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CORRECTIONS TO: "ISOMORPHIC GROUPS AND GROUP RINGS"

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ERRATA

Corrections to

ISOMORPHIC GROUPS AND GROUP RINGS

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Since the above named paper is apparently still of interest we discuss and correct two errors which occur in it.

We use the notation of the original paper [2].

1. The first error was pointed out to me many years ago by D. B. Coleman. Namely the characterization of the Frattini subgroup $\Phi(\mathscr{N})$ given in the first paragraph of page 569 is not right. What one gets is the intersection of all maximal subgroups $\mathscr{M}$ of $\mathscr{N}$ which are normal in $\mathscr{G}$ and this is just not $\Phi(\mathscr{N})$. For example if $\mathscr{G}$ is a nonabelian group of order p^3 and period p (for $p > 2$) and if $[\mathscr{G}: \mathscr{N}] = p$ then $\Phi(\mathscr{N}) = \langle 1 \rangle$ but this intersection is clearly the center of $\mathscr{G}$ which is not $\langle 1 \rangle$.

We correct this problem by essentially ignoring it. We just delete $\Phi(\mathscr{N})$ from part 4 of Theorem D. Note that if $\mathscr{G}$ is nilpotent then $\Phi(\mathscr{N})$ is of course given by $\Phi(\mathscr{N}) = \mathscr{N}'C^n(\mathscr{N})$ where n is the product of the distinct prime factors of $|\mathscr{N}|$. Thus $\Phi(\mathscr{N})$ is determined in this case and it can remain in the statement of Theorem E.

2. A more serious error was pointed out in a recent paper of T. Obayashi [1]. Namely Lemma 3 is just not right. The mistake occurs in the last line of the proof where it is assumed that $(\mathscr{N})(\mathscr{L}) = (\mathscr{L})(\mathscr{N})$. This fact is not true. For example it fails when $\mathscr{N} = \mathscr{G}$ is the dihedral group of order 8 and when $\mathscr{L}$ is the cyclic subgroup of index 2. However it is clear that no such trouble arises when $\mathscr{N}$ is abelian. Thus we replace Lemma 3 by

LEMMA 3*. *Let $\mathscr{L}, \mathscr{M}, \mathscr{N}$ be three normal subgroups of $\mathscr{G}$ with $\mathscr{L} \subseteq \mathscr{M} \subseteq \mathscr{N}$ and $\mathscr{N}$ abelian. Then*

$$(\mathscr{L}) \cap (\mathscr{M})(\mathscr{N}) \subseteq (\mathscr{L})(\mathscr{N}) .$$

Proof. We first assume that $\mathscr{G} = \mathscr{N}$ and proceed as in the original proof. Then we use the fact that $R[\mathscr{G}]$ is free over $R[\mathscr{N}]$ to obtain the general result.

One can give an alternate proof in case $R = Z$ is the ring of rational integers using results of Whitcomb. If $\mathscr{L}$ is a subgroup of $\mathscr{N}$ let $I(\mathscr{L})$ denote the augmentation ideal in the integral group ring $Z[\mathscr{L}]$. Suppose $\mathscr{N}$ is abelian. By [3], the map $g \rightarrow (1-g) + I(\mathscr{N})^2$ is an isomorphism of $\mathscr{N}$ onto the additive group $I(\mathscr{N})/I(\mathscr{N})^2$. Thus clearly

$$\mathscr{L} \simeq \frac{I(\mathscr{L}) + I(\mathscr{N})^2}{I(\mathscr{N})^2} = \frac{I(\mathscr{L})Z[\mathscr{N}] + I(\mathscr{N})^2}{I(\mathscr{N})^2}$$
$$\simeq \frac{I(\mathscr{L})Z[\mathscr{N}]}{I(\mathscr{L})Z[\mathscr{N}] \cap I(\mathscr{N})^2} .$$

On the other hand we know by [3] that

$$\mathscr{L} \simeq \frac{I(\mathscr{L})Z[\mathscr{N}]}{I(\mathscr{L})I(\mathscr{N})} .$$

Since $\mathscr{L}$ is finite and since clearly $I(\mathscr{L})Z[\mathscr{N}] \cap I(\mathscr{N})^2 \supseteqq I(\mathscr{L})I(\mathscr{N})$ this latter inclusion is therefore an equality.

If $x \in \mathscr{G}$ we let $x^{\mathscr{G}}$ denote the normal closure of the cyclic group $\langle x \rangle$ in $\mathscr{G}$. The crucial Proposition 4 is not only correct as stated but it is true in a slightly more general context.

PROPOSITION 4*. *Let K_x and K_y be two class sums in $R[\mathscr{G}]$. Suppose that $(x^{\mathscr{G}}, \mathscr{G}, y^{\mathscr{G}}) = (y^{\mathscr{G}}, \mathscr{G}, x^{\mathscr{G}}) = 1$. Then we can find $(x, y) \in \mathscr{X}$ in $R[\mathscr{G}]$.*

Proof. We proceed as in the original proof. Note that the normal subgroup $\mathscr{N}$ is defined by $\mathscr{N} = (x^{\mathscr{G}}, \mathscr{G}) \cap (y^{\mathscr{G}}, \mathscr{G})$. Since $y^{\mathscr{G}}$ centralizes $(x^{\mathscr{G}}, \mathscr{G})$ we see that $(y^{\mathscr{G}}, \mathscr{G}) \subseteqq y^{\mathscr{G}}$ also centralizes $(x^{\mathscr{G}}, \mathscr{G})$ and thus $\mathscr{N}$ is abelian. With this observation we can apply Lemma 3* instead of Lemma 3 where needed and the result follows.

Thus, the proposition is generalized by dropping the assumption $(x, \mathscr{G}) \cap (y, \mathscr{G}) \subseteqq \mathscr{Z}_\infty$ of the original. We remark that as indicated on page 572 the notation $(x, \mathscr{G})$ used in [2] is just a shorthand for $(x^{\mathscr{G}}, \mathscr{G})$. Finally we point out a misprint in equation (10) of page 573. The first part should read $m_x \gamma_x \hat{\mathscr{N}} = K_x \hat{\mathscr{N}}$.

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1. T. Obayashi, *Integral group rings of finite groups*, Osaka J. Math., **7** (1970), 253-266.
2. D. S. Passman, *Isomorphic groups and group rings*, Pacific J. Math., **15** (1965), 561-583.
3. A. R. Whitcomb, *The group ring problem*, Ph. D. thesis, Univ. of Chicago, 1968.

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