

# Pacific Journal of Mathematics

**CORRECTIONS TO: "ISOMORPHIC GROUPS AND GROUP  
RINGS"**

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# ERRATA

Corrections to

## ISOMORPHIC GROUPS AND GROUP RINGS

D. S. PASSMAN

Volume 15 (1965), 561-583

Since the above named paper is apparently still of interest we discuss and correct two errors which occur in it.

We use the notation of the original paper [2].

1. The first error was pointed out to me many years ago by D. B. Coleman. Namely the characterization of the Frattini subgroup  $\Phi(\mathcal{N})$  given in the first paragraph of page 569 is not right. What one gets is the intersection of all maximal subgroups  $\mathcal{M}$  of  $\mathcal{N}$  which are normal in  $\mathcal{G}$  and this is just not  $\Phi(\mathcal{N})$ . For example if  $\mathcal{G}$  is a nonabelian group of order  $p^3$  and period  $p$  (for  $p > 2$ ) and if  $[\mathcal{G} : \mathcal{N}] = p$  then  $\Phi(\mathcal{N}) = \langle 1 \rangle$  but this intersection is clearly the center of  $\mathcal{G}$  which is not  $\langle 1 \rangle$ .

We correct this problem by essentially ignoring it. We just delete  $\Phi(\mathcal{N})$  from part 4 of Theorem D. Note that if  $\mathcal{G}$  is nilpotent then  $\Phi(\mathcal{N})$  is of course given by  $\Phi(\mathcal{N}) = \mathcal{N}'C^n(\mathcal{N})$  where  $n$  is the product of the distinct prime factors of  $|\mathcal{N}|$ . Thus  $\Phi(\mathcal{N})$  is determined in this case and it can remain in the statement of Theorem E.

2. A more serious error was pointed out in a recent paper of T. Obayashi [1]. Namely Lemma 3 is just not right. The mistake occurs in the last line of the proof where it is assumed that  $(\mathcal{N})(\mathcal{L}) = (\mathcal{L})(\mathcal{N})$ . This fact is not true. For example it fails when  $\mathcal{N} = \mathcal{G}$  is the dihedral group of order 8 and when  $\mathcal{L}$  is the cyclic subgroup of index 2. However it is clear that no such trouble arises when  $\mathcal{N}$  is abelian. Thus we replace Lemma 3 by

LEMMA 3\*. Let  $\mathcal{L}, \mathcal{M}, \mathcal{N}$  be three normal subgroups of  $\mathcal{G}$  with  $\mathcal{L} \subseteq \mathcal{M} \subseteq \mathcal{N}$  and  $\mathcal{N}$  abelian. Then

$$(\mathcal{L}) \cap (\mathcal{M})(\mathcal{N}) \subseteq (\mathcal{L})(\mathcal{N}).$$

*Proof.* We first assume that  $\mathcal{G} = \mathcal{N}$  and proceed as in the original proof. Then we use the fact that  $R[\mathcal{G}]$  is free over  $R[\mathcal{N}]$  to obtain the general result.

One can give an alternate proof in case  $R = \mathbb{Z}$  is the ring of rational integers using results of Whitcomb. If  $\mathcal{L}$  is a subgroup

of  $\mathcal{N}$  let  $I(\mathcal{L})$  denote the augmentation ideal in the integral group ring  $Z[\mathcal{L}]$ . Suppose  $\mathcal{N}$  is abelian. By [3], the map  $g \rightarrow (1 - g) + I(\mathcal{N})^2$  is an isomorphism of  $\mathcal{N}$  onto the additive group  $I(\mathcal{N})/I(\mathcal{N})^2$ . Thus clearly

$$\begin{aligned} \mathcal{L} &\simeq \frac{I(\mathcal{L}) + I(\mathcal{N})^2}{I(\mathcal{N})^2} = \frac{I(\mathcal{L})Z[\mathcal{N}] + I(\mathcal{N})^2}{I(\mathcal{N})^2} \\ &\simeq \frac{I(\mathcal{L})Z[\mathcal{N}]}{I(\mathcal{L})Z[\mathcal{N}] \cap I(\mathcal{N})^2}. \end{aligned}$$

On the other hand we know by [3] that

$$\mathcal{L} \simeq \frac{I(\mathcal{L})Z[\mathcal{N}]}{I(\mathcal{L})I(\mathcal{N})}.$$

Since  $\mathcal{L}$  is finite and since clearly  $I(\mathcal{L})Z[\mathcal{N}] \cap I(\mathcal{N})^2 \supseteq I(\mathcal{L})I(\mathcal{N})$  this latter inclusion is therefore an equality.

If  $x \in \mathcal{G}$  we let  $x^\circ$  denote the normal closure of the cyclic group  $\langle x \rangle$  in  $\mathcal{G}$ . The crucial Proposition 4 is not only correct as stated but it is true in a slightly more general context.

**PROPOSITION 4\*.** *Let  $K_x$  and  $K_y$  be two class sums in  $R[\mathcal{G}]$ . Suppose that  $(x^\circ, \mathcal{G}, y^\circ) = (y^\circ, \mathcal{G}, x^\circ) = 1$ . Then we can find  $(x, y) \in \mathcal{X}$  in  $R[\mathcal{G}]$ .*

*Proof.* We proceed as in the original proof. Note that the normal subgroup  $\mathcal{N}$  is defined by  $\mathcal{N} = (x^\circ, \mathcal{G}) \cap (y^\circ, \mathcal{G})$ . Since  $y^\circ$  centralizes  $(x^\circ, \mathcal{G})$  we see that  $(y^\circ, \mathcal{G}) \subseteq y^\circ$  also centralizes  $(x^\circ, \mathcal{G})$  and thus  $\mathcal{N}$  is abelian. With this observation we can apply Lemma 3\* instead of Lemma 3 where needed and the result follows.

Thus, the proposition is generalized by dropping the assumption  $(x, \mathcal{G}) \cap (y, \mathcal{G}) \subseteq \mathcal{X}_\infty$  of the original. We remark that as indicated on page 572 the notation  $(x, \mathcal{G})$  used in [2] is just a shorthand for  $(x^\circ, \mathcal{G})$ . Finally we point out a misprint in equation (10) of page 573. The first part should read  $m_x \gamma_x \hat{\mathcal{N}} = K_x \hat{\mathcal{N}}$ .

#### REFERENCES

1. T. Obayashi, *Integral group rings of finite groups*, Osaka J. Math., **7** (1970), 253-266.
2. D. S. Passman, *Isomorphic groups and group rings*, Pacific J. Math., **15** (1965), 561-583.
3. A. R. Whitcomb, *The group ring problem*, Ph. D. thesis, Univ. of Chicago, 1968.

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