INTERPOLATION BY ANALYTIC FUNCTIONS

Arne Stray
It is shown that interpolation problems for $R(X), A(X)$ and $H^\infty(X^\circ)$ are local problems whenever $X$ is a compact plane set.

Introduction and notation. Let $X$ be compact plane set, $X^\circ$ its interior and $\partial X = X \setminus X^\circ$ its boundary.

$H^\infty(X^\circ)$ denotes all bounded complex-valued analytic functions on $X^\circ$. $A(X)$ is all continuous functions on $X$ which are analytic in $X^\circ$. $R(X)$ denotes the uniform closure on $X$ of the rational functions with poles outside $X$.

A subset $E$ of $X$ is an interpolation set for $A(X)$ if $A(X) \setminus E$ (the restrictions to $E$ of the functions in $A(X)$) equals the space $C(E)$ of all continuous complex-valued functions on $E$.

$E$ is called a peak set for $A(X)$ if there exists $f \in A(X)$ such that $f = 1$ on $E$ and $|f(x)| < 1$ if $x \in X \setminus E$.

A peak interpolation set for $A(X)$ is a set $E$ which has both these properties. Peak and interpolation sets for $R(X)$ are defined in the same way.

A sequence $S = \{z_n\}$ of distinct points is called an interpolating sequence for $H^\infty(X^\circ)$ if for any bounded sequence $\{w_n\}$ of complex numbers there exists $f \in H^\infty(X^\circ)$ such that $f(z_n) = w_n$ for each $n$. (For more about interpolating sequences see Ch. 10 in [3].)

If $F$ is a subset of the complex plane we give it (as a topological space) the topology induced from $C$. $C_b(F)$ is the Banach space of all bounded continuous complex-valued functions on $F$. We also consider $H^\infty(X^\circ), R(X)$ and $A(X)$ as Banach spaces with the usual sup norm.

Let us mention two other Banach-spaces of analytic functions which has not been much studied yet, but which may be useful in characterizing interpolation sets for $R(X)$ and $A(X)$ among other things.

$HR(X)$ denotes all functions on $X^\circ$ which are pointwise limits on $X^\circ$ of bounded sequences in $R(X)$. For each $f \in HR(X)$ we define

$$\|f\|_{HR} = \inf \{ \sup_{\nu_n} \|f_\nu\| : \{f_\nu\} \subset R(X), f_\nu \longrightarrow f \text{ pointwise on } X^\circ \}.$$  

With this norm $HR(X)$ clearly is a Banach space. In the same way we define $HA(X)$ corresponding to $A(X)$ and it is also a Banach space with the norm $\|\|_{HA}$. Very recently A. M. Davie has shown that the norm $\|\|_{HA}$ is the same as sup norm on $X^\circ$ and the same is proved for $\|\|_{HR}$ if almost every point of $\partial X$ (w.r.t. area) is a peak point for $R(X)$. We shall not need these interesting results.
here. (See [1] for his results.) Some results about \(HR(X)\) can be found in [2].

If \(f\) is a complex-valued function defined on a set \(F\) and \(S \subseteq F\) is a subset we define \(\|f\|_s\) as \(\sup \{ |f(z)| : z \in S \}\).

A typical problem we shall study in this paper is the following:

Let \(S\) be a sequence in \(X^0\). What local conditions on \(S\) are sufficient to conclude that \(S\) is an interpolating sequence for \(H^\infty(X^0)\)?

An obvious necessary condition is that \(S \cap D_s\) is an interpolating sequence for \(H^\infty(X^0)\) whenever \(D_s\) is an open disc centered at \(z\) for which \(D_s \cap S \neq \emptyset\).

Suppose that the following weaker condition is satisfied:

\((\ast)\): For every \(z \in \overline{S}\) (the closure of \(S\)) there exists \(\delta_z > 0\) such that \(S \cap D_s\) is an interpolating sequence for \(H^\infty(D_s \cap X^0)\) where \(D_s = \{w : |w - z| < \delta_z\}\).

We shall then by definition say that \(S\) admits local \(H^\infty\)-interpolation w.r.t. \(X^0\).

Our main result is the following:

**Theorem 1.** Let \(X\) be a compact set with nonempty interior \(X^0\). A sequence \(S\) in \(X^0\) is an interpolating sequence for \(H^\infty(X^0)\) if and only if \(S\) admits local \(H^\infty\)-interpolation w.r.t. \(X^0\).

Some time after Theorem 1 was proved we learnt about a result of J. Rainwater which has some connection with Theorem 1. If in the definition of local \(H^\infty\)-interpolation the condition \((\ast)\) had been replaced by the other necessary condition for interpolation mentioned above Theorem 1 would be a somewhat weaker result.

We want to point out this weaker result is easy to deduce from J. Rainwaters paper. (See [4].) We also want to point out that a theorem of E. L. Stout on interpolating sequences in multiply connected domains in an easy consequence of Theorem 1. (See [5].)

Interpolating sequences can clearly also be defined for \(HR(X)\) and \(HA(X)\). It should also be clear what is meant by saying that a sequence \(S \subseteq X^0\) admits local \(HR\)-interpolation (or \(HA\)-interpolation) w.r.t. \(X\).

It will follow from our proof that Theorem 1 also holds for \(HR(X)\) and \(HA(X)\). We shall give some reasons for this at the end of the proof.

**Lemma 1.** Let \(X\) be as in Theorem 1 and \(z_0 \in \partial X\). Let \(0 < r_1 < r_2\) and define \(0_1 = \{w : |w - z_0| < r_1\}\) and \(0_2 = \{w : |w - z_0| > r_2\}\). Suppose there exists \(z_i \in \overline{C \setminus X}\) such that \(r_2 > |z_i - z_0| > r_1\).

Let \(S_i\) be an interpolating sequence for \(H^\infty(X^0 \cap 0_i)\) for \(i = 1, 2\).
Suppose $S_i \subset 0$, for $i = 1, 2$.
Then $S = S_1 \cup S_2$ is an interpolating sequence for $H^\infty(X^\circ)$.

**Proof.** Put $\Gamma_i = \partial 0_i$ for $i = 1, 2$.
Then $\text{dist} \cdot (S, \Gamma_i) > 0$.
Assume $h \in H^\infty(X^\circ \cap 0_i)$. Extend it to $C$ by defining $h(z) = 0$ if $z \in X^\circ \cap 0_i$.

Let $\delta > 0$ be given. Then cover $C$ by open discs $\Delta_n = \Delta(z_n, \delta)$ (of radius $\delta$ and centered at $z_n$) and choose continuously differentiable functions $g_n$ supported on $\Delta_n$ as in the scheme for approximation described on page 210 in [2].

Let $T_{g_n}$ be the integral operator on $L^\infty(dx\,dy)$ defined by

$$T_{g_n}(f)(w) = \frac{1}{\pi} \int \frac{f(w) - f(z)}{w - z} \frac{\partial g_n}{\partial z} \, dx\,dy$$

$$= f(w) \cdot g_n(w) + \frac{1}{\pi} \int \frac{f(z)}{z - w} \frac{\partial g_n}{\partial z} \, dx\,dy .$$

We mention that $T_{g_n}(f)$ is analytic outside the support of $g_n$ and wherever $f$ is and that $T_{g_n}(f)$ is continuous wherever $f$ is.

Also $f - T_{g_n}(f)$ is analytic in the interior of the set where $g_n$ attains the value 1. (See on p. 28–29 in [2] for more details.)

Put $h_n = T_{g_n}(h)$. We are only interested in those $n$ for which $\Delta_n \cap \Gamma_1 \cap \Gamma_2 \neq \emptyset$. Assume this happens if and only if $1 \leq n \leq N$.

Then $h - \sum_n h_n = h - T_{\sum_n g_n}(h)$ is analytic near $\Gamma_1 \cap \Gamma_2$ since $\sum_n g_n$ equals 1 near $\Gamma_1 \cap \Gamma_2$.

Now there exist functions $\{H_n\}_{n=1}^N$ analytic outside a compact subset of $D_n = \{w: |w - z_n| < 2\delta\} \setminus 0_i$ such that $h_n - H_n$ has a triple zero in the Taylor expansion at infinity and in our situation we can obtain $\|H_n\| \leq c_i \|h\|$ where $c_i$ is an absolute constant. (See Theorem 7.4 on p. 213 in [2] and the proof of it.)

Now one has to observe two important facts.

(a) If $B$ is a subset of $C$ and $\text{dist} (B, \Gamma_1 \cap \Gamma_2) > 0$ and $\varepsilon > 0$ one can choose $\delta$ depending only on $\varepsilon$ and $\text{dist} (B, \Gamma_1 \cap \Gamma_2)$ so small that the sum $f_\varepsilon = h - \sum_n (h_n - H_n)$ satisfies

$$\|h - f_\varepsilon\|_B \leq \varepsilon \|h\| .$$

(b) The functions $H_n$ can be chosen such that its singularities lies on a fixed compact subset of $D_n$ independent of $h$.

In fact one can find two functions $F_{n,1}$ and $F_{n,2}$ analytic outside a compact subset of $D_n$ such that $\|F_{n,1}\| + \|F_{n,2}\| \leq 20$ and $H_n = \lambda_{n,1}(h)F_{n,1} + \lambda_{n,2}(h)F_{n,2}$. (See lemma 6.3 on page 209 in [2].)

Here $\lambda_{n,k}(h)$ is a complex number and we have

$$|\lambda_{n,k}(h)| \leq c_i \|h\| \quad \text{for } k = 1, 2 ,$$
where $c_2$ in our situation is an absolute constant. If $F_{n,k}$ is constructed as in the mentioned lemma in [2]. We also mention that the maps $h \rightarrow \lambda_{m,k}(h)$ are linear.

(Some details indicating how this can be done, can be found in the proof of Lemma 3.1 in [4].)

Given $\varepsilon > 0$ we first choose $\delta$ so small that

$$(3) \quad \|h - f\|_S < \frac{\varepsilon\|h\|}{4}$$

whenever $h$ is as above. The choose rational functions $r_{n,k}$ with poles only at $z_i$ such that

$$(4) \sum_{n=1}^{\infty} (\|F_{n,1} - r_{n,1}\|_{b_1} + \|F_{n,2} - r_{n,2}\|_{b_2}) < \frac{\varepsilon}{4c_2}.$$  

Now define $A_i: H^\omega(X^0 \cap 0_i) \rightarrow H^\omega(X^0)$ by

$$A_i(h) = [h - \sum (h_{n,i} - (\lambda_{n,i}(h)r_{n,1} + \lambda_{n,2}(h)r_{n,2}))]X^0.$$  

From (1), (2), (3) and (4) we deduce that

(i) $\|A_i(h)\| \leq c_i\|h\|$ where $c_i$ depends only on the rational functions $r_{n,k}$.

(ii) $\|A_i(h) - h\|_S \leq \varepsilon\|h\|/4 + \varepsilon\|h\|/4 = \varepsilon\|h\|/2$.

In addition we also mention that $A_i$ is linear but this fact will not be needed.

In exactly the same way we define a map $A_2: H^\omega(X^0 \cap 0_i) \rightarrow H^\omega(X^0)$.

Suppose now $f \in C_i(S)$. By the open mapping theorem applied to the restriction $H^\omega(0_i \cap X^0) \rightarrow C_i(S_i)$ for $i = 1, 2$, there exists a constant $M$ independent of $f$ and functions $h_i \in H^\omega(0_i \cap X^0)$ such that

$$\|h_i\| \leq M\|f\| \text{ and } h_i = f \text{ on } S_i \text{ for } i = 1, 2.$$  

Put $h_i = 0$ outside $0_i \cap X^0$ and define $g = A_1(h_1) + A_2(h_2)$.

Then $g \in H^\omega(X^0)$, $\|g\| \leq 2c_iM\|f\|$ and $\|f - g\|_S = \|A_1(h_1) - h_1 + A_2(h_2) - h_2\|_S \leq \varepsilon M\|f\| \leq 1/2\|f\|$ if we choose $\varepsilon \leq 1/2M$.

Put $g_1 = g$ and assume $g_1, \ldots, g_n$ constructed such that

$$\|g_k\| \leq 2^{-k+\gamma}c_iM\|f\| \text{ for } 1 \leq k \leq n$$

and

$$\left\|f - \sum_{j=1}^{n} g_j\right\|_S \leq \frac{\|f\|}{2^n}.$$  

By the approximation technique above one easily find $g_{n+1} \in H^\omega(X^0)$ such that $\|g_{n+1}\| \leq 2^{-n-\gamma}c_iM\|f\|$ and
\[ \left\| f - \sum_{i=1}^{n+1} g_i \right\|_S \leq \frac{\| f \|}{2^{n+1}}. \]

By induction the series \( \sum_{n} g_n \in H^\omega(X^0) \) interpolates \( f \) on \( S \).

**Lemma 2.** Let \( S \) be a sequence in \( X^0 \) with no clusterpoints in \( X^0 \). Assume there exist \( n \) points \( z_1, \ldots, z_{n} \) and numbers \( r_k > s_k > t_k \) for \( 1 \leq k \leq n \) such that the open discs \( \Delta(z_k, r_k) \) cover \( S \).

Assume also that \( (C\setminus X) \cap \{ w : r_k > |w - z_k| > s_k \} \) and \( (C\setminus X) \cap \{ w : s_k > |w - z_k| > t_k \} \) are nonempty for each \( k \). If for each \( k \), \( \Delta(z_k, r_k) \cap S \) is an interpolating sequence for \( H^\omega(X^0) \) then also \( S \) is.

**Proof.** We can assume \( n \geq 2 \) and by induction the lemma to be true if \( n \) is replaced by \( n - 1 \).

Put \( S_1 = S \cap \Delta(z_n, t_n) \).

By hypothesis \( S_2 = S \cap (\bigcup_{k=1}^{n-1} \Delta(z_k, s_k)) \) is an interpolating sequence for \( H^\omega(X^0) \) and given \( f \in C(S) \) we can find \( h_1 \in H^\omega(X^0) \) such that \( h_1 = f \) on \( S_2 \).

The choose \( h_2 \in H(X^0) \) equal to \( f - h_1 \) on \( \Delta(z_n, r_n) \).

By Lemma 1 we can find \( h_3 \) in \( H^\omega(X^0) \) such that \( h_3 = 1 \) on \( S_1 \) and \( h_3 = 0 \) on \( S_2 \setminus \Delta(z_n, s_n) \).

Then \( h_1 + h_2 h_3 = f \) on \( S \).

**Proof of Theorem 1.** We have to show that the local condition implies that \( S \) is an interpolating sequence.

\( S \) has no clusterpoints in \( X^0 \) and for each \( z \in (\partial X) \cap S \) we can find \( r_z > 0 \) such that \( \Delta(z, r_z) \cap S \) is an interpolating sequence for \( H^\omega(X^0) \) where \( X^0_1 = \{ w : |w - z| < 2r_z \} \cap X^0 \). By Lemma 1 \( S \cap \Delta(z, r_z) \) is an interpolating sequence for \( H^\omega(X^0) \).

Since \( z \in \partial X \) we can choose \( s_z > t_z > 0 \) such that \( (C\setminus X) \cap \{ w : r_z > |w - z| > s_z \} \) and \( (C\setminus X) \cap \{ w : s_z > |w - z| > t_z \} \) are nonempty.

Since \( S \cap (\partial X) \) is compact we can obtain the hypothesis of Lemma 2 for a set \( S' \subset S \) such that \( S \setminus S' \) is finite.

But if \( S' \) is an interpolating sequence for \( H^\omega(X^0) \) then clearly also \( S \) is.

**Remark.** To prove Theorem 1 in case \( H = HR(X) \) one must modify the arguments slightly in the proof of Lemma 1. We use the notation from that lemma.

Given \( f \in C(S) \) one finds \( h_i \in HR(\overline{0}_i \cap X) \) equal to \( f \) on \( S_i \) such that \( \| h_i \|_{HR} \leq M \| f \| \) where \( M \) is a constant independent of \( S \) found by using the open mapping theorem.

Then we find a sequence \( \{ g_i \}_{n=1}^{\infty} \subset C(S^0) \) analytic in a neighbourhood
of $X \cap \bar{0}$ (depending on $n$) such that $\sup_n \| g_n^i \| \leq 2M \| f \|$ and such that $g_n^i \to h_i$ pointwise on the interior of $X \cap \bar{0}$. ($S^2$ denotes the extend complex plane with the usual topology.) We can also assume $g_n^i$ converges in the $w^*$-topology of $L^\infty(dx\,dy)$ to a function $\tilde{h}_i$ equal to $h_i$ on $0 \cap X^o$ such that $\| \tilde{h}_i \|_{L^\infty} \leq 2M \| f \|$.

We can assume $\tilde{h}_i = 0$ outside $\bar{0}$.

Then it is easy to see that $\sum_{i=1}^n A_i(\tilde{h}_i)$ will approximate $f$ well on $S$ and that $A_i(g_n^i) \mid X$ belongs to $R(X)$ for all $n$ and that $A_i(g_n^i) \to A_i(\tilde{h}_i)$ pointwise on $X^o$. Also $\| A_i(\tilde{h}_i) \|_{HR} \leq k \cdot M \| f \|$. ($k$ is independent of $f$.)

With these remarks Lemma 1 also applies for $HR(X)$. It is clear that similar modifications give Lemma 1 also for $HA(X)$.

But then the rest of the proof of Theorem 1 including the proof of Lemma 2 applies almost directly.

**Corollary 1.** Let $X$ be a compact plane set and $E$ a closed subset.

Then $E$ is an interpolation set for $R(X)$ if and only if for each $z \in E$ there exists a closed disc $N_z = \{ w : |w - z| \leq r \}$ such that $E \cap N_z$ is an interpolation set for $R(X \cap N_z)$.

**Proof.** Clearly $E_z = E \cap \{ w : |w - z| \leq z/2 \}$ is an interpolation set for $R(X \cap N_z)$.

The approximation technique used in the proof of Lemma 1 shows that $E_z$ then is an interpolation for $R(X)$.

But then the corollary follows from Rainwaters result.

**Remark.** A similar corollary also clearly holds for $A(X)$.

Finally we state a theorem for $R(X)$ which is not difficult to prove. Perhaps it makes the space $HR(X)$ a little more attractive.

**Theorem 3.** Let $S$ be a closed subset of a compact plane set $X$.

Suppose that

(i) $S \cap \partial X$ is a peak interpolation set for $R(X)$

(ii) $S \cap X^o$ is an interpolating sequence for $HR(X)$.

Then $R(X) \mid S = C(S)$.

One proves Theorem 3 by showing that for every $f \in C(S)$ there exists $g \in R(X)$ such that

$$\| f - g \|_S \leq \frac{1}{2} \| f \| \quad \text{and} \quad \| g \| \leq M \| f \|$$

where $M$ is independent of $f$. This is sufficient by the approximation
argument at the end of the proof of Lemma 1. 
First choose $f_1 \in R(X)$ such that $f_1 = f$ on $S \cap \partial X$ and $\|f_1\| \leq \|f\|$. 
Interpolate then $f - f_1$ on $S \cap X^o$ by $f_2 \in HR(X)$ such that $\|f_2\|_{HR} \leq M_i \|f\|$ where $M_i$ is independent of $f$.
If $\varepsilon > 0$ choose an open set $V, S \cap \partial X$ such that $|f_2| < \varepsilon$ on $S \cap X^o \cap V$.
Choose also $f_3 \in R(X)$ such that $\|f_3\| \leq 2$, $f_3 = 0$ on $S \cap \partial X$ and $|1 - f_3| < \varepsilon$ on $X \setminus V$, and $f_4 \in R(X)$ such that $|f_4(z) - f_3(z)| \leq \varepsilon$ for all $z \in S$ where $|f_3(z)| \geq \varepsilon$ and such that $\|f_4\| \leq 2\|f_2\|_{HR} \leq 2M_i \|f\|$.
Then put $g = f_1 + f_3 f_4$. We have $\|g\| \leq (1 + 4M_i) \|f\|$ and $\|f - g\| \leq \varepsilon \|f\| (2 + 3M_i)$.
So with $\varepsilon = 1/(4 + 6M_i)$ we have what we want.

Finally I want to thank Dr. A. M. Davie for some very helpful correspondence which gave our results considerably greater generality.

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Stephen Richard Bernfeld, *The extendability of solutions of perturbed scalar differential equations* ........................................ 277
James Edwin Brink, *Inequalities involving \( f_{-p} \) and \( f^{(n)}_q \) for \( f \) with \( n \) zeros* .......................................................... 289
Orrin Frink and Robert S. Smith, *On the distributivity of the lattice of filters of a groupoid* .................................................. 313
Donald Goldsmith, *On the density of certain cohesive basic sequences* ....... 323
Charles Lemuel H Africans, *Planar images of decomposable continua* ........ 329
W. N. Hudson, *A decomposition theorem for biadditive processes* ........... 333
W. N. Hudson, *Continuity of sample functions of biadditive processes* ...... 343
Masako Izumi and Shin-ichi Izumi, *Integrability of trigonometric series. II* .......................................................... 359
H. M. Ko, *Fixed point theorems for point-to-set mappings and the set of fixed points* .......................................................... 369
Gregers Louis Krabbe, *An algebra of generalized functions on an open interval: two-sided operational calculus* .............. 381
Thomas Latimer Kriete, III, *Complete non-selfadjointness of almost selfadjoint operators* .................................................. 413
Shiva Narain Lal and Siya Ram, *On the absolute Hausdorff summability of a Fourier series* .................................................. 439
Ronald Leslie Lipsman, *Representation theory of almost connected groups* .......... 453
James R. McLaughlin, *Integrated orthonormal series* ................................ 469
H. Minc, *On permanents of circulants* ........................................ 477
Akihiro Okuyama, *On a generalization of \( \Sigma \)-spaces* ...................... 485
Norberto Salinas, *Invariant subspaces and operators of class \( (S) \)* .......... 497
James D. Stafney, *The spectrum of certain lower triangular matrices as operators on the \( l_p \) spaces* ........................................ 515
Arne Stray, *Interpolation by analytic functions* .................................. 527
Li Pi Su, *Rings of analytic functions on any subset of the complex plane* .... 535
R. J. Tondra, *A property of manifolds compactly equivalent to compact manifolds* .................................................. 539