ON ARITHMETIC PROPERTIES OF THE TAYLOR SERIES OF RATIONAL FUNCTIONS. II

DAVID GEOFFREY CANTOR
ON ARITHMETIC PROPERTIES OF THE TAYLOR SERIES OF RATIONAL FUNCTIONS, II

DAVID G. CANTOR

Suppose \( a_n, b_n, \) and \( c_n = a_n b_n \) are sequences of algebraic integers and that all \( b_n \) are nonzero. It is easy to verify that if both \( a(z) = \sum_{n=0}^{\infty} a_n z^n \) and \( b(z) = \sum_{n=0}^{\infty} b_n z^n \) are rational functions, then so is \( c(z) = \sum_{n=0}^{\infty} c_n z^n \). We are interested in studying the conjecture that if \( b(z) \) and \( c(z) \) are rational functions, then so is \( a(z) \). We shall prove this in the case that \( b(z) \) has no more than three distinct singularities.

Let \( k \) be an algebraic number field; denote by \( M_k \) the set of valuations of \( k \), normalized so as to satisfy the Artin product-formula. We assume, whenever convenient, that each valuation in \( M_k \) has been extended in some fashion to \( \Omega \), the algebraic closure of \( k \). Let \( S \) be a finite subset of \( M_k \) containing all Archimedean valuations. We say that \( \alpha \in k \) is an \( S \)-integer if \( |\alpha|_v \leq 1 \) for all \( v \in M_k - S \) and that \( \alpha \) is an \( S \)-unit if \( \alpha \) and \( 1/\alpha \) are both \( S \)-integers. Let \( a_n \) be a sequence of \( S \)-integers of \( k \). Suppose there exist rational functions \( b(z) = \sum_{n=0}^{\infty} b_n z^n \) and \( c(z) = \sum_{n=0}^{\infty} c_n z^n \) whose coefficients lie in an extension field \( K \) (possibly transcendental) of \( k \); suppose that none of the \( b_n \) are 0 and that \( a_n^c = c_b \) for \( n \geq 0 \). In [1], I showed that if \( b(z) \) has only one singularity (possibly a pole of high multiplicity) then \( a(z) = \sum_{n=0}^{\infty} a_n z^n \) is a rational function. In [6] G. Pathiaux extended this result by showing that, under the additional assumption that \( K \) is algebraic, if \( b(z) \) has at most two distinct singularities, then \( a(z) \) is rational.

Here we shall study various extensions of these results. In particular we shall show that if \( b(z) \) has at most three distinct singularities, then \( a(z) \) is rational.

We note that since \( b(z) \) and \( c(z) \) are rational functions, we may write \( b_n \) and \( c_n \) as exponential polynomials:

\[
\begin{align*}
(1) & \quad b_n = \sum_{i=1}^{r} \lambda_i(n) \theta_i^n \\
(2) & \quad c_n = \sum_{i=1}^{s} \mu_i(n) \varphi_i^n
\end{align*}
\]

for all sufficiently large \( n \). Here the \( \lambda_i(n) \) and \( \mu_i(n) \) are polynomials in \( n \). By appropriately enlarging \( K \), if necessary, we may assume that the \( \theta_i \), the \( \varphi_i \), and the coefficients of the polynomials \( \lambda_i(n) \) and \( \mu_i(n) \) all lie in \( K \). By omitting a finite number of terms from each of the sequences \( a_n, b_n, c_n \) we may assume that (1) and (2) hold for all \( n \geq 0 \). The purpose of the first lemma is to show that we may
assume that $K$ is algebraic over $k$.

**LEMMA 1.** Suppose $a_n$, $b_n$, $c_n$ are sequences as above. There exist sequences $\bar{b}_n$, $\bar{c}_n$ lying in a finite algebraic extension of $k$ with $\bar{b}(z) = \sum_{n=0}^{\infty} \bar{b}_n z^n$ and $\bar{c}(z) = \sum_{n=0}^{\infty} \bar{c}_n z^n$ rational functions such that $a_n \bar{b}_n - c_n$ for all integral $n \geq 0$ and such that only finitely many $\bar{b}_n$ are 0.

**Proof.** As above we may write $b_n = \sum_{i=1}^{\infty} \lambda_i(n) \theta_i^e$ and $c_n = \sum_{i=1}^{\infty} \mu_i(n) \varphi_i^e$.

If all the coefficients of the $\lambda_i$ and the $\mu_i$, and the $\theta_i$ and $\varphi_i$ are in $k$, then the Lemma is true with the $\bar{b}_n = b_n$ and $\bar{c}_n = c_n$. We henceforth assume this not the case. Let $R$ be the ring generated by adjoining the $\theta_i$, the $\varphi_i$, the ratios $\theta_i/\theta_j$, and the coefficients of the $\lambda_i$ and the $\mu_i$ to $k$. By the assumption above the transcendence degree $t$ of $R/k$ is $\geq 1$. We are going to construct a homomorphism $\tau$ of $R$ into a finite algebraic extension of $k$ such that $\tau$, when restricted to $k$, will be the identity. If $\tau \alpha$ is abbreviated $\bar{\alpha}$ then $\bar{b}_n = \sum_{i=1}^{\infty} \lambda_i(n) \theta_i^e$ and thus $\sum \bar{b}_n \bar{\alpha}^e$ is rational; similarly $\sum \bar{c}_n \bar{\alpha}^e$ is rational and since $a_n \bar{b}_n = c_n$ clearly $a_n \bar{b}_n = \bar{c}_n$. The remainder of this proof is devoted to constructing such a homomorphism $\tau$ for which only finitely many $\bar{b}_n$ are zero. By the Noether normalization lemma [3], there exists a transcendence basis $x = (x_1, x_2, \cdots, x_t)$ for $R/k$ such that each element of $R$ is integral over $k[x]$. Since $R/k[x]$ is algebraic and finitely generated, its degree $d$ is finite. Each element $\alpha$ in $R$ satisfies a polynomial equation $f(\alpha) = 0$, where

$$f(Y) = \sum_{i=0}^{e} p_i(x) Y^{e-i}$$

is a polynomial with coefficients $p_i(x)$ in $k[x]$, of degree $e \leq d$, and monic ($p_0(x) = 1$). Any homomorphism $\tau$ of $k[x]$ into $k$, which is the identity on $k$, has the form $p(x) \rightarrow p(u)$ where $u = (u_1, u_2, \cdots, u_t)$ is a $t$-tuple of elements of $k$ and $p(x)$ is in $k[x]$. Such a homomorphism $\tau$ can be extended to a homomorphism of $R$ into $\Omega$, the algebraic closure of $k [3]$. The image $\bar{\alpha}$ of $\alpha$ will satisfy the monic polynomial $\sum_{i=0}^{e} p_i(u) Y^{e-i}$ and hence have degree $\leq e$ over $k$. Since $e \leq d$, every element in $\tau R$ will have degree $\leq d$ over $k$ and hence $\tau R$ will be contained in a finite algebraic extension of $k$. Moreover if $p_e(u) \neq 0$, then $\bar{\alpha} \neq 0$. Denote by $\Phi_k(h)$ the degree of the field generated by the primitive $h^{th}$ roots of unity over $k$. It is easy to verify that $\Phi_k(h) \geq \Phi_Q(h)/[k: Q]$ where $Q$ is the field of rational numbers and $\Phi_Q(h)$ is, of course, Euler's phi-function. Since $\Phi_Q(h) \rightarrow \infty$ as $h \rightarrow \infty$, so does $\Phi_k(h)$. Let $h$ be the largest integer for which $\Phi_k(h) \leq d$. Let $m$ be the least common multiple of all of the orders of all of the roots of unity which can be written in the form $\theta_i/\theta_j$. We can write
where the $\sigma_i$ are the distinct $m^{th}$ powers of the $\theta_i$, and the $\gamma_{is}(n)$ are polynomials, not all 0 (for each value of $s$). Let $\alpha$ be the product of all the nonzero coefficients of the $\gamma_{is}(n)$ and the elements $(\sigma_i/\sigma_j)^{k_1} - 1$ for $i \neq j$ (the latter quantities are not 0 since the ratios $\sigma_i/\sigma_j$ cannot be roots of unity). Now let $u = (u_1, u_2, \ldots, u_t)$ be elements of $h$ for which $p_s(u) \neq 0$. Then under the homomorphism $\tau$, defined above, $\alpha = \tau\alpha$ will be nonzero, and $\overline{\gamma}_{is}(n)$ (the polynomial obtained by applying $\tau$ to each coefficient of the polynomial $\gamma_{is}(n)$) will be the zero-polynomial if and only if $\gamma_{is}(n)$ is the zero-polynomial. None of the ratios $\overline{\sigma}_i/\overline{\sigma}_j$, with $i \neq j$, are roots of unity, for since $(\overline{\sigma}_i/\overline{\sigma}_j)^{k_1} \neq 1$, if $\overline{\sigma}_i/\overline{\sigma}_j$ were a root of unity, it would have to have order $> h$ and hence degree $> d$ over $k$; but the latter is not the case. If any of the $m$ sequences $\overline{b}_{mn+\pm}$ had infinitely many zeros then either all of the polynomials $\overline{\gamma}_{is}(n)$ would be zero or by a theorem of Mähler [4] and Lech [5] the zeros would be periodic, and two of the $\overline{\sigma}_i$ would have root a unity. Thus the sequence $b_n$ has only finitely many zeros.

**Lemma 2.** Suppose $a_n$ is a sequence of $S$-integers of $k$, that $a_n = c_n/b_n$ where $b_n = \sum \lambda_i(n)\theta_i^n$ is never 0 and $c_n = \sum \mu_i(n)\varphi_i^n$; suppose the $\theta_i$, $\varphi_i$ and the coefficients of the $\lambda_i(n)$ and the $\mu_i(n)$ are integers of $k$. Suppose there exists a valuation $v_0 \in S$ such that $|\theta_i|_{v_0} > |\theta_i|_{v_0}$ for $i \geq 2$. Then $\sum_{n=0}^{\infty} a_nz^n$ is rational.

**Proof.** Elementary estimates show there exist $M > 0$ and $R > 0$ such that $|b_n|_v$ and $|c_n|_v$ are $\leq MR^n$ for all $v \in S$ and $n \geq 0$, and that $|b_n|_v \leq 1$ for all $v \notin S$ and $n \geq 0$. Since $\prod_{v \in S} |b_n|_v \geq 1$, if $w \in S$, then

$$\left|\frac{1}{b_n}\right|_w \leq \prod_{v \in S} |b_n|_v \leq M^{s-1}R^{(s-1)n}$$

where $s$ is the cardinality of $S$. Then $|a_n|_w = |c_n/b_n|_w \leq M^sR^n$. It follows that $\sum_{n=0}^{\infty} a_nz^n$ has positive radius of convergence in $k_w$, the completion of $k$ under the valuation $w$. Let $\overline{k}_w$ be the algebraic closure of $k_w$, and assume that $w$ has been extended to $\overline{k}_w$. Let $R_w$ be the radius convergence of $a(z) = \sum_{n=0}^{\infty} a_nz^n$ in $k_w$. Then $a(z)$ is analytic in $\overline{k}_w$ for $|z|_w < R_w$. Now

$$\lambda_i(n)\theta_i^\tau a_n = c_n - \sum_{i=2}^{r} \lambda_i(n)\theta_i^\tau a_n$$

or

$$\sum_{n=0}^{\infty} \lambda_i(n)a_nz^n = c\left(\frac{z}{\theta}\right) - \sum_{i=2}^{r} \lambda_i(n)\left(\frac{\theta_i}{\theta_1}\right)^n a_nz^n.
$$

In the field $\overline{k}_{v_0}$, the algebraic closure of $k_{v_0}$, the last equation expresses
\[ \sum_{n=0}^{\infty} \lambda_i(n) a_n z^n \] as a rational function plus a sum of functions each meromorphic for \( |z|_v \leq \delta R_v \) where \( \delta = \min_{i \geq 2} |\theta_i|_v |v_0| \) is \( > 1 \). Thus by analytic extension \( \sum_{n=0}^{\infty} \lambda_i(n) a_n z^n \) is meromorphic for \( |z|_v < \delta R_v \).

Repeated applications of the above transformation show that \( \sum_{n=0}^{\infty} \lambda_i(n) a_n z^n \) is meromorphic for \( |z|_v < \delta R_v \). Elementary estimates show that \( \sum_{n=0}^{\infty} \lambda_i(n) a_n z^n \) has radius of convergence \( R_v \) for all \( v \in S \). Choosing \( j \) so large that \( \delta_j \Pi_{v \in S} R_v \) is \( > 1 \), we find, by a theorem of Dwork [2], that \( \sum_{n=0}^{\infty} \lambda_i(n) a_n z^n \) is a rational function. By [1] so is \( \sum_{n=0}^{\infty} a_n z^n \).

**Lemma 3.** Suppose \( a_n \) is a sequence of \( S \)-integers of \( k \), that \( a_n = c_n/b_n \) where \( b_n = \sum \lambda_i(n) \theta_i^n \) is never zero and \( c_n = \sum \mu_i(n) \varphi_i^n \), suppose the \( \theta_i \), \( \varphi_i \) and the nonzero coefficients of the \( \lambda_i(n) \) and the \( \mu_i(n) \) are \( S \)-units of \( h \). Suppose there exists a valuation \( v_0 \) of \( h \) such that \( |\theta_i|_{v_0} < |\theta_i|_v \) for \( i \geq 2 \). Then \( \sum_{n=0}^{\infty} a_n z^n \) is rational.

**Proof.** Extend the definition of \( c_n \) and \( b_n \) to negative \( n \) by their formulas. If infinitely many such \( b_n \) were zero, then by a theorem of Lech [4] and Mährer [5], \( b_n \) would be zero for all \( n \) in a doubly infinite arithmetic progression, contradicting the hypotheses. Extend the definition of \( a_n \) to negative \( n \) by putting \( a_n = c_n/b_n \) if \( b_n \neq 0 \) and otherwise put \( a_n = 0 \). Now let \( v \) be any valuation of \( M_k \) not in \( S \). Then \( v \) is an extension of a \( p \)-adic valuation \( | \cdot |_p \) of \( Q \). There exists an integer \( f \) such that if \( \alpha \in k \) and \( |\alpha|_v = 1 \) then \( |\alpha^{p^f} - 1|_v < 1 \).

Letting \( m \) be an integer of the form \( p^h(p^f - 1) \), where \( h \) is large, we find that \( |\alpha^m - 1|_v \) can be made very small. In particular we can choose \( m \) so large that if \( b_n \neq 0 \) then \( |b_{n+m}|_v = |b_n|_v \) and that \( |c_{n+m} - c_n|_v < |b_n|_v \). We can choose \( m \) so large that \( m + n \geq 0 \) and then \( |c_{n+m} - b_{m+n}|_v \leq 1 \). Thus \( |a_n|_v \leq 1 \). Restating all this, we have shown that there exists \( n_0 \) such that if \( n \leq n_0 \) then \( b_n \neq 0 \) and if \( v \in S \) then \( |a_n|_v \leq 1 \). We apply Lemma 2 to the sequences \( a'_n = a_{n_0 - n}, b'_n = b_{n_0 - n} \) and \( c'_n = c_{n_0 - n} \), to conclude that \( \sum_{n=0}^{\infty} a'_n z^n \) is rational. It follows that \( a_n \) can be written in the form

\[ a_n = \sum_{i=1}^{s} \gamma_i(n) \sigma_i^n \]

for \( n \leq n_0 \). Then the exponential polynomial

\[ \sum_{i=1}^{s} \mu_i(n) \varphi_i^n - \sum_{i=1}^{s} \lambda_i \theta_i^n \sum_{i=1}^{t} \gamma_i(n) \sigma_i^n \]

is 0 for \( n \leq n_0 \). By the theorem of Mährler [5] and Lech [4], it is identically 0. Thus \( a_n = \sum_{i=1}^{t} \gamma_i(n) \sigma_i^n \) for \( n \geq 0 \) and \( a(z) = \sum_{n=0}^{\infty} a_n z^n \) is a rational function.

We now come to the result mentioned at the beginning of this
ON ARITHMETIC PROPERTIES OF THE TAYLOR SERIES

THEOREM 4. Suppose \( a_n \) is a sequence of \( S \)-integers of \( k \) and that \( b_n \) and \( c_n \) are sequences of elements of an extension field \( K \) of \( k \) such that \( \sum_{n=0}^{\infty} b_n z^n \) and \( \sum_{n=0}^{\infty} c_n z^n \) are rational functions and \( b_n \) is never zero. If \( a_n = c_n/b_n \) and the rational function \( \sum_{n=0}^{\infty} b_n z^n \) has at most 3 distinct singularities then \( \sum_{n=0}^{\infty} a_n z^n \) is rational.

Proof. By Lemma 1, we may assume \( K \) is algebraic over \( k \) and that \( b_n = \sum_{i=1}^{\infty} \lambda_i(n) \theta_i^i \) and that \( c_n = \sum_{i=1}^{\infty} \mu_i(n) \varphi_i^i \) where the \( \theta_i, \varphi_i \) and all coefficients of the \( \lambda_i \) and \( \mu_i \) are algebraic over \( k \). By replacing \( k \) by a larger field and \( S \) by the set of extensions of the valuations in \( S \) to this new field, we may assume that the above quantities are, in fact, in \( k \). By increasing \( S \) appropriately, we may assume that those of the above quantities which are not zero are \( S \)-units. Now if \( r = 1 \), the theorem follows immediately from [1]. If \( r = 2 \) then either \( \theta_1/\theta_2 \) is a root of unity, in which case the theorem follows from the case \( r = 1 \) or there is a valuation \( v \) such that \( |\theta_1|_v > |\theta_2|_v \), and the theorem follows from Lemma 2. If \( r = 3 \) then either \( |\theta_1|_v = |\theta_2|_v = |\theta_3|_v \), for all \( v \in S \) and \( \theta_1/\theta_2 \) and \( \theta_1/\theta_3 \) are roots of unity, so the theorem follows from the case \( r = 1 \), or there is a valuation \( v_0 \in S \) for which not all of the three values are equal. In the latter case we may assume that \( |\theta_1|_{v_0} \leq |\theta_2|_{v_0} \leq |\theta_3|_{v_0} \) and \( |\theta_1|_{v_0} < |\theta_3|_{v_0} \). If \( |\theta_2|_{v_0} = |\theta_3|_{v_0} \) then the theorem follows from Lemma 3, and otherwise from Lemma 2.

It is worth noting that the method of the theorem cannot be extended to the case where \( b(z) \) has 4 singularities. In fact, consider the case where \( k \) is the field \( Q(i) \) where \( i = \sqrt{-1} \) and \( \theta_1 = (1 + 2i) \times (1 + 4i), \theta_2 = (1 + 2i)(1 - 4i), \theta_3 = (1 - 2i)(1 + 4i), \theta_4 = (1 - 2i)(1 - 4i) \). The ideals generated by \( (1 + 2i), (1 - 2i), (1 + 4i), (1 - 4i) \) are prime and give rise to 4 valuations of \( Q(i) \). At each of these valuations, two of the \( \theta_j \) take one value and two another. For example at the valuation corresponding to the prime ideal generated by \( 1 - 2i \), \( \theta_1 \) and \( \theta_2 \) both have value 1, while \( \theta_3 \) and \( \theta_4 \) both have the same value which is less than 1. All 4 \( \theta_j \) take the same value at all other valuations. Thus the hypotheses of Lemma 2 or Lemma 3 cannot be met.

REFERENCES


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<table>
<thead>
<tr>
<th>Author</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tom M. (Mike) Apostol</td>
<td>Arithmetical properties of generalized Ramanujan sums</td>
</tr>
<tr>
<td>David Lee Armacost and William Louis Armacost</td>
<td>On p-thetic groups</td>
</tr>
<tr>
<td>Janet E. Mills</td>
<td>Regular semigroups which are extensions of groups</td>
</tr>
<tr>
<td>Gregory Frank Bachelis</td>
<td>Homomorphisms of Banach algebras with minimal ideals</td>
</tr>
<tr>
<td>John Allen Beachy</td>
<td>A generalization of injectivity</td>
</tr>
<tr>
<td>David Geoffrey Cantor</td>
<td>On arithmetic properties of the Taylor series of rational functions. II</td>
</tr>
<tr>
<td>Václav Chvátal and Frank Harary</td>
<td>Generalized Ramsey theory for graphs. III. Small off-diagonal numbers</td>
</tr>
<tr>
<td>Frank Rimi DeMeyer</td>
<td>Irreducible characters and solvability of finite groups</td>
</tr>
<tr>
<td>Robert P. Dickinson</td>
<td>On right zero unions of commutative semigroups</td>
</tr>
<tr>
<td>John Dustin Donald</td>
<td>Non-openness and non-equidimensionality in algebraic quotients</td>
</tr>
<tr>
<td>John D. Donaldson and Qazi Ibadur Rahman</td>
<td>Inequalities for polynomials with a prescribed zero</td>
</tr>
<tr>
<td>Robert E. Hall</td>
<td>The translational hull of an N-semigroup</td>
</tr>
<tr>
<td>John P. Holmes</td>
<td>Differentiable power-associative groupoids</td>
</tr>
<tr>
<td>Steven Kenyon Ingram</td>
<td>Continuous dependence on parameters and boundary data for nonlinear two-point boundary value problems</td>
</tr>
<tr>
<td>Robert Clarke James</td>
<td>Super-reflexive spaces with bases</td>
</tr>
<tr>
<td>Gary Douglas Jones</td>
<td>The embedding of homeomorphisms of the plane in continuous flows</td>
</tr>
<tr>
<td>Mary Joel Jordan</td>
<td>Period H-semigroups and t-semisimple periodic H-semigroups</td>
</tr>
<tr>
<td>Ronald Allen Knight</td>
<td>Dynamical systems of characteristic 0</td>
</tr>
<tr>
<td>Kwangil Koh</td>
<td>On a representation of a strongly harmonic ring by sheaves</td>
</tr>
<tr>
<td>Hui-Hsiung Kuo</td>
<td>Stochastic integrals in abstract Wiener space</td>
</tr>
<tr>
<td>Thomas Graham McLaughlin</td>
<td>Supersimple sets and the problem of extending a retracing function</td>
</tr>
<tr>
<td>William Nathan</td>
<td>Open mappings on 2-manifolds</td>
</tr>
<tr>
<td>M. J. O’Malley</td>
<td>Isomorphic power series rings</td>
</tr>
<tr>
<td>Sean B. O’Reilly</td>
<td>Completely adequate neighborhood systems and metrization</td>
</tr>
<tr>
<td>Qazi Ibadur Rahman</td>
<td>On the zeros of a polynomial and its derivative</td>
</tr>
<tr>
<td>Russell Daniel Rupp, Jr.</td>
<td>The Weierstrass excess function</td>
</tr>
<tr>
<td>Hugo Teufel</td>
<td>A note on second order differential inequalities and functional differential equations</td>
</tr>
<tr>
<td>M. J. Wicks</td>
<td>A general solution of binary homogeneous equations over free groups</td>
</tr>
</tbody>
</table>