$. By Theorem 3.1, there exists a $g \in SH(F)$ such that $g|_{\cup_{i=1}^{n} C_i} = h|_{\cup_{i=1}^{n} C_i}$. Then $g^{-1}h(C_i) \subset U_i$ for $i \leqq n$, so that $g^{-1}h \in SH(F)$. Therefore $h = g(g^{-1}h) \in SH(F)$.

The following corollary to Theorem 3.2 then is true because infinite-dimensional Fréchet spaces are homogeneous with respect to compact sets, which in turn follows from Michael's version of the Bartle-Graves Theorem, found for example in [15], and from the fact that separable infinite-dimensional Fréchet spaces are homeomorphic to separable Hilbert space, which can be found in [3].

COROLLARY. *Let F be a Fréchet space such that $F \sim F \times F$. Then $SH(F) = H(F)$ if and only if $SH(F)$ is open in $H(F)$ under the compact-open topology.*

Kirby showed in [11] that if E is finite-dimensional, then $SH(E)$ is open in $H(E)$ under the compact-open topology. But he made use of the fact that $H(E)$ with the compact-open topology forms a topological group. This is not the case for infinite-dimensional E. We might ask the following questions. If $H_{\mathscr{C}}(E)$ is a topological group, is $SH_{\mathscr{C}}(E)$ open in $H_{\mathscr{C}}(E)$? Which classes, $\mathscr{C}$, make $H_{\mathscr{C}}(E)$ into a topological group? One answer to this last question is the following theorem.

THEOREM 3.3. *Let E be an infinite-dimensional normed linear space, and let M be a connected manifold modeled on E. If $\mathscr{C}$ consists of the collared cells in E or M, respectively, then $H_{\mathscr{C}}(E)$ is a topological group and $H_{\mathscr{C}}(M)$ is a topological semigroup. If $\mathscr{C}$ consists of the collared cells in M and the complements of the interiors of the collared cells in M, then $H_{\mathscr{C}}(M)$ is a topological group.*

Proof. Let $h_1, h_2 \in H(E)$ (or $H(M)$). Let $\bigcap_{i=1}^n [B_i, U_i]$ be an open set in $H(E)$ containing h_2h_1, where each $B_i \in \mathscr{C}$. For each $i \leqq n$, let $C_i \in \mathscr{C}$ which is contained in $h_2^{-1}(U_i)$, such that $h_1(B_i) \subset \operatorname{Int} C_i$. Such a C_i can be found since a collared cell is collared in every open set containing it [17]. Then $h_2(C_i) \subset U_i$. Let $g_1 \in \bigcap_{i=1}^n [B_i, \operatorname{Int} C_i]$ and $g_2 \in \bigcap_{i=1}^n [C_i, U_i]$. Then $g_2g_1(B_i) \subset g_2(\operatorname{Int} C_i) \subset U_i$.

Let $h \in H(E)$ (or $H(M)$). Let $\bigcap_{i=1}^n [B_i, U_i]$ be an open set in $H(E)$ containing h^{-1}, where each $B_i \in \mathscr{C}$. For each $i \leqq n$, let $D_i \in \mathscr{C}$ which is contained in U_i, such that $h^{-1}(B_i) \subset \operatorname{Int} D_i$. Let $C_i = E - \operatorname{Int} D_i$ which is an element of $\mathscr{C}$(see the proof of Lemma 1.6). Then $h(C_i) = h(E - \operatorname{Int} D_i) \subset h(E - h^{-1}(B_i)) = E - B_i$. Let $g \in \bigcap_{i=1}^n [C_i, E - B_i]$. Then $C_i \subset g^{-1}(E - B_i) = E - g^{-1}(B_i)$, so that $g^{-1}(B_i) \subset E - C_i = \operatorname{Int} D_i \subset U_i$.

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