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Let F be an unequal characteristic local field. The aim of this paper is to outline a block form of the Cartan-Brauer modular decomposition theory which incorporates the notion of defect groups. The irreducible F-representations of a finite group G are associated with blocks in the group algebra over the residue field $\bar{F}$. The defect groups of a block to which an irreducible F-representation T belongs are shown to coincide with the defect groups of the block to which any absolutely irreducible constituent of T belongs. A result on the Schur index of an absolutely irreducible representation belonging to a block of defect zero is proven which yields an analogue to the Brauer-Nesbitt Theorem on blocks of defect zero. The number of $\bar{F}$-blocks of highest defect is shown to be equal to the number of p-regular $\bar{F}$-conjugacy classes of highest defect.

Let G be a finite group and let F be an unequal characteristic local field; i.e., a field of characteristic zero, complete with respect to a discrete non-Archimedean valuation, whose residue field $\bar{F}$ is of characteristic p. The aim of this paper is to initiate a study of a block form of the Cartan-Brauer decomposition theory [2, §83A] which incorporates the notion of defect groups. Defect groups for $\bar{F}$-blocks are defined as in [9]. The irreducible F-representations and the indecomposable $\bar{F}$-representations are associated with $\bar{F}$-blocks. In Theorem 2 we show that the defect groups of a block to which an irreducible F-representation T belongs are the same as the defect groups of the blocks to which the absolutely irreducible constituents of T belong. Using Theorem 2 and assuming that the residue field $\bar{F}$ is perfect, we obtain a result on the Schur index which yields an analogue to the Brauer-Nesbitt Theorem on blocks of defect zero. [2, 86.3]. In § 4 we use some lifting techniques together with a permutation lemma of Brauer to show that the number of $\bar{F}$-blocks of highest defect is equal to the number of p-regular $\bar{F}$-conjugacy classes of highest defect. This last result has been obtained independently by W. F. Reynolds as a consequence of some work on blocks in twisted group algebras [8]. Also L. G. Kovacs in some unpublished notes has obtained the result for blocks in group algebras over finite fields using the theory of vertices and sources.

Some of the results in this paper are essentially contained in a thesis presented to the Department of Mathematics and the Graduate School of the University of Oregon in partial fulfillment of the requirements for the degree of Doctor of Philosophy. The author wishes to take this opportunity to thank Professor C. W. Curtis for his valu-

able assistance and kind encouragement.

First we make some preliminary definitions and observations. Denote the conjugacy classes of G by $\mathscr{C}_1, \cdots, \mathscr{C}_t$ and let C_i be the class sum corresponding to the class $\mathscr{C}_i$. Let F be a field of characteristic p. Primitive idempotents in the center $Z(FG)$ of the group algebra FG are called *F-block idempotents*, or more simply, *F-blocks*. Suppose E is a Galois extension of F. Denote the Galois group of E over F by $\mathscr{G}(E/F)$. Each $\sigma \in \mathscr{G}(E/F)$ induces an automorphism of the group algebra EG which permutes the E-block idempotents. The following result characterizes F-blocks with a given defect group in terms of E-blocks.

PROPOSITION 1. [7, Theorem 2] *Let $\mathscr{D}$ be the set of E-blocks whose defect groups are conjugate to some p-subgroup D of G. Then $\mathscr{G}(E/F)$ acts as a permutation group on $\mathscr{D}$ and the orbits of $\mathscr{D}$ relative to $\mathscr{G}(E/F)$ are in one-to-one correspondence with the set of E-blocks whose defect groups are conjugate to D. If $\mathscr{O}$ is an orbit, then $e = \Sigma d$, where the sum is over $\mathscr{O}$, is the F-block corresponding to $\mathscr{O}$.*

Proof. The fact that F-blocks correspond to orbits of E-blocks relative to the action of $\mathscr{G}(E/F)$ is a consequence of a well-known result about idempotents in commutative algebras [10, § 124]. If an E-block d is written as in [9, 3.2], it is easy to see that d and $\sigma(d)$ have the same defect groups, and hence that each $\sigma \in \mathscr{G}(E/F)$ permutes $\mathscr{D}$.

Let Ω be an algebraic closure of E. If e is an F-block, then $Z(FG)e$ is a completely primary algebra and so $Z(FG)e/\text{rad}\, Z(FG)e$ is a finite extension of F. The natural homomorphism ψ of $Z(FG)$ onto $Z(FG)e/\text{rad}\, Z(FG)e$ has as its kernel the maximal ideal $Z(FG)(1-e) + \text{rad}\, Z(FG)$. Any homomorphism from $Z(FG)$ into Ω with kernel $Z(FG)(1-e) + \text{rad}\, Z(FG)$ is said to be *associated with e*. Observe that if ψ is associated with e, then $\psi(e) = 1$.

Let $e = \Sigma \sigma_i(d)$, where the sum is over an orbit $\mathscr{O}$, be an F-block. Write

$$e = \sum_{i \in A} f_i C_i + \sum_{j \in B} f_j C_j$$

and

$$d = \sum_{i \in A'} b_i C_i + \sum_{j \in B'} b_j C_j$$

as in [9, 3.2]. To show that e and d have defect groups conjugate

to D it suffices to show that $A \cap A' \neq \emptyset$. We have $f_i = \Sigma b_i{}^{\sigma_j}$. Thus if $f_i \neq 0$, then $b_i \neq 0$, and so $A \cup B \subseteq A' \cup B'$. Therefore, $A \subseteq A' \cup B'$.

Now assume $A \subseteq B'$. Let η be a homomorphism from $Z(EG)$ into Ω associated with d. Since $A \subseteq B'$, we have by [9, 3.2]

$$\eta(e) = \sum_{i \in A} f_i \eta(C_i) + \sum_{j \in B} f_j \eta(C_j) = 0 .$$

But the restriction of η to $Z(FG)$ is a homomorphism associated with e, a contradiction. Thus $A \subsetneq B'$ and $A \cap A' \neq \emptyset$ and the proof is complete.

2. **Block idempotents and representations.** Let F be a local field of unequal characteristic. Denote the ring of integers in F by O_F the unique maximal ideal by P_F and the residue field O_F/P_F by $\bar{F}$. The natural map $\alpha \rightarrow \bar{\alpha}$ from O_F onto $\bar{F}$ induces a homomorphism from the group algebra $O_F G$ onto $\bar{F}G$. By [2, § 77] there is a one-to-one correspondence between O_F-blocks and $\bar{F}$-blocks, the correspondence being given by $e \leftrightarrow \bar{e}$. Here O_F-blocks are primitive central idempotents in the group algebra $O_F G$.

An irreducible FG-module X is isomorphic to a minimal left ideal in some simple component A of FG. The identity ε of A is a primitive central idempotent in FG. The module X is said to be *associated with* ε. Observe thet an O_F-block e is a central idempotent in FG, and hence e is the sum of a uniquely determined set of primitive central idempotents in FG. An irreducible FG-module X *belongs to* an $\bar{F}$-block $\bar{e}$ provided the primitive central idempotent ε, with which X is associated, is a summand of e. If X affords a character χ, we shall also say that χ belongs to $\bar{e}$. An indecomposable $\bar{F}G$-module M belongs to $\bar{e}$ in case $\bar{e}M \neq 0$.

Let X be an indecomposable $O_F G$-submodule of an irreducible FG-module X^* obtained by [2, 73.6]. As in [2, § 85] we form the $\bar{F}G$-module $\bar{X}$; it can be shown that the $\bar{F}$-block to which the $\bar{F}G$-composition factors of $\bar{X}$ belong is the same as the $\bar{F}$-block to which X^* belongs. Thus, we can establish a block form of the decomposition theory of [2, § 83A]. The hypothesis that F is complete is necessary for such a block decomposition theory; for if T is the irreducible Q-representation of a cyclic group of order $p-1$ associated with the cyclotomic polynomial $\Phi_{p-1}(x)$, then the composition factors of $\bar{T}$ lie in different $\bar{Q} = Z_p$ blocks.

We return to the case where F is complete. All algebraic extensions of F will be assumed to lie in a fixed algebraic closure Ω of F. Denote the unique extension of the valuation on F to Ω by ϕ, and for an extension L set

$$O_L = \{a \in L \mid \phi(a) \leqq 1\}$$
$$P_L = \{a \in L \mid \phi(a) < 1\}$$
$$\bar{L} = O_L/P_L .$$

For $\alpha \in O_L$ set $\bar{\alpha} = \alpha + P_L$. We fix the following notation for the remainder of the paper. Let ω be a primitive nth root of unity where $n = p^a m$ with $(p, m) = 1$ is the exponent of G. Set $E = F(\omega)$ and $K = F(\omega^{p^a})$. From the extension theory of complete fields [11, Chapter 2] it follows that elements σ of $\mathscr{G}(K/F)$ map O_K onto $O_{\sigma K} = O_K$ and P_K onto $P_{\sigma K} = P_K$. The map $\bar{\sigma}$ defined by $\bar{\sigma}(\bar{\alpha}) = \overline{\sigma(\alpha)}$ is an automorphism of $\bar{K}$. Since $\bar{K}$ is separable over $\bar{F}$, the map $\sigma \to \bar{\sigma}$ is a homomorphism from $\mathscr{G}(K/F)$ onto $\mathscr{G}(\bar{K}/\bar{F})$, [11, 3-5-6]. The fact that K is unramified over F implies that $\mathscr{G}(K/F)$ and $\mathscr{G}(\bar{K}/\bar{F})$ are isomorphic. Also since E is totally ramified over K, it follows that $\bar{E} = \bar{K}$; thus, the $\bar{E}$-blocks are the same as the $\bar{K}$-blocks. Since E is a splitting field for G, the absolutely irreducible representations of G belong to $\bar{E}$-blocks, which by the preceding observation are actually $\bar{K}$-blocks.

THEOREM 2. *Let χ be a irreducible F-character with an absolutely irreducible constituent ζ. Then the defect groups of the $\bar{F}$-block to which χ belongs are the same as the defect groups of the $\bar{K}$-block to which ζ belongs.*

Proof. Since E is a splitting field for G, ζ is afforded by an irreducible EG-module. By [3, Theorem 1.4] $\chi = m_F(\zeta)\Sigma\zeta^{\sigma_i}$ where the sum is over a full set of conjugates of ζ by $\mathscr{G}(E/F)$and $m_F(\zeta)$ is the Schur index of ζ relative to F.

If δ and ε are the primitive central idempotents in EG and FG with which ζ and χ are associated respectively, then $\varepsilon = \Sigma\sigma_i(\delta)$.

New let $\bar{e}$ and $\bar{d}$ be the $\bar{F}$ and $\bar{K}$-blocks to which χ and ζ belong respectively. Since δ is an E-summand of ε, and ε is an F-summand of the O_F-block e; it follows that δ is an E-summand of e. If $e = \Sigma d_j$ is a decomposition of e into O_K-blocks, then one easily verifies that δ is an E-summand of some d_j. Thus $d = d_j$ and by Proposition 1, $\bar{e}$ and $\bar{d}$ have the same defect groups.

In the following two sections we shall say that a block $\bar{e}$ has defect a in case each of its defect groups have order p^a.

3. Defect zero. Throughout this section we assume that the residue field $\bar{F}$ is perfect. Since $\bar{K}$ is the residue field of E and E is a splitting field for G, it follows that the Brauer-Nesbitt Theorem [2, 86.3] holds for $\bar{K}$-blocks. Let $\bar{e}$ be an $\bar{F}$-block of defect zero. By Proposition 1, we have $\bar{e} = \sum_{i=1}^{r} \bar{\sigma}_i(\bar{d})$ where the $\bar{\sigma}_i$'s are chosen so that $\{\bar{\sigma}_i(\bar{d})\}_{i=1}^{r}$ is an orbit of $\bar{K}$-blocks each of defect zero.

LEMMA 3. *The $\bar{F}$-block $\bar{e}$ contains exactly one irreducible FG-module, one irreducible $\bar{F}G$-module, and one projective indecomposable module. If M is the unique irreducible $\bar{F}G$-module belonging to $\bar{e}$, then*

$$M^{\bar{K}} \cong \sum_{i=1}^{r} V^{\bar{\sigma}_i}$$

where V is the unique irreducible $\bar{K}G$-module belonging to $\bar{d}$ and the $\bar{\sigma}_i$'s are determined by the $\bar{K}$-decomposition of $\bar{e}$.

Proof. Observe that $\bar{e} = \sum_{i=1}^{r} \bar{\sigma}_i(\bar{d})$ where $\{\sigma_i(d)\}_{i=1}^{r}$ is an orbit of O_K-blocks relative to $\mathscr{G}(K/F)$. Since $\bar{d}$ contains only one irreducible EG-module [2, 86.3], it follows that d is itself a primitive central idempotent in both EG and KG. Therefore, e is a primitive central idempotent in FG; hence, $\bar{e}$ contains only the irreducible FG-module associated with the primitive central idempotent e.

Let V be the unique irreducible $\bar{K}G$-module which belongs to $\bar{d}$ [2, 86.3]. For $\bar{\sigma} \in \mathscr{G}(\bar{K}/\bar{F})$, let $V^{\bar{\sigma}}$ be a $\bar{K}G$-module affording the representation $T^{\bar{\sigma}}$ where T is a representation afforded by V. Here $T^{\bar{\sigma}}(g)$ is the matrix obtained by conjugating the entries of $T(g)$ by $\bar{\sigma}$. Now each $V^{\bar{\sigma}_i}$ belongs to $\bar{\sigma}_i(\bar{d})$, and by the fact that each $\bar{\sigma}_i(\bar{d})$ has defect zero and hence [2, 86.3] contains only $V^{\bar{\sigma}_i}$, it follows that $\{V^{\bar{\sigma}_i}\}_{i=1}^{r}$ is a full set of distinct conjugates of V by $\mathscr{G}(\bar{K}/\bar{F})$.

By [3, Theorem 1.4] and the fact that the Schur index $m_{\bar{F}}(V) = 1$, it follows that

$$M^{\bar{K}} \cong \sum_{i=1} V^{\bar{\sigma}_i}$$

for some irreducible $\bar{F}G$-module M. Since the $V^{\bar{\sigma}_i}$ are uniquely determined up to isomorphism by the $\bar{K}$-blocks $\bar{\sigma}_i(\bar{d})$, it follows by [2, 29.11] that M is the unique irreducible $\bar{F}G$-module belonging to $\bar{e}$. The fact that $\bar{e}$ contains only one projective indecomposable $\bar{F}G$-module follows from [2, § 55].

LEMMA 4. *Let L be an arbitrary field and let N be an arbitrary irreducible LG-module having an absolutely irreducible constituent W which affords a character ω. Then* $(\mathrm{Hom}_{LG}(N, N): L) = m_L^2(W)(L(\omega): L)$, *where $m_L(W)$ is the Schur index of W relative to L.*

Proof. For characteristic zero see [2, § 70 especially p. 469]. For characteristic p, let C be a Galois extension of L so that C is a splitting field for G and suppose that W is an irreducible CG-module. Then by [3, Theorem 1.4] $N^C \cong \Sigma W^{\sigma_i}$ where the sum is over a full set of conjugates of W by $\mathscr{G}(C/L)$.

$$\mathrm{Hom}_{LG}(N, N) \otimes_L C \cong \mathrm{Hom}_{CG}(N^c, N^c)$$

by [2, 29.5]. Since $W^{\sigma_i} \not\cong W^{\sigma_j}$ for $i \neq j$, $(\mathrm{Hom}_{CG}(N^c, N^c): C)$ is equal to the number of distinct conjugates of W. By [2, 30.15] this is the number of distinct conjugates of W by $\mathscr{G}(C/L)$. But from Galois theory this is $(L(\omega): L)$. Thus $(\mathrm{Hom}_{LG}(N, N): L) = (L(\omega): L)$ and the lemma is proven.

Let X^* be the unique irreducible FG-module belonging to $\bar{e}$. Choose an indecomposable O_FG-submodule X with $X^* = FX$ and $(X: O_F) = (X^*: F)$ [2, 73.6]. Denote by M and U respectively the unique irreducible and projective indecomposable $\bar{F}G$-modules belonging to $\bar{e}$.

THEOREM 5. *Let ζ be an absolutely irreducible character belonging to a block of defect zero. Then $m_F(\zeta) = 1$.*

Proof. Let χ be the irreducible F-character of which ζ is a constituent. By [3, Theorem 1.4] $\chi = m_F(\zeta) \sum_{i=1}^{r} \zeta^{\sigma_i}$ where $\{\zeta^{\sigma_i}\}_{i=1}^{r}$ is a full set of conjugates of ζ by $\mathscr{G}(E/F)$. Since ζ belongs to a block of defect zero, we have by [2, 83.6] $\zeta(g) = 0$ for non p-regular g. Since $\zeta(g) \in K$ for p-regular g, we may assume that each $\sigma_i \in \mathscr{G}(K/F)$. Let $\bar{d}$ be the $\bar{K}$-block to which ζ belongs. Observe that the restriction of ζ to the p-regular element of G is the Brauer character of the unique irreducible $\bar{K}G$-module V which belongs to $\bar{d}$. [2, 83.6] Also since each $\bar{\sigma}_i(\bar{d})$ has defect zero, ζ is the Brauer character of $V^{\bar{\sigma}_i}$. It follows by Lemma 3 that $\mu = \Sigma\zeta^{\rho_i}$ is the Brauer character of the unique irreducible FG-module M which belongs to $\bar{e}$. Since $\chi = m_F(\zeta)\mu$, it follows that the multiplicity of M in $\bar{X}$ is $m_F(\zeta)$.

Since X^* belongs to a block of defect zero, the O_FG-module X is projective [5, Corollary to 4.1a]. Select a primitive idempotent $\varepsilon \in O_FG$ so that $X \cong O_FG\varepsilon$. Since $X^* = FX$, it follows that $X^* \cong FG\varepsilon$, and so by [2, 54.19] we have

$$(3.1) \qquad (\varepsilon X^*: F) = (\mathrm{Hom}_{FG}(X^*, X^*): F) .$$

Also by [2, § 77] $\bar{\varepsilon}$ is primitive idempotent in $\bar{F}G$ and hence $U \cong \bar{F}G\bar{\varepsilon}$. Using the fact that $\bar{F}$ is perfect, we apply [2, 54.19] again to conclude that

$$(3.2) \qquad (\bar{\varepsilon}\bar{X}: \bar{F}) = d \cdot (\mathrm{Hom}_{\bar{F}G}(M, M): \bar{F})$$

where $d = m_F(\zeta)$ is the multiplicity of M in $\bar{X}$. Now by Lemma 4, $(\mathrm{Hom}_{FG}(X, X): F) = m_F^2(\zeta)(F(\zeta): F)$ and $(\mathrm{Hom}_{\bar{F}G}(M, M): \bar{F}) = (\bar{F}(\bar{\zeta}): \bar{F})$. Since $(\varepsilon X^*: F) = (\bar{\varepsilon}\bar{X}: \bar{F})$ and $(F(\zeta): F) = (\bar{F}(\bar{\zeta}): \bar{F})$, we have by (3.1), and (3.2),

$$m_F^2(\zeta) = m_F(\zeta) .$$

Thus, $m_F(\zeta) = 1$ and Theorem 5 is proven.

COROLLARY. *Let G be a finite group with a unique $\bar{K}$-block $\bar{d}$ of defect zero, and let ζ be the absolutely irreducible character belonging to $\bar{d}$. Then ζ is a rational-valued and the representation affording ζ is realizable in the p-adic completion Q_p of the rational field.*

THEOREM 6. *Let $\bar{e}$ be an $\bar{F}$-block of defect zero. Then $\bar{e}$ contains exactly one irreducible FG-module X^*, one irreducible $\bar{F}G$-module M, and one projective indecomposable module U. Moreover, $\bar{X} \cong U \cong M$.*

Proof. The first part is Lemma 3. That $\bar{X} \cong U \cong M$ follows from the fact that $m_F(\zeta) = 1$. For then Brauer character μ of M is equal to the Brauer character χ of $\bar{X}$.

4. **Highest defect.** Our notation is as in § 2. Observe that the Galois group $\mathscr{G}(\bar{K}/\bar{F})$ is isomorphic to a subgroup of U_m, the units modulo m. In § 4 we let σ_i denote the element of $\mathscr{G}(\bar{K}/\bar{F})$ which maps a primitive m^{th} root of one to its i^{th} power. Two p-regular elements x and y of G are said to be *$\bar{F}$-conjugate* in case $x^i = gyg^{-1}$ for some $g \in G$ and $\sigma_i \in \mathscr{G}(\bar{K}/\bar{F})$. $\bar{F}$-conjugacy is an equivalence relation on the p-regular elements of G and the equivalence classes are called *p-regular $\bar{F}$-conjugacy* classes. A defect group of an F-class $\mathscr{X}$ is any p-Sylow subgroup of the centralizer $C(x)$ where $x \in \mathscr{X}$. We estabilsh the following result using an argument similar to that in [4, 12.3].

THEOREM 7. *The number of $\bar{F}$-blocks of highest defect is equal to the number of p-regular $\bar{F}$-conjugacy classes of highest defect.*

Proof. By [9, Theorem 6.1] the number of $\bar{K}$-blocks of highest defect is equal to the number of p-regular conjugacy classes of highest defect. Let $\{\mathscr{C}_1, \cdots, \mathscr{C}_t\}$ be the ordinary p-regular conjugacy classes of highest defect and let $\{\bar{d}_1, \cdots, \bar{d}_t\}$ be the $\bar{K}$-blocks of highest defect. Now write each $\bar{d}_i$ as in [9, 3.2].

$$\bar{d}_i = \sum_{j=1}^{t} b_i(g_j)C_j + \sum_{k \in B} b_k C_k, \qquad g_j \in \mathscr{C}_j . \tag{4.1}$$

Let B be the $t \times t$ matrix

$$B = (b_i(g_j)) .$$

Using the homomorphisms associated with the $\bar{d}_i$'s in the proof of Pro-

position 1, one can show that the rows of B are independent, and thus that B is nonsingular.

By Proposition 1, $\mathscr{G}(\bar{K}/\bar{F})$ acts as a permutation group on the set $\{\bar{d}_i\}$ and the orbits of $\{\bar{d}_i\}$ relative to $\mathscr{G}(\bar{K}/\bar{F})$ are in one-to-one correspondence with the $\bar{F}$-blocks of highest defect. We observe that the action of $\mathscr{G}(\bar{K}/\bar{F})$ on $\{\bar{d}_i\}$ induces an action of $\mathscr{G}(\bar{K}/\bar{F})$ on the rows of B, and thus the orbits of the rows of B relative to $\mathscr{G}(\bar{K}/\bar{F})$ are in one-to-one correspondence with the $\bar{F}$-blocks of highest defect.

For a p-regular class $\mathscr{C}_j$ and $\sigma_i \in \mathscr{G}(\bar{K}/\bar{F})$ define $\sigma_i(\mathscr{C}_j) = \mathscr{C}_k$ where $x^i \in \mathscr{C}_k$ for $x \in \mathscr{C}_j$. We observe that the action of $\mathscr{G}(\bar{K}/\bar{F})$ on the p-regular classes preserves defect groups. The orbits of the set $\{\mathscr{C}_i\}_{i=1}^t$ relative to $\mathscr{G}(\bar{K}/\bar{F})$ are in one-to-one correspondence with p-regular $\bar{F}$-conjugacy classes of highest defect. Also the action of $\mathscr{G}(\bar{K}/\bar{F})$ on $\{\mathscr{C}_i\}_{i=1}^t$ induces an action of $\mathscr{G}(\bar{K}/\bar{F})$ on the columns of the matrix B and thus the p-regular $\bar{F}$-classes of highest defect are in one-to-one correspondence with the orbits of columns relative to $\mathscr{G}(\bar{K}/\bar{F})$.

By a well-known permutation lemma of Brauer [1, Lemma 3], Theorem 7 is proven once we establish that $b_i^{\sigma_k}(g_j) = b_i(g_j^k)$ for each $b_i(g_j)$ in the sum (4.1) and $\sigma_k \in \mathscr{G}(\bar{K}/\bar{F})$. For non-$p$-regular g, $b_i(g) = 0$. For p-regular g, we have $b_i(g) = \overline{c_i(g)}$ where $d_i = \sum_{g \in G} c_i(g)g$. But $d = \Sigma\delta(\zeta)$ where $\delta(\zeta)$ is the primitive central idempotent in EG with which ζ is associated and the sum is over all ζ belonging to $\bar{d}$. By [2, § 33]

$$\delta(\zeta) = \frac{\zeta(1)}{|G|} \sum_{g \in G} \zeta(g^{-1})g \ .$$

Thus

$$c(g) = \sum_{\zeta \in \bar{d}} \frac{\zeta(1)\zeta(g^{-1})}{|G|} \ .$$

K is unramified over F and therefore $\mathscr{G}(K/F) \cong \mathscr{G}(\bar{K}/\bar{F})$ under the map $\sigma \longrightarrow \bar{\sigma}$. It is easy to check that σ_k is the element $\mathscr{G}(K/F)$ corresponding to the unit k in U_m if and only if $\bar{\sigma}_k \in \mathscr{G}(\bar{K}/\bar{F})$ corresponds to k. Thus

$$\begin{aligned} c_i^{\sigma_k}(\delta) &= \sum_{\zeta \in \bar{\omega}_i} \frac{\zeta(1)\zeta^{\sigma_k}(g^{-1})}{|G|} \\ &= c_i(g^k) \end{aligned}$$

and therefore

$$b_i^{\bar{\sigma}_k}(g) = b_i(g^k) \ .$$

REMARK. If L is any field of characteristic p, then $L = \bar{F}$ for some

F where F is an unequal characteristic local field. [6, Theorem 2] Thus, Theorem 7 is actually a result on the number of blocks over an arbitrary field of characteristic p.

REFERENCES

1. Richard Brauer, *On the connection between the ordinary and modular characters of groups of finite order*, Ann. Math., **42** (1941), 926-935.
2. C. W. Curtis and I. Reiner, *Representations Theory of Finite Groups and Associative Algebras*, Interscience, New York. 1962.
3. Burton Fein, *Representations of direct products of finite groups*, Pacific J. Math., **20** (1967), 45-58.
4. Walter Feit, *Characters of Finite Groups*, W. A. Benjamin, New York, 1967.
5. J. A. Green, *Blocks of modular representations*, Math. Zeit., **79** (1962), 100-115.
6. S. MacLane, *Subfields and automorphism groups of p-adic fields*, Ann. Math., **40** (1939), 423-442.
7. W. F. Reynolds, *Block idempotents and normal p-subgroups*, Nagoya Math. J., **28** (1966), 1-13.
8. _____, *Blocks and F-class algebras of finite groups*, Pacific J. Math., **38** (1971), 193-205.
9. A. Rosenberg, *Blocks and centres of group algebras*, Math. Zeit., **76** (1961), 209-216.
10. B. L. Van der Waerden. *Modern Algebra*, Vol. II, Ungar, New York, 1949.
11. E. Weiss, *Algebraic Number Theory*, McGraw-Hill, New York, 1963.

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