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## **A GENERALIZATION OF THE PRIME RADICAL IN NONASSOCIATIVE RINGS**

HYO CHUL MYUNG

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In [5] Tsai defined the Brown-McCoy prime radical for Jordan rings in terms of the quadratic operation and proved basic results for the radical. In this paper we give a definition of the prime radical for arbitrary nonassociative rings in terms of a  $*$ -operation defined on the family of ideals and of a function  $f$  of the ring into the family of ideals in the ring. The prime radical for Jordan or standard rings is obtained by a particular choice of the  $*$ -operation and the function  $f$ . We also extend the results for the Jordan case to weakly  $W$ -admissible rings which include the generalized standard rings and therefore alternative and standard rings as well as Jordan rings.

1. Let  $K$  be any nonassociative ring and let  $\mathcal{I}(K)$  denote the family of ideals of  $K$ .

DEFINITION 1. We define a  $*$ -operation as a mapping of  $\mathcal{I}(K) \times \mathcal{I}(K)$  into the family of additive subgroups of  $K$  such that

(\*1) for  $A, B, C$ , and  $D$  in  $\mathcal{I}(K)$  if  $A \subseteq C$  and  $B \subseteq D$ , then  $A*B \subseteq C*D$ ,

(\*2)  $(0)*A = B*(0) = (0)$  for all  $A, B$  in  $\mathcal{I}(K)$ ,

(\*3)  $\overline{A*B} = \overline{A}*\overline{B}$  for any homomorphic images  $\overline{A}$  and  $\overline{B}$  of  $A$  and  $B$  in  $\mathcal{I}(K)$ .

If  $K$  is a Jordan ring, let  $U_x \equiv 2R_x^2 - R_{x^2}$  be the quadratic operation and  $AU_B$  be the additive subgroup of  $K$  generated by  $xU_y$ ,  $x \in A$  and  $y \in B$ . Then the  $U$ -operation satisfies the conditions above. If the characteristic is not 2, it is shown in [5] that  $AU_A = AA^2$  and is an ideal of  $K$  for  $A$  in  $\mathcal{I}(K)$ .

For any ring  $K$  and  $A, B$  in  $\mathcal{I}(K)$ , if we define  $A*B$  as the additive subgroup  $AB^2 + B^2A + (AB)B + (BA)B$ , then  $A*B$  also satisfies the conditions in Definition 1. In case  $K$  is a standard ring, it is shown in [6] that  $A*B$  is an ideal of  $K$  for  $A, B$  in  $\mathcal{I}(K)$ . If  $K$  is commutative or anticommutative, then  $A*B = AB^2 + (AB)B$ . In particular, if  $K$  is a Lie ring,  $A*B$  is an ideal of  $K$ . Since  $A^2$  is not in general an ideal of  $K$  for  $A$  in  $\mathcal{I}(K)$ , but there are considerably broad classes of nonassociative rings in which  $A^3 \equiv AA^2 + A^2A$  is an ideal of  $K$  for every ideal  $A$ , this example will be particularly interesting.

We recall that a noncommutative Jordan ring  $K$  is one satisfying

the flexible law  $(x, y, x) = 0$  and the Jordan identity  $(x, y, x^2) = 0$  for all  $x, y$  in  $K$ , where  $(x, y, z) = (xy)z - x(yz)$ . Most of the well known nonassociative rings are included in the class of noncommutative Jordan rings. Recently Thedy [4] defined a considerably broad class of algebras that generalizes many of the well known algebras.

DEFINITION 2. A noncommutative Jordan ring  $K$  is called weakly  $W$ -admissible if it satisfies

$$[(a, b, c), c] - ([a, c], c, b) = 0,$$

and

$$\begin{aligned}
 & ([a, b], d, c) + ([b, c], d, a) + ([c, a], d, b) \\
 & = p[(a, b, c), d] + q[S(a, b, c), d] + r[d, [b, [a, c]]]
 \end{aligned}$$

for some integers  $p, q, r$  such that either  $m(p, q, r) \equiv 3 + 2p + 6q - 4r \neq 0$ , or  $n(p, r) \equiv p + 4r \neq 0$ , where  $[a, b] = ab - ba$  and  $S(a, b, c) = (a, b, c) + (b, c, a) + (c, a, b)$ .

Thedy called a noncommutative Jordan algebra over a field  $W$ -admissible if it satisfies the identity  $[a, (a, a, b)] = 0$  and the two identities above for  $p, q, r$  in the field such that either  $m(p, q, r) \neq 0$  or  $n(p, r) \neq 0$ . He proved that if the characteristic is not 2, then any generalized standard ring of Schafer [2] is  $W$ -admissible with  $p = -2$  and  $q = r = 0$ . Therefore, weakly  $W$ -admissible rings include generalized standard rings and hence alternative and standard rings as well as Jordan rings. In case the characteristic is not 2, it is also shown in [4, p. 192] that in any weakly  $W$ -admissible ring  $K$ ,  $A^3$  is an ideal of  $K$  for  $A$  in  $\mathcal{S}(K)$ .

LEMMA 1.1. Let  $K$  be any ring. Then the conditions (\*2) and (\*3) imply

- (i)  $(A + C)*(B + C) \subseteq A*B + C$ , and
- (ii)  $A*B \subseteq A \cap B$

for ideals  $A, B, C$  of  $K$ .

*Proof.* Consider the quotient ring  $\bar{K} = K/C$ , then by (\*3)  $\overline{(A + C)*(B + C)} = \bar{A}*\bar{B} = \overline{A*B}$ , and hence (i). Let  $\bar{K} = K/A$ , then  $\bar{A}*\bar{B} = \bar{A}*\bar{B} = (\bar{0})*\bar{B} = (\bar{0})$  by (\*2) and so  $A*B \subseteq A$ . Similarly  $A*B \subseteq B$  and  $A*B \subseteq A \cap B$ .

DEFINITION 3. Let  $K$  be any ring. Then  $f$  is defined as a function of  $K$  into  $\mathcal{S}(K)$  such that for every  $a$  in  $K$

- (f 1)  $a \in f(a)$ ,
- (f 2) if  $x \in f(a)$ , then  $f(x) \subseteq f(a)$ ,

(f 3)  $\overline{f(a)} = f(\bar{a})$ , where  $\bar{a}$  is a homomorphic image of  $a$ .

The principal ideal  $(a)$  generated by  $a$  in  $K$  is an example of  $f(a)$ . Now let  $S$  be a subset of  $K$  and define  $f(a)$  to be the ideal  $(a, S)$  generated by  $a$  and  $S$ . Then  $f$  satisfies the conditions above. A similar function to  $f$  has been defined in [1] for the associative case and in [3].

Henceforth we assume that  $f$  denotes a function of  $K$  into  $\mathcal{S}(K)$  satisfying (f 1), (f 2), and (f 3). Then clearly  $(a) \subseteq f(a)$ . For an ideal  $A$  of  $K$ , we denote the ideal  $\sum_{a \in A} f(a)$  by  $f(A)$ . Then  $A \subseteq f(A)$  and  $f(A) \subseteq f(B)$  if  $A \subseteq B$ , and also  $f((a)) = f(a)$ . But in general  $f(A) \neq A$  as shown by the example  $f(a) = (a, S)$  for a subset  $S$  of  $K$ . Let  $\mathcal{S}'(K)$  denote the family of ideals  $f(A)$  for  $A$  in  $\mathcal{S}(K)$ . Then  $\mathcal{S}'(K) \subseteq \mathcal{S}(K)$  and in particular, if  $f$  is such that  $f(a) = (a)$  for all  $a$  in  $K$ , then  $f(A) = A$  and  $\mathcal{S}'(K) = \mathcal{S}(K)$ .

2. In this section we give a definition of the prime radical for any ring in terms of the  $*$ -operation and the function  $f$ .

LEMMA 2.1. *Let  $K$  be any ring where the  $*$ -operation and the function  $f$  are defined. For an ideal  $P$  of  $K$ , the following are equivalent:*

- (i) *If  $f(A)*f(B) \subseteq P$  for  $A, B$  in  $\mathcal{S}(K)$ , then either  $f(A) \subseteq P$  or  $f(B) \subseteq P$ .*
- (ii) *If we have  $f(A) \cap c(P) \neq \emptyset$  and  $f(B) \cap c(P) \neq \emptyset$ , then  $f(A)*f(B) \cap c(P) \neq \emptyset$ .*
- (iii) *If  $a$  and  $b$  are in  $c(P)$ , then  $f(a)*f(b) \cap c(P) \neq \emptyset$ .*

*Proof.* We need only to show that (ii) and (iii) are equivalent. Let  $a$  and  $b$  be in  $c(P)$ , then  $f(a) \cap c(P) \neq \emptyset$  and  $f(b) \cap c(P) \neq \emptyset$ . Hence (ii) implies (iii). Now let  $A$  and  $B$  be ideals of  $K$  with  $f(A) \cap c(P) \neq \emptyset$  and  $f(B) \cap c(P) \neq \emptyset$ . Let  $a \in f(A) \cap c(P)$  and  $b \in f(B) \cap c(P)$ . Assuming (iii), we get  $f(a)*f(b) \cap c(P) \neq \emptyset$  and by  $(*)$   $f(A)*f(B) \cap (P) \neq \emptyset$ , thus (ii) holds.

DEFINITION 4. (i) An ideal  $P$  of  $K$  is called  $f^*$ -prime if it satisfies any one of Lemma 2.1. A nonempty subset  $M$  of  $K$  is called an  $f^*$ -system if, for  $A, B$  in  $\mathcal{S}(K)$ ,  $f(A) \cap M \neq \emptyset$  and  $f(B) \cap M \neq \emptyset$  imply  $f(A)*f(B) \cap M \neq \emptyset$ .

(ii) An ideal  $P$  of  $K$  is called  $f^*$ -semiprime if, for any ideal  $A$  of  $K$ ,  $f(A)*f(A) \subseteq P$  implies  $f(A) \subseteq P$ . A nonempty subset  $M$  of  $K$  is called an  $sf^*$ -system if, for  $A$  in  $\mathcal{S}(K)$ ,  $f(A) \cap M \neq \emptyset$  implies  $f(A)*f(A) \cap M \neq \emptyset$ .

An ideal  $P$  is  $f^*$ -prime if and only if  $c(P)$  is an  $f^*$ -system. Similarly, an ideal  $P$  is  $f^*$ -semiprime if and only if  $c(P)$  is an  $sf^*$ -

system. Let  $K$  be a Jordan or standard ring. If we define  $A*B$  as  $AU_B$  or as  $AB^2 + B^2A + (AB)B + (BA)B$  and define  $f(a)$  as  $(a)$  for every  $a$  in  $K$ , then the definition of  $f^*$ -prime and  $f^*$ -semiprime ideals coincide with those in [5] or in [6].

DEFINITION 5. For  $A$  in  $\mathcal{S}(K)$ ,  $A^* = \{x \in K \mid \text{any } f^*\text{-system containing } x \text{ meets } A\}$  is called the  $f^*$ -radical of  $A$ . Similarly,  $A_* = \{y \in K \mid \text{any } sf^*\text{-system containing } y \text{ meets } A\}$  is called the  $sf^*$ -radical of  $A$ .

THEOREM 2.2. *Let  $A$  be an ideal of  $K$ . Then*

- (i)  $A^*$  is the intersection of all the  $f^*$ -prime ideals  $P_i$  containing  $A$ .
- (ii)  $A_*$  is the intersection of all  $f^*$ -semiprime ideals containing  $A$ .
- (iii)  $A_*$  is an  $f^*$ -semiprime ideal of  $K$ .
- (iv)  $A$  is  $f^*$ -semiprime if and only if  $A = A_*$ .

*Proof.* The proofs are essentially the same as in [5]. But to emphasize use of the  $*$ -operation and the function  $f$  we prove only (i). Let  $\bigcap_i P_i$  be the intersection of all the  $f^*$ -prime ideals  $P_i$  of  $K$  containing  $A$ . If  $a \notin P_i$  for some  $i$ , then  $a \in c(P_i)$ , being an  $f^*$ -system, and  $c(P_i) \cap A = \emptyset$ . Hence  $a \notin A^*$  and  $A^* \subseteq \bigcap_i P_i$ . Conversely, if  $a \notin A^*$ , then there exists an  $f^*$ -system  $M$  with  $a \in M$  but  $A \cap M = \emptyset$ . By Zorn's lemma we find a maximal ideal  $P$  such that  $P \supseteq A$  but  $P \cap M = \emptyset$ . Let  $B, C$  be ideals of  $K$  such that  $f(B) \cap c(P) \neq \emptyset$  and  $f(C) \cap c(P) \neq \emptyset$ . By the maximality of  $P$ ,  $(f(B) + P) \cap M \neq \emptyset$  and  $(f(C) + P) \cap M \neq \emptyset$ . Since  $M$  is an  $f^*$ -system,  $\emptyset \neq (f(B) + P) * (f(C) + P) \cap M \subseteq (f(B) * f(C) + P) \cap M$  by Lemma 1.1 (i), thus  $f(B) * f(C) \cap c(P) \neq \emptyset$ . Hence  $P$  is  $f^*$ -prime and  $a \notin P$ .

LEMMA 2.3. *Let  $a$  be an element of  $K$  and  $S$  be an  $sf^*$ -system containing  $a$ . Then there exists an  $f^*$ -system  $M$  such that  $a \in M$  and  $M \subseteq S$ .*

*Proof.* Let  $a_1 = a$ , then  $a_1 \in f(a_1) \cap S$  and so  $f(a_1) * f(a_1) \cap S \neq \emptyset$ . Hence we obtain a set  $M = \{a_1, a_2, \dots, a_n, \dots\}$  such that  $a_{k+1} \in f(a_k) \cap S$  and  $M \subseteq S$ . By Lemma 1.1 (ii) we note that  $a_{k+1} \in f(a_k) * f(a_k) \subseteq f(a_k)$  and so  $f(a_{k+1}) \subseteq f(a_k)$ . Let  $p = \max(i, j)$ , then  $a_{p+1} \in f(a_p) * f(a_p) \cap S \subseteq f(a_i) * f(a_j) \cap S$ . Hence  $f(a_i) * f(a_j) \cap M \neq \emptyset$  and  $M$  is an  $f^*$ -system.

Therefore, as in [5], we have

THEOREM 2.4. *For any ideal  $A$  of  $K$ ,  $A^* = A_*$ .  $A^*$  is called the  $f^*$ -prime radical of  $A$ .*

DEFINITION 6. The  $f^*$ -prime radical,  $R^*(K)$ , of  $K$  is the  $f^*$ -prime radical of the ideal  $(0)$ . A ring  $K$  is said to be  $f^*$ -semisimple if  $R^*(K) = (0)$ .

LEMMA 2.5. Let  $\bar{K}$  be a homomorphic image of  $K$ . If  $M$  is an  $f^*$ -system of  $K$ , then so is  $\bar{M}$  in  $\bar{K}$ .

*Proof.* Let  $\bar{A}, \bar{B}$  be ideals of  $\bar{K}$  such that  $f(\bar{A}) \cap \bar{M} \neq \emptyset$  and  $f(\bar{B}) \cap \bar{M} \neq \emptyset$ , where  $A$  and  $B$  are ideals in  $K$  containing the kernel. Recalling (f 3) and  $A \subseteq f(A)$ , these imply  $f(A) \cap M \neq \emptyset$  and  $f(B) \cap M \neq \emptyset$ . Since  $M$  is an  $f^*$ -system, by (\*3) and (f 3) we see that  $f(\bar{A}) * f(\bar{B}) \cap \bar{M} \neq \emptyset$ .

Therefore, by Lemma 2.3 we easily see that any homomorphic image of an  $f^*$ -prime ideal containing the kernel is also  $f^*$ -prime. Hence we obtain

THEOREM 2.6. Let  $K$  be a ring and  $R^*(K)$  be the  $f^*$ -prime radical of  $K$ , then  $R^*(K/R^*(K)) = (0)$ , that is,  $K/R^*(K)$  is  $f^*$ -semisimple.

DEFINITION 7. A ring  $K$  is called an  $f^*$ -prime ring if  $(0)$  is an  $f^*$ -prime ideal in  $K$ .

Clearly, an  $f^*$ -prime ring is  $f^*$ -semisimple. Since any homomorphic image of an  $f^*$ -prime ideal is  $f^*$ -prime, if  $P$  is an  $f^*$ -prime ideal in  $K$  then  $K/P$  is an  $f^*$ -prime ring. Let  $\bar{K} = K/P$  be an  $f^*$ -prime ring and let  $f(A) * f(B) \subseteq P$ , then  $f(\bar{A}) * f(\bar{B}) \subseteq (\bar{0})$  and so  $f(A) \subseteq P$  or  $f(B) \subseteq P$ , thus  $P$  is  $f^*$ -prime in  $K$ . Hence  $P$  is an  $f^*$ -prime ideal of  $K$  if and only if  $K/P$  is an  $f^*$ -prime ring. Therefore, as for Jordan rings, we obtain

THEOREM 2.7. A ring  $K$  is isomorphic to a subdirect sum of  $f^*$ -prime rings if and only if  $K$  is  $f^*$ -semisimple.

3. Throughout this section we assume that the  $*$ -operation satisfies the following additional condition:

$$(*4) \quad A * A = A^3 \text{ and } A * A \text{ is an ideal of } K \text{ for } A \text{ in } \mathcal{S}(K).$$

We recall that if  $K$  is a weakly  $W$ -admissible or Lie ring then  $A * B = AB^2 + B^2A + (AB)B + (BA)B$  satisfies (\*4).

THEOREM 3.1. Let  $A$  be an ideal of a ring  $K$  and  $r \in A_*$ . Then a power of  $r$  belongs to  $A$ . Furthermore if  $K$  is power-associative, then the  $f^*$ -radical  $R^*(K)$  is a nil ideal in  $K$ .

*Proof.* Let  $M$  be the multiplicatively closed system generated

by  $r$  in  $K$ . Then it follows from (\*4) that  $M$  is an  $sf^*$ -system containing  $r$ . Hence  $M \cap A \neq \emptyset$ . If  $K$  is power-associative and  $r \in R^*(K)$ , then  $r^k \in (0)$  for some  $k$  and so  $R^*(K)$  is nil.

Therefore, the  $f^*$ -radical  $R^*(K)$  is contained in the nil radical  $N(K)$  (the maximal nil ideal in  $K$ ).

Let  $\mathcal{S}'(K)$  denote the set of ideals  $f(A)$  for  $A$  in  $\mathcal{S}(K)$ . Then  $\mathcal{S}'(K) \subseteq \mathcal{S}(K)$ .

**THEOREM 3.2.** *A ring  $K$  is  $f^*$ -semisimple if and only if  $\mathcal{S}'(K)$  contains no nonzero nilpotent ideal.*

*Proof.* It follows from Theorem 2.2 (iv) that  $K$  is  $f^*$ -semisimple if and only if the ideal  $(0)$  is  $f^*$ -semiprime. If  $f(A)$  is a nonzero nilpotent ideal for  $A$  in  $\mathcal{S}(K)$ , there exist positive integers  $u = 3^t$  and  $v = 3^{t-1}$  such that  $f(A)^u = (0)$  but  $f(A)^v \neq (0)$ . But then since  $f(A)^u * f(A)^v \subseteq f(A)^{3v} = f(A)^u = (0)$ ,  $(0)$  is not  $f^*$ -semiprime. Conversely, if  $(0)$  is not  $f^*$ -semiprime, then there exists an ideal  $f(A) \neq (0)$  such that  $f(A) * f(A) = f(A)^3 = (0)$ , thus  $f(A)$  is nilpotent.

**COROLLARY 3.3.** *The  $f^*$ -radical  $R^*(K)$  contains all the nilpotent ideals in  $\mathcal{S}'(K)$ .*

*Proof.* Let  $f(A)$  be a nilpotent ideal in  $\mathcal{S}'(K)$  and  $\bar{K} = K/R^*(K)$ , then  $\overline{f(A)} = f(\bar{A}) \in \mathcal{S}'(\bar{K})$ , and  $f(\bar{A})$  is nilpotent in  $\bar{K}$ . Since  $\bar{K}$  is  $f^*$ -semisimple, by Theorem 3.2  $f(\bar{A}) = (\bar{0})$ , thus  $f(A) \subseteq R^*(K)$ .

**THEOREM 3.4.** *If  $K$  is a ring and  $\mathcal{S}'(K)$  contains a maximal nilpotent ideal  $S'(K)$ , then  $R^*(K) = S'(K)$ .*

*Proof.* By Corollary 3.3,  $S'(K) \subseteq R^*(K)$ . Let  $\bar{K} = K/S'(K)$ , then  $\mathcal{S}'(\bar{K})$  contains no nonzero nilpotent ideal and by Theorem 3.2  $R^*(\bar{K}) = (0)$ . If  $r \in S'(K)$ , then  $\bar{r} \neq \bar{0}$  and so there exists an  $f^*$ -prime ideal  $\bar{P}$  in  $\bar{K}$  with  $\bar{r} \in \bar{P}$ . From (\*3) and (f3) it follows that the inverse image  $P$  of  $\bar{P}$  is an  $f^*$ -prime ideal in  $K$ . But since  $\bar{r} \in \bar{P}$ ,  $r \in P$  and so  $r \in R^*(K)$ , thus  $R^*(K) \subseteq S'(K)$ .

Now suppose that  $f(a) = (a)$  for every element  $a$  in  $K$ . Then  $\mathcal{S}(K) = \mathcal{S}'(K)$ . Hence by Theorem 3.2  $K$  is  $f^*$ -semisimple if and only if  $K$  has no nonzero nilpotent ideal, and  $R^*(K)$  contains all nilpotent ideals of  $K$ . In this case the ideal  $S'(K)$  is a maximal nilpotent ideal  $S(K)$  in  $K$  and by Theorem 3.4  $R^*(K) = S(K)$ .

Let  $K$  now be a finite dimensional  $W$ -admissible or Lie algebra over a field. Let  $f(a) = (a)$  for all  $a$  in  $K$ . If  $K$  is  $W$ -admissible, then it is shown in [4] that the nil radical  $N(K)$  is nilpotent and so the

unique maximal nilpotent ideal  $S(K)$ . Hence by Theorem 3.4  $R^*(K) = N(K) = S(K)$ . If  $K$  is a Lie algebra, it is well known that  $K$  has a maximal nilpotent ideal  $S(K)$  and hence  $R^*(K) = S(K)$ .

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