

# Pacific Journal of Mathematics

**NORMPRESERVING EXTENSIONS IN SUBSPACES OF  $C(X)$**

EGGERT BRIEM AND MURALI RAO

## NORMPRESERVING EXTENSIONS IN SUBSPACES OF $C(X)$

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**If  $B$  is a subspace of  $C(X)$  and  $F$  is a closed subset of  $X$ , this note gives sufficient conditions in order that every function in the restriction subspace  $B|_F$  has an extension in  $B$  with no increase in norm.**

**Introduction.** Let  $X$  be a compact Hausdorff space,  $C(X)$  the Banach algebra of all continuous complex-valued functions on  $X$  and let  $B$  be a closed linear subspace of  $C(X)$  separating the points of  $X$  and containing the constants. A closed subset  $F$  of  $X$  is said to have the normpreserving extension property w.r.t.  $B$  if any function  $b_0$  in the restriction subspace  $B|_F$  has an extension  $b \in B$  (i.e.  $b|_F = b_0$ ) such that  $\|b\| = \|b_0\|_F$  ( $\|\cdot\|$  (resp.  $\|\cdot\|_F$ ) denotes the supremum norm on  $X$  (resp.  $F$ )). The main result is the following:

*Let  $F$  be a closed subset of  $X$  and suppose there is a map  $T$  (not necessarily linear) from  $M(X)$  into  $M(X)$  satisfying the following conditions*

- (i)  $m - Tm \in B^\perp$  for all  $m \in M(X)$
- (ii)  $T\lambda$  is a probability measure when  $\lambda$  is
- (iii) If  $s_i \in \mathbb{C}$  and  $m_i \in M(X)$   $i = 1, \dots, n$  and  $\sum_{i=1}^n s_i m_i \in k(F)^\perp$  then  $\sum_{i=1}^n s_i (Tm_i)|_{X \setminus F} \in B^\perp$ .

*Then  $F$  has the normpreserving extension property.*

$M(X)$  denotes the set of regular Borel measures on  $X$ , and if  $A$  is a subset of  $B$  then  $A^\perp$  is the set of those measures in  $M(X)$  which annihilate  $A$ .  $k(F)$  consists of those functions in  $B$  which are identically 0 on  $F$ . Also if  $G$  is a Borel subset of  $X$  and  $m \in M(X)$  then  $m|_G$  is the measure  $\chi_G m$  where  $\chi_G$  is the characteristic function for  $G$ .

Two conditions, either of which is known to imply that a closed subset  $F$  of  $X$  has the normpreserving extension property are the following:

*Condition 1.* For all  $\sigma \in B^\perp$ ,  $\sigma|_F \in B^\perp$ .

*Condition 2.*  $F$  is a compact subset of the Choquet boundary  $\Sigma_B$  for  $B$  and for all  $\sigma \in M(\Sigma_B) \cap B^\perp$ ,  $\sigma|_F \in B^\perp$ .

$(M(\Sigma_B))$  denotes the set of those  $\sigma \in M(X)$  for which the total variation  $|\sigma|$  is maximal in Choquet's ordering for positive measures (see [1])

Ch. I §3 and [6] p. 24).

In Chapter 2 of this note we show that when either Condition 1 or Condition 2 is satisfied there exists a map  $T$  with the above properties.

Actually, when Condition 1 or Condition 2 is satisfied stronger extension properties than the normpreserving one hold. (In the case of Condition 1 see [4] Theorem 3 and [5] Theorem 4.8 in the case of Condition 2 see [2] Theorem 4.5 and [3] Theorem 2). But as we show in Chapter 2 these stronger extension properties are corollaries to theorems based on the existence of a map  $T$  described above. Thus we are able to deal simultaneously with Conditions 1 and 2.

**1. A condition for the normpreserving extension property.**

*Throughout this chapter  $F$  is a fixed closed subset of  $X$  and  $T$  is a map from  $M(X)$  into  $M(X)$  satisfying*

- (i)  $m - Tm \in B^\perp$  for all  $m \in M(X)$
- (ii)  $T\lambda$  is a probability measure when  $\lambda$  is.
- (iii) If  $s_i \in C$  and  $m_i \in M(X)$  and  $\sum_{i=1}^n s_i m_i \in k(F)^\perp$  then

$$\sum_{i=1}^n s_i (Tm_i)|_{X \setminus F} \in B^\perp .$$

**REMARK 1.1.** It follows from conditions (i) and (iii) that if  $\sum s_i \sigma_i \in B^\perp$  then  $\sum s_i (T\sigma_i)|_F \in B^\perp$ . Also if  $\lambda$  is a probability measure and  $\lambda = \lambda|_F$  then  $T\lambda = (T\lambda)|_F$ , because  $\lambda \in k(F)^\perp$  hence by (iii)  $(T\lambda)|_{X \setminus F} \in B^\perp$ . Since  $B$  contains the constants and  $T\lambda$  is a positive measure  $(T\lambda)|_{X \setminus F} = 0$ .

We let  $S_B$  denote the state space of  $B$  s.e.  $S_B = \{p \in B^*: \|p\| = p(1) = 1\}$ .  $S_B$  is a convex set which is compact in the  $w^*$ -topology and the natural map of  $X$  into  $S_B$  is a homeomorphism. We shall frequently think of  $X$  as embedded in  $S_B$ . A representing measure for  $p \in S_B$  is a probability measure  $v_p$  on  $X$  such that  $p(f) = \int f dv_p$  for all  $f \in B$ .

**DEFINITION 1.2.** For each  $b_0 \in B|_F$  we define a function  $\bar{b}_0$  on  $S_B$  as follows. If  $p \in S_B$  put

$$\bar{b}_0(p) = \int_F b_0 dTv_p$$

where  $v_p$  is any representing measure for  $p$  on  $X$ .

**REMARK 1.3.** The above definition is meaningful because if  $v'_p$  is another representing measure for  $p$  on  $X$  then  $v_p - v'_p \in B^\perp$ ; hence by Remark 1.1  $(Tv_p)|_F - (Tv'_p)|_F \in B^\perp$ .

**LEMMA 1.4.**  $\bar{b}_0$  has the following properties:

- (1)  $\bar{b}_0$  is an affine function
- (2)  $|\bar{b}_0(p)| \leq \|b_0\|_F$  for all  $p \in S_B$
- (3)  $\bar{b}_0(p) = b_0(p)$  if  $p \in F$
- (4)  $\bar{b}_0$  is a linear combination of upper semicontinuous affine functions.
- (5)  $\int \bar{b}_0 d\sigma = 0$  for all  $\sigma \in B^\perp$ .

*Proof* 1. follows from the definition of  $\bar{b}_0$  and remark 1.1. (2) is trivial: To prove (3) observe that if  $x \in F$  then by remark 1.1  $T\delta_x = (T\delta_x)|_F$  ( $\delta_x$  is point mass at  $x$ ). But  $T\delta_x$  is a representing measure for  $x$ . (4) Observe that if  $b_0 \in B|_F$  and  $f_0 = Re b_0$ , we can define  $\bar{f}_0$  in exactly the same way as we defined  $\bar{b}_0$ . Then  $\bar{f}_0$  is affine on  $S_B$  and  $\bar{f}_0 = Re \bar{b}_0$ . First assume that  $f_0 \geq 0$ . We want to show that  $\bar{f}_0$  is upper semi-continuous. For each  $t \geq 0$  put  $K_t = \{p \in S_B: \bar{f}_0(p) \geq t\}$  we must show that  $K_t$  is closed. Let  $\{p_\alpha\}$  be a net from  $K_t$  with limit point  $p_0$ , and  $v_\alpha$  a representing measure for  $p_\alpha$  on  $X$  for each  $\alpha$ . Write  $Tv_\alpha = u_\alpha + w_\alpha$  where  $u_\alpha = (Tv_\alpha)|_F$ . Let  $u_0$  be a  $w^*$ -clusterpoint for  $\{u_\alpha\}$  and let  $\{u_\beta\}$  be a subnet from  $\{u_\alpha\}$  converging to  $u_0$ . Also let  $w_0$  be a clusterpoint for  $\{w_\beta\}$ . Then  $v_0 = u_0 + w_0$  is a representing measure for  $p_0$  and since

$$u_0 = u_0|_F, T\left(\frac{u_0}{\|u_0\|}\right) = T\left(\frac{u_0}{\|u_0\|}\right)|_F.$$

(Remark 1.1). Using this and Remark 1.1 once more we get:

$$\begin{aligned} \bar{f}_0(p_0) &= \int_F f_0 dTv_0 = \|u_0\| \int_F f_0 dT\left(\frac{u_0}{\|u_0\|}\right) + \|w_0\| \cdot \int_F f_0 dT\left(\frac{w_0}{\|w_0\|}\right) \\ &\geq \|u_0\| \int_F f_0 dT\left(\frac{u_0}{\|u_0\|}\right) = \int_F f_0 du_0 \geq t. \end{aligned}$$

Hence  $p_0 \in K_t$ .

In general take a positive number  $k$  such that  $f_0 + k \geq 0$ . Then  $\bar{f}_0 = \overline{f_0 + k} - \bar{k}$  is the difference of upper semi-continuous functions. Since this holds for any  $f_0 \in Re B|_F$  (4) is proved.

Since  $\bar{b}_0$  is a linear combination of real valued affine upper semi-continuous functions it satisfies the barycenter formula i.e. if  $p \in S_B$  and  $v_p$  is a representing measure for  $p$  then

$$\int \bar{b}_0 dv_p = \bar{b}_0(p)$$

(See [1] Cor. I 1.4)

Now we consider a measure  $\sigma \in B^\perp$  with a decomposition  $\sigma = \sum_{i=1}^4 t_i \sigma_i$  into probability measures  $\sigma_i$  representing points  $p_i \in S_B$  for

$i = 1, 2, 3, 4$ . By axiom (i) the measure  $T\sigma_i$  also represent  $p_i$  for  $i = 1, 2, 3, 4$ . Applying the above result together with the definition of  $\bar{b}_0$  and axiom (iii), we obtain:

$$\begin{aligned} \int \bar{b}_0 d\sigma &= \sum_{i=1}^4 t_i \int \bar{b}_0 d\sigma_i = \sum_{i=1}^4 t_i \bar{b}_0(p_i) \\ &= \sum_{i=1}^4 t_i \int_F b_0 d(T\sigma_i) = \int_F b_0 d\left[\sum_{i=1}^4 t_i(T\sigma_i)\right] = 0. \end{aligned}$$

This completes the proof of (5).

**PROPOSITION 1.5.**  $B|_F$  is closed in  $C(F)$

*Proof.* Let  $\sigma \in B^\perp$ , and consider a  $b_0 \in B|_F$  such that  $\|b_0\|_F \leq 1$ . By statement (5) of Lemma 1.4:

$$0 = \int \bar{b}_0 d\sigma = \int_F b_0 d\sigma + \int_{X \setminus F} \bar{b}_0 d\sigma.$$

Hence

$$\left| \int_F b_0 d\sigma \right| = \left| \int_{X \setminus F} \bar{b}_0 d\sigma \right| \leq \|\sigma|_{X \setminus F}\|,$$

and so  $\|\sigma|_F\| \leq \|\sigma|_{X \setminus F}\|$ .

By a result of Gamelin [4] and Glicksberg [5] (see also [3, Prop. 1]) this implies that  $B|_F$  is almost normpreserving, or what is equivalent, that  $B|_{k(F)}$  is isometric to  $B|_F$ . Hence  $B|_F$  is complete in uniform norm, and we are done.

**PROPOSITION 1.6.** Let  $b_0 \in B|_F$  and let  $\psi$  be a strictly positive lower semi-continuous function on  $X$  such that  $\psi(x) > |\bar{b}_0(x)|$  for all  $x \in X$ . Then there is a function  $b \in B$  such that  $b|_F = b_0$  and  $|b(x)| < \psi(x)$  for all  $x \in X$ .

*Proof.* Apply Theorem 2.2 of [2].

**THEOREM 1.7.** Let  $F$  and  $T$  be as in the beginning of this chapter and let  $b_0 \in B|_F$  with  $\|b_0\|_F \leq 1$  and let  $\psi$  be a strictly positive lower semi-continuous function such that  $\psi(x) > |\bar{b}_0(x)|$  for all  $x \in X$ . Then there is a function  $b \in B$  such that

$$b|_F = b_0, \|b\| = \|b_0\|_F \text{ and } |b(x)| < \psi(x) \text{ for all } x \in X.$$

*Proof.* The proof is exactly the same as proof of [3] Theorem 2 after replacing the function  $A$  from [3] by  $\bar{b}_0$  and Lemma 1 of [3] by Proposition 1.6 of this note.

**COROLLARY 1.8.**  *$F$  and  $T$  as before. Then  $F$  has the normpreserving extension property w.r.t.  $B$ .*

**THEOREM 1.9.** *Let  $F$  and  $T$  be as before let  $b_0 \in B|_F$  and let  $\psi$  be a strictly positive lower semi-continuous function such that  $\psi(x) \geq |\bar{b}_0(x)|$  for all  $x \in X$ . Suppose furthermore that  $\psi(x) \geq \int \psi dT\lambda_x$  for all  $x \in X \setminus F$  for which  $\bar{b}_0(x) \neq 0$  ( $\lambda_x$  is a representing measure for  $x$ ). Then there is a function  $b \in B$  such that*

$$b|_F = b_0 \text{ and } |b(x)| \leq \psi(x) \text{ for all } x \in X.$$

*Proof.* The proof is the same as the proof of [2] Theorem 4.5 replacing in the proof of Theorem 2.1 of [2] by Proposition 1.6 of this note.

**2. Relations to conditions 1 and 2.** We start by showing the equivalence of condition 1 to a condition involving  $k(F)^\perp$

**PROPOSITION 2.1.** *Let  $F$  be a closed subset of  $X$ . Then the following conditions are equivalent:*

- 1. For all  $\sigma \in B^\perp, \sigma|_F \in B^\perp$
- 1'. For all  $\sigma \in k(F)^\perp, \sigma|_{X \setminus F} \in B^\perp$ .

*Proof.* Condition 1' trivially implies 1. Suppose Condition 1 is satisfied and let  $\sigma \in k(F)^\perp$ . Let  $b_0 \in B|_F$  and let  $b \in B$  be any extension of  $b_0$ . Since  $\sigma \in k(F)^\perp$  the quantity  $\int b d\sigma$  is independent of the choice of the extension  $b$ . Thus  $b_0 \rightarrow \int b d\sigma$  is a well defined linear functional on  $B|_F$ . By [4] Theorem 1,  $B|_F$  is closed in  $C(F)$ . It then follows from the open mapping theorem that  $b_0 \rightarrow \int b d\sigma$  is a continuous linear functional. Thus we can find a measure  $\sigma_1 = \sigma_1|_F$  such that  $\sigma_1 - \sigma \in B^\perp$ . But then  $\sigma|_{X \setminus F} = (\sigma_1 - \sigma)|_{X \setminus F} \in B^\perp$ .

Let again  $F$  be a closed subset of  $X$  and suppose that Condition 1 is satisfied. Let  $T$  be the identity map from  $M(X)$  to  $M(X)$ . By the above proposition  $T$  satisfies requirements (i) (ii) and (iii) from the beginning of Chapter 1. In this case if  $b_0 \in B|_F, \bar{b}_0(x) = 0$  for all  $x \in X \setminus F$ . From Theorem 1.9 we can then deduce the following well known theorem.

**THEOREM 2.2.** *Let  $F$  be a closed subset of  $X$  and suppose that  $\mu|_F \in B^\perp$  for all  $\mu \in B^\perp$ . If  $b_0 \in B|_F$  and  $\psi$  is a strictly positive lower semi-continuous function with  $\psi(x) \geq |b_0(x)|$  for all  $x \in F$  then there is function  $b \in B$  such that*

$b|_F = b_0$  and  $|b(x)| \leq \psi(x)$  for all  $x \in X$ .

We now look at Condition 2. Let  $F$  be a compact subset of the Choquet boundary  $\Sigma_B$  and suppose Condition 2 is satisfied i.e. for all  $\sigma \in B^\perp \cap M(\Sigma_B)$ ,  $\sigma|_F \in B^\perp$ . We need the following lemma

**LEMMA 2.3.** *Under the above hypotheses  $B|_F$  is closed in  $C(F)$ .*

*Proof.* By [5] Theorem 3.1 we must show the existence of a constant  $c \geq 1$  such that  $\|\mu - (B|_F)^\perp\| \leq c \|\mu - B^\perp\|$  for all  $\mu \in M(F)$ . Let  $\mu \in M(F)$  and  $\sigma \in B^\perp$ . We write  $\sigma = \sigma|_F + \sigma|_{X \setminus F}$  and further write  $\sigma|_{X \setminus F} = t_1\lambda_1 - t_2\lambda_2 + i(t_3\lambda_3 - t_4\lambda_4)$  where the  $t_i$ 's are positive numbers and the  $\lambda$ 's are probability measures such that  $\lambda_1$  and  $\lambda_2$  (resp.  $\lambda_3$  and  $\lambda_4$ ) live on disjoint subsets of  $X$ . For  $i = 1, \dots, 4$  let  $v_i$  be a maximal measure such that  $\lambda_i - v_i \in B^\perp$ . Put  $w = t_1v_1 - t_2v_2 + i(t_3v_3 - t_4v_4)$ . Then  $\sigma|_{X \setminus F} - w \in B^\perp$  and  $\|w\| \leq \sum_{i=1}^4 t_i \|v_i\| = \sum_{i=1}^4 t_i \|\lambda_i\| \leq 2\|\sigma|_{X \setminus F}\|$ . Now  $\sigma|_F + w \in B^\perp \cap M(\Sigma_B)$  so that  $\sigma|_F + w|_F \in B^\perp$ . Hence  $\|\mu - (A|_F)^\perp\| \leq \|\mu - (\sigma|_F + w|_F)\| \leq \|\mu - \sigma|_F\| + 2\|\sigma|_{X \setminus F}\| \leq 2\|\mu - \sigma\|$ . Thus we can take  $c = 2$  and the lemma is proved.

As above let  $F$  be a compact subset of  $\Sigma_B$  and suppose that for all  $\sigma \in M(\Sigma_B) \cap B^\perp$ ,  $\sigma|_F \in B^\perp$ . We define a map  $T$  from  $M(X)$  to  $M(X)$  as follows. If  $\lambda$  is a probability measure on  $X$  pick a maximal measure  $v$  with  $\lambda - v \in B^\perp$  and put  $T\lambda = v$ . If  $\lambda$  is already maximal put  $T\lambda = \lambda$ . If  $\sigma \in M(X)$  write  $\sigma = t_1\lambda_1 - t_2\lambda_2 + i(t_3\lambda_3 - t_4\lambda_4)$  where the  $t_i$ 's are positive numbers and where  $\lambda_1$  and  $\lambda_2$  (resp.  $\lambda_3$  and  $\lambda_4$ ) are probability measures living on disjoint subsets of  $X$ . Then put  $T\sigma = t_1T\lambda_1 - t_2T\lambda_2 + i(t_3T\lambda_3 - t_4T\lambda_4)$ . The map  $T$  from  $M(X)$  to  $M(X)$  we get in this way obviously has properties (i) and (ii) from the beginning of Chapter 1. Observe that  $T\sigma = \sigma$  if  $\sigma = \sigma|_F$  since  $F \subset \Sigma_B$ . To see that  $T$  also has property (iii) let  $\Sigma s_i \sigma_i \in k(F)^\perp$ . By Lemma 2.3  $B|_F$  is closed in  $C(F)$ . Just as in the proof of Proposition 2.1 we can find a measure  $\mu = \mu|_F$  such that  $\mu - \Sigma s_i \sigma_i \in B^\perp$ . Then  $\mu - \Sigma s_i T\sigma_i \in B^\perp \cap M(\Sigma_B)$  so that  $\mu - \Sigma s_i (T\sigma_i)|_F \in B^\perp$ , but then  $\Sigma s_i (T\sigma_i)|_{X \setminus F} \in B^\perp$ . We can then using Theorems 1.7 and 1.9 deduce the same interpolation theorems as in [2] and [3]. In particular we get from Theorem 1.7:

**THEOREM 2.4.** *Let  $F$  be a compact subset of the Choquet boundary  $\Sigma_B$  and suppose that for all  $\sigma \in B^\perp \cap M(\Sigma_B)$ ,  $\sigma|_F \in B^\perp$ . Then  $F$  has the normpreserving extension property w.r.t.  $B$ .*

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