

Pacific Journal of Mathematics

**CORRECTIONS TO: "ISOMORPHIC GROUPS AND GROUP
RINGS"**

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ERRATA

Corrections to

ISOMORPHIC GROUPS AND GROUP RINGS

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Since the above named paper is apparently still of interest we discuss and correct two errors which occur in it.

We use the notation of the original paper [2].

1. The first error was pointed out to me many years ago by D. B. Coleman. Namely the characterization of the Frattini subgroup $\Phi(\mathcal{N})$ given in the first paragraph of page 569 is not right. What one gets is the intersection of all maximal subgroups \mathcal{M} of \mathcal{N} which are normal in \mathcal{G} and this is just not $\Phi(\mathcal{N})$. For example if \mathcal{G} is a nonabelian group of order p^3 and period p (for $p > 2$) and if $[\mathcal{G} : \mathcal{N}] = p$ then $\Phi(\mathcal{N}) = \langle 1 \rangle$ but this intersection is clearly the center of \mathcal{G} which is not $\langle 1 \rangle$.

We correct this problem by essentially ignoring it. We just delete $\Phi(\mathcal{N})$ from part 4 of Theorem D. Note that if \mathcal{G} is nilpotent then $\Phi(\mathcal{N})$ is of course given by $\Phi(\mathcal{N}) = \mathcal{N}'C^n(\mathcal{N})$ where n is the product of the distinct prime factors of $|\mathcal{N}|$. Thus $\Phi(\mathcal{N})$ is determined in this case and it can remain in the statement of Theorem E.

2. A more serious error was pointed out in a recent paper of T. Obayashi [1]. Namely Lemma 3 is just not right. The mistake occurs in the last line of the proof where it is assumed that $(\mathcal{N})(\mathcal{L}) = (\mathcal{L})(\mathcal{N})$. This fact is not true. For example it fails when $\mathcal{N} = \mathcal{G}$ is the dihedral group of order 8 and when \mathcal{L} is the cyclic subgroup of index 2. However it is clear that no such trouble arises when \mathcal{N} is abelian. Thus we replace Lemma 3 by

LEMMA 3*. *Let $\mathcal{L}, \mathcal{M}, \mathcal{N}$ be three normal subgroups of \mathcal{G} with $\mathcal{L} \subseteq \mathcal{M} \subseteq \mathcal{N}$ and \mathcal{N} abelian. Then*

$$(\mathcal{L}) \cap (\mathcal{M})(\mathcal{N}) \subseteq (\mathcal{L})(\mathcal{N}).$$

Proof. We first assume that $\mathcal{G} = \mathcal{N}$ and proceed as in the original proof. Then we use the fact that $R[\mathcal{G}]$ is free over $R[\mathcal{N}]$ to obtain the general result.

One can give an alternate proof in case $R = \mathbb{Z}$ is the ring of rational integers using results of Whitcomb. If \mathcal{L} is a subgroup

of \mathcal{N} let $I(\mathcal{L})$ denote the augmentation ideal in the integral group ring $Z[\mathcal{L}]$. Suppose \mathcal{N} is abelian. By [3], the map $g \rightarrow (1 - g) + I(\mathcal{N})^2$ is an isomorphism of \mathcal{N} onto the additive group $I(\mathcal{N})/I(\mathcal{N})^2$. Thus clearly

$$\begin{aligned} \mathcal{L} &\simeq \frac{I(\mathcal{L}) + I(\mathcal{N})^2}{I(\mathcal{N})^2} = \frac{I(\mathcal{L})Z[\mathcal{N}] + I(\mathcal{N})^2}{I(\mathcal{N})^2} \\ &\simeq \frac{I(\mathcal{L})Z[\mathcal{N}]}{I(\mathcal{L})Z[\mathcal{N}] \cap I(\mathcal{N})^2}. \end{aligned}$$

On the other hand we know by [3] that

$$\mathcal{L} \simeq \frac{I(\mathcal{L})Z[\mathcal{N}]}{I(\mathcal{L})I(\mathcal{N})}.$$

Since \mathcal{L} is finite and since clearly $I(\mathcal{L})Z[\mathcal{N}] \cap I(\mathcal{N})^2 \cong I(\mathcal{L})I(\mathcal{N})$ this latter inclusion is therefore an equality.

If $x \in \mathcal{G}$ we let $x^\mathcal{G}$ denote the normal closure of the cyclic group $\langle x \rangle$ in \mathcal{G} . The crucial Proposition 4 is not only correct as stated but it is true in a slightly more general context.

PROPOSITION 4*. *Let K_x and K_y be two class sums in $R[\mathcal{G}]$. Suppose that $(x^\mathcal{G}, \mathcal{G}, y^\mathcal{G}) = (y^\mathcal{G}, \mathcal{G}, x^\mathcal{G}) = 1$. Then we can find $(x, y) \in \mathcal{X}$ in $R[\mathcal{G}]$.*

Proof. We proceed as in the original proof. Note that the normal subgroup \mathcal{N} is defined by $\mathcal{N} = (x^\mathcal{G}, \mathcal{G}) \cap (y^\mathcal{G}, \mathcal{G})$. Since $y^\mathcal{G}$ centralizes $(x^\mathcal{G}, \mathcal{G})$ we see that $(y^\mathcal{G}, \mathcal{G}) \subseteq y^\mathcal{G}$ also centralizes $(x^\mathcal{G}, \mathcal{G})$ and thus \mathcal{N} is abelian. With this observation we can apply Lemma 3* instead of Lemma 3 where needed and the result follows.

Thus, the proposition is generalized by dropping the assumption $(x, \mathcal{G}) \cap (y, \mathcal{G}) \subseteq \mathcal{X}_\infty$ of the original. We remark that as indicated on page 572 the notation (x, \mathcal{G}) used in [2] is just a shorthand for $(x^\mathcal{G}, \mathcal{G})$. Finally we point out a misprint in equation (10) of page 573. The first part should read $m_x \gamma_x \hat{\mathcal{N}} = K_x \hat{\mathcal{N}}$.

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1. T. Obayashi, *Integral group rings of finite groups*, Osaka J. Math., **7** (1970), 253-266.
2. D. S. Passman, *Isomorphic groups and group rings*, Pacific J. Math., **15** (1965), 561-583.
3. A. R. Whitcomb, *The group ring problem*, Ph. D. thesis, Univ. of Chicago, 1968.

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