PEAK INTERPOLATION SETS FOR SOME ALGEBRAS OF
ANALYTIC FUNCTIONS

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For certain algebras of analytic functions on holomorphically convex sets in $C^n$ metric sufficient conditions are given for a set (not necessarily compact) to be an interpolation set. The results extend the Rudin-Carleson theorem for the disc algebra.

Let $K$ be a compact subset of $C^n$ which is holomorphically convex, i.e. $K$ is the intersection of a decreasing sequence of pseudoconvex domains (see [4], Ch. 2). We denote by $H(K)$ the uniform closure on $K$ of the algebra of all functions analytic in a neighborhood of $K$, and by $A(K)$ the algebra of all continuous functions on $K$ analytic on $K^\circ$ (the interior of $K$). If $E$ is any subset of the boundary $\partial K$ of $K$ then we denote by $H^\infty_E$ the algebra of all bounded continuous functions on $K^\circ \cup E$ which are analytic on $K^\circ$. We show that if the boundary of $K$ is well behaved at each point of $E$, and $E$ satisfies a metric condition which says roughly that $E$ has zero 2-dimensional measure in the directions of the complex tangent and zero one dimensional measure in the orthogonal direction, then $E$ is a peak interpolation set (in an appropriate sense) for $H^\infty_{E \cup (\partial K \setminus E)}$. If $E$ is compact then it is a peak interpolation set in the usual sense ([2], p. 59) for the uniform algebra $H(K)$. We show also that if $E$ has zero one-dimensional measure then the conditions on $\partial K$ can be relaxed.

We say that $\partial K$ is strictly pseudoconvex in a neighborhood of a point $\zeta \in \partial K$ if there is an open neighborhood $V$ of $\zeta$ such that $V \cap \partial K$ is a $C^2$-submanifold of $V$ and the Levi form is positive definite at $\zeta$. Then we can find an open neighborhood $V$ of $\zeta$ and a $C^2$ strictly plurisubharmonic function $\rho$ in $V$ such that $K \cap V = \{z \in V; \rho(z) \leq 0\}$ and $\text{grad } \rho \neq 0$ on $V \cap \partial K$. (See [3] Prop. IX. A4).

**Lemma 1.** Let $K$ be a holomorphically convex compact set in $C^n$ and let $\zeta$ be a point of $\partial K$ in a neighborhood of which $\partial K$ is strictly pseudoconvex. We can find positive numbers $m_\zeta$ and $M_\zeta$ and $G_\zeta \in H(K)$, such that

(a) $\text{Re } G_\zeta(z) \geq m_\zeta |\zeta - z|^2, z \in K$

(b) $\text{Re } G_\zeta(z) \leq M_\zeta |\zeta - z|^2, z \in \partial K$

(c) $\text{grad } (\text{Re } G_\zeta)(\zeta) = - \text{grad } \rho(\zeta)$.

**Proof.** Put
Then the Taylor expansion of $\rho$ about $\zeta$ is

$$p(z) = 2\text{Re} F(z) + \sum_{i,j=1}^{n} \frac{\partial^2 \rho(\zeta)}{\partial \zeta_i \partial \zeta_j} (z_i - \zeta_i)(z_j - \zeta_j) + o(|z - \zeta|^2).$$

Since $\rho$ is strictly plurisubharmonic at $\zeta$ it follows that, shrinking $V$ if necessary, we can find $m > 0$ with $\text{Re} F(z) \leq -m|\zeta - z|^2$ for $z \in K \cap V$. Since $\rho = 0$ on $\partial K \cap V$ we also deduce that for some constant $M$

$$\text{Re} F(z) \leq -M|\zeta - z|^2; z \in \partial K \cap V.$$

Choose a pseudoconvex open neighborhood $U$ of $K$ so that $\text{Re} F < 0$ on an open neighborhood $W$ of $\partial V \cap U$ in $U$. Let $W_1 = W \cup (V \cap U)$ and $W_2 = W \cup (U \cap V)$, so that $W_1 \cup W_2 = U$ and $W_1 \cap W_2 = W$. By solving a Cousin problem in $U$ (see [4], Theorem 5.5.1) we can find analytic functions $g_1$ and $g_2$ on $W_1$ and $W_2$ respectively such that $g_2 - g_1 = F^{-1} \log F$ on $W$.

Put $h(z) = \begin{cases} F(z) \exp (F(z)^2 g_1(z)), & z \in W_1 \\ \exp (F(z)^2 g_2(z)), & z \in W_2. \end{cases}$

The definitions agree on $W$ so $h$ is an analytic function on $U$, $h(z) = 0$ only when $z = \zeta$, and in a neighborhood of $\zeta$, $h(z) = F(z) + O(|z - \zeta|^2)$. Thus $\text{Re} h \leq 0$ near $\zeta$, so there exists $\varepsilon > 0$ such that if $z \in K$ and $|h(z) - \varepsilon| \leq \varepsilon$ then $z = \zeta$. Put

$$G(z) = -\frac{h(z)}{\varepsilon - h(z)}, & z \in K.$$  

Then $G \in \bar{H}(K)$, $\text{Re} G(z) > 0$ for $z \in K \setminus \{\zeta\}$. Finally, near $\zeta$, $\text{Re} G(z) = -\varepsilon^{-1} \text{Re} F(z) + \varepsilon^{-1}(\text{Im} F)^2 + O(|z - \zeta|^2)$ from which it follows that $G_\zeta = 2\varepsilon G$ has the required properties.

If $S$ is a real subspace of $C^a$ and $Y$ is any subset we denote by $d_S(Y)$ the diameter (in the Euclidean metric) of the (real) orthogonal projection of $Y$ on $S$.

Let $K$ be a compact holomorphically convex subset of $C^a$ and suppose $\partial K$ is strictly pseudoconvex in a neighborhood of a point $\zeta \in \partial D$. Then in a neighborhood of $\zeta$ we can write $\partial K = \{z: \rho(z) = 0\}$ where $\rho(z)$ is strictly plurisubharmonic in a neighborhood of $\zeta$ and $\text{grad} \rho \neq 0$. The vector $i \text{ grad} \rho$ is orthogonal to $\text{grad} \rho$ and so lies in the (real) tangent space to $\partial K$ at $\zeta$; let $T(\zeta)$ be the orthogonal complement to $i \text{ grad} \rho$ in this space. Then $T(\zeta)$ is the unique complex subspace of the real tangent space with complex dimension $n - 1$. Let $L(\zeta)$ be the real
line spanned by the vector $i \langle i \nabla p \rangle$.

If $E$ is any subset of $\partial K$ we denote by $H^p_E$ the set of all bounded continuous functions on $K^e \cup E$ which are analytic on $K^e$. We define $A(K) = H^p_{\partial K}$.

**Theorem 1.** Let $F$ be a subset of $\partial K$ such that $\partial K$ is strictly pseudoconvex in a neighborhood of $F$. Suppose that for every $\varepsilon > 0$ the set $F$ can be covered by a sequence $\{V_i\}$ of open sets with diameter $< \varepsilon$ such that if $\zeta_i \in F \cap V_i$ for each $i$ then $\sum_i d_{L,\zeta_i}^p(V_i) < \varepsilon$ and $\sum_i (d_{T,\zeta_i}^p(V_i))^2 < \varepsilon$. Let $V$ be a neighborhood of $F$, let $\eta > 0$, and let $g$ be a bounded continuous function on $F$ with $\|g\| \leq 1$.

Then we can find $f \in H^p_{r(U \setminus (K \setminus F))}$ with $f|_F = g$, $\|f\| \leq 1$, and $|f| < \eta$ on $K \setminus V$.

The proof will be split up into lemmas.

**Lemma 2.** Let $F, V$ and $\eta$ be as in the theorem. Then we can find $f \in H^p_{r(U \setminus (K \setminus F))}$ with $f = 1$ on $F$, $\|f\| \leq 2$ and $|f| < \eta$ on $K \setminus V$.

**Proof.** For each $\zeta \in F$ we choose $m_\zeta, M_\zeta > 0$, and a function $G_\zeta \in H(K)$ as in Lemma 1.

If $W_\zeta$ is a sufficiently small open neighborhood of $\zeta$, then whenever $\zeta \in U \supseteq W_\zeta$ and $z \in U \cap \partial K$ we have

$$|G_\zeta(z)| \leq \text{Re} \ G_\zeta(z) + |\text{Im} \ G_\zeta(z)|$$
$$\leq A_\zeta|z - \zeta|^2 + |\langle \nabla \text{Im} \ G_\zeta(\zeta), z - \zeta \rangle|$$
$$\leq 2A_\zeta(d_1^2 + d_2^2) + |\nabla \rho(\zeta)| d_1$$
$$\leq B_\zeta(d_1^2 + d_2^2)$$

where $d_1 = d_{L,\zeta}(U)$, $d_2 = d_{T,\zeta}(U)$, $A_\zeta, B_\zeta$ do not depend on $z$, and $\langle \cdot, \cdot \rangle$ denotes the real scalar product.

For each positive integer $n$ let

$$F_n = \{\zeta \in F: B_\zeta < n, \|A_\zeta, 1/n\| \leq W_\zeta, m_\zeta d(\zeta, K \setminus V)^2 > 1/n, m_\zeta > 1/n\}.$$

Then $F = \bigcup_n F_n$. For each $n$ we choose a sequence $\{V_i^{(n)}\}$ of open sets with diameter less than $1/n$ such that each point of $F_n$ is contained in infinitely many $V_i^{(n)}$, and $\sum_i (d_{L,\zeta_i}^p(V_i^{(n)}) + (d_{T,\zeta_i}^p(V_i^{(n)}))^2) < \eta n^{-2-2}$ for some choice of $\zeta_i^{(n)} \in V_i^{(n)} \cap F_n$. Renumber the collection of all $V_i^{(n)}$ as $V_1, V_2, \ldots$. For each $j$ choose $n_j$ so that $V_j = V_i^{(n_j)}$ for some $i$, and let $\zeta_j = \zeta_i^{(n_j)}$. Let $G_j = G_{\zeta_j}$. Writing $c_j = d_{L,\zeta_j}(V_j) + (d_{T,\zeta_j}(V_j))^2$ we define

$$B_n(z) = \prod_{j=1}^r \frac{G_j(z)}{2n_j c_j + G_j(z)}$$
for $z \in K$, $r = 1, 2, \ldots$. 

Then $B_r \in H(K)$ and $|B_r| \leq 1$ on $K$. We claim that $\{B_r\}$ converges pointwise on $F \cup (K \setminus \overline{F})$ to a limit $B$ which is continuous on $F \cup (K \setminus \overline{F})$, analytic on $K^o$, zero at each point of $F$, with $\|B\| \leq 1$ and $|1 - B| < \eta$ on $K \setminus V$.

If $z \in K \setminus V$ then $\text{Re } G_j(z) \geq m_{\zeta_j} |z - \zeta_j|^2 > 1/n_j$, so

$$\sum_{j=1}^{\infty} \left| 1 - \frac{G_j(z)}{2n_j c_j + G_j(z)} \right| = \sum_{j=1}^{\infty} \frac{2n_j c_j}{2n_j c_j + G_j(z)} \leq \sum_{j=1}^{\infty} 2n_j^2 c_j < \eta/2,$$

which proves that $B_r(z)$ converges to a limit $B(z)$ with $|1 - B(z)| < \eta$. If $z \in K \setminus \overline{F}$ then

$$\sum_{j=0}^{\infty} \frac{2n_j c_j}{2n_j c_j + G_j(z)} \leq \sum_{j=0}^{\infty} \frac{2n_j^2 c_j}{|z - \zeta_j|^2} \leq d(z, F)^{-1} \sum_{j=0}^{\infty} 2n_j^2 c_j.$$

The series on the right converges, so $B_r$ converges uniformly to a limit $B$ on sets at positive distance from $F$, so $B$ is continuous on $K \setminus \overline{F}$ and analytic on $K^o$.

Finally let $z \in F$. Then $z \in V_j$ for infinitely many $j$. For each such $j$ we have $V_j \subseteq W_{\zeta_j}$ and for all $w \in W_{\zeta_j}$,

$$\left| \frac{G_j(w)}{2n_j c_j + G_j(w)} \right| \leq \frac{n_j c_j}{2n_j c_j} = \frac{1}{2}.$$

It follows that $B_r(z) \to 0$ and $\lim |B_r|$ is continuous at $z$. Thus $B$ has the asserted properties, and $f = 1 - B$ satisfies the requirements of the theorem.

**Lemma 3.** Let $X$ be a compact subset of $K$, $W$ a neighborhood of $X$, and $h$ a continuous function on $K$ with support in $X$ such that $\|h\| \leq 1$. Let $\eta > 0$.

Then there exists $f \in H_{F \cup (K \setminus \overline{F})}^\infty$ such that $|f - h| < \eta$ on $F$, $\|f\| \leq 3$, and $|f| < \eta$ on $K \setminus W$.

**Proof.** Choose $0 < \delta < d(X, K \setminus W)$ so small that $|h(x) - h(y)| < \eta/8$ whenever $x, y \in K$, $|x - y| < \delta$. We can easily find an integer $N > 0$, compact sets $X_1, \ldots, X_r$ contained in $X$, and open sets $W_1, \ldots, W_r$, with diameters $< \delta$, with $W_i \supseteq X_i$, $W_i \subseteq W$, such that

(a) if $x \in X$ and $N_x$ is the number of integers $i$ in $\{1, \ldots, r\}$ for which $x \in X_i$, $|N_x - N| < \eta N/8$

(b) if $x \in C^n$ the number of integers $i$ for which $x \in W_i \setminus X_i$ is less than $\eta N$.

Let $F_i = F \cap X_i$. For $i = 1, 2 \cdots r$ we can find by Lemma 2 functions $f_i \in H_{F_i \cup (K \setminus \overline{F})}^\infty$ with $f_i = 1$ on $F_i$, $\|f_i\| \leq 2$ and $|f_i| < \eta/3r$ on $K \setminus W_i$. **
Choose $x_i \in X_i$ for each $i$ and put $f(z) = \frac{1}{N} \sum_{i=1}^{N} f_i(z) h(x_i), z \in F \cup (K \setminus \bar{F})$. Clearly $f \in H^\infty_{F \cup (K \setminus \bar{F})}$ and $\|f\| \leq 3$ by (a). If $z \in K \setminus W$ then $|f_i(z)| < \eta/r$ for each $i$ so $|f(z)| < \eta$.

Finally let $z \in F$. Then

$$f(z) = \frac{1}{N} \left( \sum_{x \in X} \sum_{i \in W \setminus X} f_i(z) h(x) \right)$$

$$= f_1(z) + f_2(z) + f_3(z), \text{ say.}$$

We have

$$|f_1(z) - h(z)| \leq \frac{1}{N} \sum_{x \in X_i} f_i(z) (h(z) - h(x))$$

$$+ \left| 1 - \frac{N_x}{N} \right| < \frac{\eta N_x}{8N} + \left| 1 - \frac{N_x}{N} \right| < \frac{\eta}{3},$$

by (a), since $|z - x| < \delta$. Moreover, $|f_2(z)| < \sum_{i=1}^{N} \eta/3$, by (b) and $|f_3(z)| < \sum_{i=1}^{N} \eta/3r = \eta/3$, so that we have $|f(z) - h(z)| < \eta$ as required.

**Lemma 4.** With $F$ as in the theorem, if $W$ is any open neighborhood of $F$ and $h$ a bounded continuous function on $W$ with $\|h\| \leq 1$, we can find $G \in H^\infty_{F \cup (K \setminus \bar{F})}$ with $|G - h| < \eta$ on $F$, $\|G\| \leq 7$, and $|G| < \eta$ outside $W$.

*Proof.* Choose a sequence $\{W_n\}$ of relatively compact open subsets of $W$ with $W = \bigcup_{n=1}^{\infty} W_n$, such that $\bar{W}_m \cap \bar{W}_n = \emptyset$ if $|m - n| > 1$. We can write $h = \sum_{n=1}^{\infty} h_n$ on $W$ where $h_n \in C(K)$ has support in $W_n$ and $\|h_n\| \leq 1$. By Lemma 3 for each $n$ we can find $f_n \in H^\infty_{F \cup (K \setminus \bar{F})}$ with $|f_n - h_n| < 2^{-n} \eta$ on $F$, $\|f_n\| < 2^{-n} \eta$ on $K \setminus W$, and $\|f_n\| \leq 3$. Then $G = \sum_{n=1}^{\infty} f_n$ has the required properties.

*Proof of Theorem 1.* By Lemma 4 and using the fact that $g$ can be approximated uniformly by functions continuous in a neighborhood of $F$, we can construct by induction on $n$ a sequence $\{G_n\}_{n=0}^{\infty}$ in $H^\infty_{F \cup (K \setminus \bar{F})}$ such that, writing $f_n = G_0 + \cdots + G_n$ we have:

1. $|G_0 - g| < \lambda/7$ on $F$,

$$|G_n + f_{n-1} - (1 + \lambda + \cdots + \lambda^n)g| < \frac{\lambda^{n+1}}{7}$$

on $F$, $n > 1$, where $\lambda = 9/10$

2. $\|G_n\| \leq 7\|f_{n-1} - (1 + \lambda + \cdots + \lambda^n)g\|_F < 8\lambda^n$

3. $\|f_n\| \leq 1 + \lambda + \cdots + 9\lambda^n$. 

(To get (3), observe that by (1) we have $|f_{n-1}| < 1 + \lambda + \cdots + \lambda^{n-1} + \lambda^n/7$ on $F$, and hence on a neighborhood of $F$; if we make $|G_n| < \lambda^{n-1}/10$ outside this neighborhood then (3) follows from (2) and (3).

(4) $|G_n| < 2^{-n}$ on $K\setminus V$.

Then (2) shows that $f_n \to G$ say uniformly on $K$, so $G \in H^r_{F \cup \overline{K} \setminus F}$; by (1) $G = 10g$ on $F$ and by (3) $\|G\| \leq 10$. Finally by (4) $|G| < \eta$ on $K\setminus V$. Then $f = (1/10)G$ is the required function.

REMARK. The metric condition on $F$ in Theorem 1 is clearly satisfied if $F$ has zero one-dimensional Hausdorff measure; however it is also satisfied by sets which are thicker in the direction of the complex tangent space, e.g. any smooth arc in $\partial K$ whose tangent at each point lies in the complex tangent space.

If $F$ is compact then of course it is a peak interpolation set, so Theorem 1 extends the Rudin-Carleson theorem. The extension to non-closed sets in the case of the disc has been obtained independently by Detraz [1], and subsequently generalized to other domains in the plane by A. Stray (private communication).

If we assume that $F$ has zero one-dimensional Hausdorff measure then we can make do with a weaker pseudoconvexity hypothesis at the points of $F$. We say that $\partial K$ is point pseudoconvex at $z$ if there exists a neighborhood $N$ of $z$ and a real $C^2$ strictly plurisubharmonic function $\rho$ in $N$ such that $\rho(z) = 0$ and $\rho(z) \leq 0$ in $N \cap K$.

THEOREM 2. Let $K$ be holomorphically convex, and let $F$ be a subset of $\partial K$ with zero one-dimensional Hausdorff outer measure such that $\partial K$ is point pseudoconvex at each point of $F$. Let $V$ be a neighborhood of $F$ in $K$, let $\eta > 0$, and let $g$ be a bounded continuous function on $F$ with $\|g\| \leq 1$.

Then we can find $f \in H^r_{F \cup \overline{K} \setminus F}$ with $f|F = g$, $\|f\| \leq 1$ and $|f| < \eta$ on $K\setminus V$.

Proof. We show that the conclusion of Lemma 2 holds; the rest of the proof is just as before. As in the proof of Lemma 2 for each $\xi \in F$ we can find positive constants $m_\xi$ and $M_\xi$, a neighborhood $W_\xi$ of $\xi$, and $G_\xi \in H(K)$ such that

(a) $m_\xi |\xi - z| \leq \text{Re } G_\xi(z)$, $z \in K$
(b) $|G_\xi(z)| \leq M_\xi |\xi - z|$, $z \in K$.

Then whenever $\xi \in U \subset W_\xi$ and $z \in U$ we have $|G_\xi(z)| \leq M_\xi \text{diam } U$. We define $F_n$ as before and cover $F_n$ by balls $\mathcal{A}_n$ such that $\sum \text{diam } (\mathcal{A}_n) < \varepsilon n^{-2^{-n-2}}$. The rest of the proof goes just as before, with $c_j$ replaced by $\text{diam } (\mathcal{A}_j)$. 
COROLLARY. Let $F$ be a compact subset of $\partial K$ with zero 1-dimensional Hausdorff measure and assume $\partial K$ is point pseudoconvex at each point of $F$. Then $F$ is a peak interpolation set for $A(K)$.

Finally we remark that the functions obtained in Theorem 1 and 2 are actually pointwise limits on $K^0$ of bounded sequences in $H(K)$; this follows from the construction. If $F$ is compact the peak-interpolating functions constructed are in $\bar{H}(K)$; in this case the proof simplifies somewhat since it is only necessary to take finite products in Lemma 2 and the theorem follows from Lemma 2 by general theorems on peak interpolation sets.

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