FACTORED CODIMENSION ONE CELLS IN EUCLIDEAN $n$-SPACE

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Seebeck has proved that if the m-cell C in Euclidean n-space $E^n$ factors k times, where $m \leq n - 2$ and $n \geq 5$, then every embedding of a compact k-dimensional polyhedron in C is tame relative to $E^n$. In this note we prove the analogous result for the case $m + 1 = n \geq 5$ and $n - k \geq 3$. In addition we show that if C factors 1 time, then each $(n - 3)$-dimensional polyhedron properly embedded in C can be homeomorphically approximated by polyhedra in C that are tame relative to $E^n$.

Following Seebeck [8] we say that an m-cell C in $E^n$ factors k times if for some homeomorphism h of $E^n$ onto itself and some $(m - k)$-cell B in $E^{n-k}$, $h(C) = B \times I^k$, where $I^k$ denotes the k-fold product of the interval I naturally embedded in $E^k$ and where

$$B \times I^k \subseteq E^{n-k} \times E^k = E^n$$

is the product embedding.

In another paper [6] the author has studied results comparable to Seebeck's for factored cells in $E^4$, but the techniques employed here differ slightly from those used in [6] and [8]. The main result generalizes work of Bryant [2], and the final section here expands on his methods to obtain a strong conclusion about tameness of all subpolyhedra in certain factored cells.

1. Definitions and Notation. For any point p in a metric space S and any positive number $\delta$, $N_\delta(p)$ denotes the set of points in S whose distance from p is less than $\delta$.

The symbol $d^2$ denotes a 2-simplex fixed throughout this paper, $\partial d^2$ its boundary, and $\text{Int } d^2$ its interior.

Let A denote a subset of a metric space X and p a limit point of A. We say that A is locally simply connected at p, written 1-LC at p, if for each $\epsilon > 0$ there is a $\delta > 0$ such that each map of $\partial d^2$ into $A \cap N_\epsilon(p)$ can be extended to a map of $d^2$ into $A \cap N_\epsilon(p)$. Furthermore, we say that A is uniformly locally simply connected, written 1-ULC, if for each $\epsilon > 0$ there is a $\delta > 0$ such that each map of $\partial d^2$ into a $\delta$-subset of A can be extended to a map of $d^2$ into an $\epsilon$-subset of A. Similarly, we say that A is locally simply connected in X at p, written 1-LC in X at p, if for each $\epsilon > 0$ there is a $\delta > 0$ such that each map of $\partial d^2$ into $A \cap N_\epsilon(p)$ extends to a map of $d^2$ into $N_\epsilon(p)$, and we say that A is uniformly locally simply connected in X (1-ULC in X) if the corresponding uniform property is satisfied.
Suppose \( f \) and \( g \) are maps of a space \( X \) into a space \( Y \) that has a metric \( \rho \). The symbol \( \rho(f, g) < \varepsilon \) means that \( \rho(f(x), g(x)) < \varepsilon \) for each \( x \) in \( X \).

A subset \( S \) of a metric space is called an \( \varepsilon \)-\textit{subset} if the diameter of \( S \), written \( \text{diam} \, S \), is less than \( \varepsilon \).

A compact 0-dimensional subset \( X \) of a cell \( C \) is said to be \textit{tame (relative to} \( C \)\textit{)} if \( X \cap \partial C \) is tame relative to \( \partial C \) and \( X \cap \text{Int} \, C \) is tame relative to \( \text{Int} \, C \). In addition, a 0-dimensional \( F \)-set \( F \) in \( C \) is said to be \textit{tame (relative to} \( C \)\textit{)} if \( F \) can be expressed as a countable union of tame (relative to \( C \)) compact subsets.

For definitions of other terms used here the reader is referred to such papers as [3, 8].

2. Tame polyhedra in factored cells. The goal of this section is to show that for any \( k \)-dimensional polyhedron \( P \) in a cell \( C \) that factors \( k \) times, \( E^n - P \) is 1-ULC. However, instead of arguing this directly, we prove first that \( E^n - C \) is 1-ULC in \( E^n - P \).

\textbf{Proposition 1.} If \( C \) is an \((n - 1)\)-cell in \( E^n \) that factors \( k \) times \((k \leq n - 3)\) and \( P \) a \( k \)-dimensional polyhedron (topologically) embedded in \( C \), then \( E^n - C \) is 1-ULC in \( E^n - P \).

\textit{Proof.} Suppose \( C = B \times I^k \subset E^{n-k} \times E^k \). Define a subset \( Z \) of \( P \) as the set of all points \( p \) of \( P \) for which there exist a neighborhood \( N_p \) of \( p \) (relative to \( P \)) and a point \( b \) in \( B \) such that \( N_p \subset \{b\} \times I^k \), and define \( Q = P - Z \). We prove first that, for each point \( c \) in \( C \), \( E^n - C \) is 1-LC in \( E^n - Q \) at \( c \).

Consider \( c \) to be of the form \((b, y)\), where \( b \in B \) and \( y \in \text{Int} \, I^k \) (the case \( y \in \partial I^k \) is similar and easier). Suppose \( N \) is a neighborhood of \((b, y)\) such that \( N \cap (B \times \partial I^k) = \emptyset \). There exist an open subset \( U \) of \( E^{n-k} \) and a contractible open subset \( V \) of \( I^k \) such that \((b, y) \in U \times V \subset N \). By the construction of \( Q \) there exists a point \( y' \in V \) such that \((b, y') \in Q \). Let \( U' \) be an open subset of \( E^{n-k} \) such that

\[ b \in U' \subset U \text{ and } (U' \times \{y'\}) \cap Q = \emptyset . \]

Now we obtain an open subset \( W \) of \( E^{n-k} \) such that \( b \in W \subset U' \) and the inclusion map \( i: W \to U' \) is homotopic to a constant map.

Let \( L \) be a loop in \((W \times V) - C \). Since \( V \) is contractible to \( y' \), \( L \) is homotopic in \((W \times V) - C \) to a loop \( L' \) in \( W \times \{y'\} \). But \( L' \) is contractible in

\[ U' \times \{y'\} \subset N - Q . \]

Thus, \( E^n - C \) is 1-LC in \( E^n - Q \) at \( c \).
The definition of \( Z \) implies that \( P \) is locally tame at each point of \( Z \). Hence, if \( f: \partial \Delta^2 \to E^n - Q \) is a map such that \( f(\partial \Delta^2) \subset E^n - P \), then \( f \) can be approximated arbitrarily closely by maps \( g: \partial \Delta^2 \to E^n \) such that \( g| \partial \Delta^2 = f| \partial \Delta^2 \) and \( g(\partial \Delta^2) \subset E^n - P \). Thus, \( E^n - C \) is 1-LC in \( E^n - P \) at each point \( c \) of \( C \). Since \( C \) is compact, the corresponding uniform property holds as well.

There may be some value in observing that this argument also gives the following result.

**Proposition 2.** Let \( B \times I^k \subset E^{n-k} \times E^k = E^n \) be an \( m \)-cell (\( m < n, \quad k \leq n-3 \)) and \( X \) a compactum in \( B \times I^k \) such that \( \dim (X \cap \{(b) \times I^2\}) < k \) for each \( b \) in \( B \). Then \( E^n - (B \times I^k) \) is 1-ULC in \( E^n - X \).

**Theorem 3.** If \( C \) is an \((n-1)\)-cell in \( E^n \) that factors \( k \) times (\( k \leq n-3 \)) and \( X \) is either a \( k \)-dimensional polyhedron or a \((k-1)\)-dimensional compactum in \( C \), then \( E^n - X \) is 1-ULC.

This theorem follows immediately from [1, Prop. 1] and either Proposition 1 or Proposition 2.

**Corollary 4.** If \( C \) is an \((n-1)\)-cell in \( E^n(n \geq 5) \) that factors \( k \) times (\( k \leq n-3 \)), then each \( k \)-dimensional polyhedron \( P \) in \( C \) is tame.

The corollary is a straightforward application of the Bryant-Seebeck characterization of tameness [3] for codimension 3 polyhedra in terms of the 1-ULC property.

3. Approximations in cells that factor 1 time. This section contains a proof of the analogue of Seebeck's Corollary 5.1 [8] for codimension one cells.

**Proposition 5.** If \( C \) is an \((n-1)\)-cell in \( E^n \) that factors 1 time, then there exists a tame 0-dimensional \( F \), set \( F \) in \( \text{Int} \, C \) such that, for each point \( c \) of \( \text{Int} \, C \), \( E^n - C \) is 1-LC in \( (E^n - C) \cup F \) at \( c \).

**Proof.** Assume \( C = B \times I \subset E^{n-1} \times E^1 = E^n \). Let \( c = (b, t) \) be a point of \( \text{Int} \, C \) and \( U \) a neighborhood of \( c \) such that \( U \cap C \subset \text{Int} \, C \). We assume further that \( U \) is a product neighborhood \( U = U' \times J \), where \( U' \subset E^{n-1} \) and \( J \subset E^1 \). Corresponding to \( U \) is a neighborhood \( V \) of \( c \) such that any map \( f': \partial \Delta^2 \to V - C \) extends to a map \( f: \Delta^2 \to U \) such that \( f^{-1}(f(\Delta^2) \cap C) \) is 0-dimensional ([4, Cor. 2C, 2.1] or [5, Th. 3.2]). We can change this map \( f \) near \( C \), altering only the \( E^1 \) coordi-
nates of points in the range, so that in addition $f(d^2) \cap C \subset B \times \{t\}$. We shall obtain a map $g: \mathcal{A} \to \bigcup\{\mathcal{A}^i\}$ satisfying

1. $g|_{\partial \mathcal{A}} = f|_{\partial \mathcal{A}} = f'$,
2. $g(d^2) \cap C$ is a tame (relative to $C$) 0-dimensional subset of $\text{Int}\ C$.

Let $\varepsilon$ be a positive number such that if $g: \mathcal{A} \to E^n$ and $\rho(f, g) < \varepsilon$, then $g(d^2) \subset U$.

Cover $f^{-1}(f(d^2) \cap C)$ by the interiors of a collection of small, pairwise disjoint 2-cells $D_1, D_2, \ldots, D_{k(1)}$ in $\mathcal{A}$. Slide the sets $f^{-1}(D_i)$ vertically to define a map $g: \mathcal{A} \to E^n$ satisfying

1. $g|_d \mathcal{A} = f|_d \mathcal{A} = f$,
2. $g(D_i) \cap C \subset B \times \{t_i\}$, where $t_i \neq t_j$ whenever $i \neq j$,
3. The $D_i$'s must be chosen with sufficiently small diameters that each set $f^{-1}(D_i) \cap C$ is contained in the interior of a small $(n - 2)$-cell in $B \times \{t\}$. Thus,

- (En) there exist pairwise disjoint $(n - 1)$-cells $K_1, K_2, \ldots, K_{k(1)}$ in $\text{Int}\ C$, each of diameter $< \varepsilon/2$, such that $\bigcup\{K_i\} \supset g_1(d^2) \cap C$.

The remaining approximations $g_1$ will be so close to $g$, that $\bigcup\{K_i\} \supset g(g_1(d^2) \cap C)$.

Let $\varepsilon_2 = \min\{\varepsilon/4, 1/2\rho(g_1(d^2) \cap C, C - \bigcup\{K_i\})\}$. To repeat this process, cover $g_1^{-1}(g_1(d^2) \cap C)$ by the interiors of a collection of a very small, pairwise disjoint 2-cells $D_1, D_2, \ldots, D_{k(2)}$ in $\bigcup\{K_i\} \subset \text{Int}\ C$.

Slide the sets $g_1(D_i)$ vertically to define a map $g_2: \mathcal{A} \to E^n$ satisfying

1. $g_2|_d \mathcal{A} = \bigcup\{D_i\} = g_1|_d \mathcal{A} = \bigcup\{D_i\}$,
2. $\rho(g_2, g_1) < \varepsilon_2$,
3. $g_2(D_i) \cap C \subset B \times \{t_i\}$, where $t_i \neq t_j$ whenever $i \neq j$,
4. $g_2^{-1}(g_2(D_i) \cap C)$ is 0-dimensional.

The $D_i$'s must be chosen with sufficiently small diameters that each set $g_1(D_i)$ is contained in a small $(n - 2)$-cell in $B \times \{t_i\}$. Thus,

- (En) there exist pairwise disjoint $(n - 1)$-cells $K_1, K_2, \ldots, K_{k(2)}$ in $\bigcup\{K_i\}$, each of diameter $< \varepsilon_2$, such that $\bigcup\{K_i\} \supset g_2(d^2) \cap C$.

By continuing in this manner we construct a sequence of maps $g_n: \mathcal{A} \to E^n$ satisfying analogous conditions $(A_n) - (B_n)$ and an associated sequence of collections $\{K_i\}$ of $n - 1$ cells in $C$ satisfying an analogous condition $(E_n)$. The restrictions of condition $(B_n)$ guarantee that $g = \lim g_n$ is a continuous function of $\mathcal{A}$ into $U$, and the restrictions of $(E_n)$ guarantee that

$$ g(d^2) \cap C \subset \bigcap_{n=1}^{\infty} \bigcup_{i=1}^{k(n)} \text{Int}_n K_i. $$

Thus, $g(d^2) \cap C$ is a tame (relative to $C$) 0-dimensional subset of $C$ [7, Lemma 2].
To prove the theorem from this fact, observe that for each $\varepsilon > 0$ there exists a countable collection $\{V_i\}$ of open sets covering $\text{Int} C$ such that any map $f' : \partial \Delta^2 \to V_i - C$ extends to a map $g$ of $\Delta^2$ into an $\varepsilon$-subset of $E^n$ such that $g(\Delta^2) \cap C$ is a tame 0-dimensional subset of $\text{Int} C$. Since there are only countably many homotopy classes of maps of $\partial \Delta^2$ into $V_i - C$, the desired set $F$ can be defined as the countable union of sets $g(\Delta^2) \cap C$.

**Theorem 6.** Suppose $C$ is an $(n - 1)$-dimensional cell in $E^n$ that factors 1 time, $P$ is an $(n - 3)$-dimensional polyhedron properly embedded in $C$, and $\varepsilon > 0$. There exists an $\varepsilon$-push $h$ of $(C, P)$ such that $h(P)$ is tame relative to $E^n$.

**Proof.** The case $n = 4$ is trivial, and no push is needed [6]; hence, we assume $n \geq 5$. By [8, Cor. 5.1] there exists an $\varepsilon/2$ push $h_i$ of $(C, P)$ such that $h_i(P \cap \partial C)$ is tame. Let $F$ denote the 0-dimensional $F_\sigma$ set of Proposition 5. There exists an $\varepsilon/2$ push $h_\sigma$ of $(C, h_i(P))$ such that $h_\sigma h_i(P) \cap F = \emptyset$ and $h_\sigma | \partial C = 1$. Let $h$ denote the $\varepsilon$-push $h_\sigma h_i$. It follows that $E^n - C$ is 1-LC in $E^n - h(P)$ at each point of $\text{Int} C$, and in stronger form, as shown in § 2, that $E^n - h(P)$ is 1-LC at each point of $\text{Int} C$. The tameness of $h(P) \cap \partial C$ then implies that $E^n - h(P)$ is 1-LC at every point of $h(P)$. Thus, $h(P)$ is tame [3].

**Corollary 7.** Let $S$ denote an $(n - 2)$ sphere in $S^{n-1}$, the $(n - 1)$-sphere, and $\Sigma$ the suspension of $S$ in $S^n$, the suspension of $S^{n-1}$. Then there exists a tame (relative to $\Sigma$) 0-dimensional $F_\sigma$ set $F$ in $\Sigma$ such that $S^n - \Sigma$ is 1-ULC in $(S^n - \Sigma) \cup F$. Furthermore, if $P$ is an $(n - 3)$-dimensional polyhedron in $\Sigma$ and $\varepsilon > 0$, there exists an $\varepsilon$-push $h$ of $(\Sigma, P)$ such that $h(P)$ is tame relative to $S^n$.

4. Factored cells in which all lower dimensional compacta are locally nice. Let $C = B \times I^k \subset E^{n-k} \times E^k = E^n$ be an $r$-cell ($r < n$). Although the low dimensional polyhedra in $C$ are nicely embedded, some $(k + 1)$-cell in $C$ may be wild. In this section we mention a property of certain cells $B$ that implies every $(r - 1)$-dimensional polyhedron in $C$ is nicely embedded.

**Theorem 8.** Let $B$ denote an $m$-cell in $E^n(m \leq n - 2)$ such that, for each $(m - 1)$-dimensional compactum $X \subset B$, $E^n - X$ is 1-ULC, and let $C$ denote $B \times I^l$, contained in $E^n \times E^k = E^{n+k}$. Then, for each $(m + k - 1)$-dimensional compactum $Y \subset C$, $E^{n+k} - Y$ is 1-ULC.

**Proof.** It suffices to consider only the case $k = 1$. Let $\varepsilon > 0$ and $w \in \text{Int} I$. We shall construct an $\varepsilon$-push $h$ of $(E^{n-1}, Y)$ such that...
\((E^n \times \{w\}) - h(Y)\) is 1-ULC. Let \(V\) denote the \(\varepsilon\)-neighborhood of \(Y\), \(\{b_j\}_{\in \mathbb{N}}\) a countable dense subset of \(B\), and \(\pi\) the natural projection of \(E^{n+1} = E^n \times E^1\) onto the first factor. For any open subset \(N\) of \(E^{n+1}\) containing \((b, w) \in B \times I\) there exists a point \((b', w') \in N \cap (B \times I - Y)\). If \(N\) is a connected open set of the form \(N = W \times J\), then there exists a homeomorphism \(g\) of \(E^{n+1}\) onto \(E^{n+1}\) such that

1. \(g|E^{n+1} - N = \text{identity}\),
2. \(g((b', w')) = (b', w)\),
3. \(g(C) = C\) and
4. \(\pi g = g\).

Consequently, there exist a sequence \(\{h_i\}\) of homeomorphisms of \(E^{n+1}\) onto itself and a sequence of points \(\{b_i\}\) in \(B\) such that for \(i = 1, 2, \ldots\)

1. \(\rho(x, h_i(x)) < \varepsilon/2^i\) for all \(x\) in \(E^n\),
2. \(\rho(b_i, b'_i) < 1/i\),
3. \((b_i', w') \in h_i \circ h_{i-1} \circ \cdots \circ h_1(Y)\),
4. \(h_{i+k}(b_i', w') = (b_i', w')\) for all \(k > 0\),
5. \(h_i(C) = C\)
6. \(\pi h_i = h_i\).

Furthermore, using Condition (a) and careful epsilonics we can construct the sequence \(\{h_i\}\) so that the function \(h = \lim_{n \to \infty} h_n \circ \cdots \circ h_1\) is an \(\varepsilon\)-homeomorphism of \(E^{n+1}\) onto itself. Then Condition (6) implies that \(h\) is an \(\varepsilon\)-push of \((E^{n+1}, Y)\).

Condition (1) implies that \(\{b_i\}\) is a dense subset of \(B\), and Conditions (2) and (3) yield that \((b_i', w') \in h(Y)\) \((i = 1, 2, \ldots)\). Thus, \(h(Y) \cap (B \times \{w\})\) is nowhere dense in \(B \times \{w\}\). Consequently, \(E^n \times \{w\} - h(Y)\) is 1-ULC by hypothesis (since \(h(Y) \subset B \times I\)), and we obtain the desired conclusion by appealing to Theorem 1 of [1].

We exploit the construction of the push \(h\) a second time in proving the following:

**Theorem 9.** Let \(B\) denote an \((n - 1)\)-cell in \(E^n\) such that, for each \((n - 2)\)-dimensional compactum \(X \subset B, E^n - B\) is 1-ULC in \(E^n - X\), and let \(C\) denote \(B \times I\), contained in \(E^n \times E^n\). Then for each \((n + k - 2)\)-dimensional compactum \(Y \subset C, E^{n+k} - C\) is 1-ULC in \(E^{n+k} - Y\).

**Proof.** Simplifying as before, we consider \(k = 1\) and \(c \in C\) a point of the form \((b, w)\), where \(b \in B\) and \(w \in \text{Int} I\), and we shall show that \(E^{n+1} - C\) is 1-LC in \(E^{n+1} - Y\) at \(c\).

Let \(\varepsilon > 0\). Choose a countable dense subset \(\{b_j\}\) of \(B\). Then reapplying the techniques found in the proof of Theorem 8, we find an \((\varepsilon/6)\)-homeomorphism \(h\) of \(E^{n+1}\) onto itself and a sequence \(\{b'_j\}\) of points in \(B\) satisfying Conditions (0)-(6) stated there. Let \(U\) denote the \(\varepsilon/6\)-neighborhood of \(b\) in \(E^n\) and \(V\) the \((\varepsilon/3)\)-neighborhood of \(w\) in \(E^n\).
Int $I$. Then both $(b, w)$ and $h((b, w))$ are contained in $U \times V$, and $\text{diam}(U \times V) < \varepsilon/2$. Since $B$ is an $(n - 1)$-cell there exists a neighborhood $U'$ of $b$ in $E^n$ such that $b \in U' \subset U$ and each map $f$ of $\partial A^2$ into $U' - B$ can be extended to a map $F$ of $A^2$ into $U$ such that $F^{-1}(F(A^2)) \cap B$ is $0$-dimensional.

In this paragraph we prove that $U' \times V$ is a neighborhood of $h(c)$ such that any loop in $(U' \times V) - C$ is contractible in an $\varepsilon/2$-subset of $E^{n+1} - h(Y)$. If $f: \partial A^2 \to (U' \times V) - C$, $f$ is homotopic in $(U' \times V) - C$ to a map $f': \partial A^2 \to U' \times \{w\}$. Let $F: A^2 \to U \times \{w\}$ be an extension of $f'$ such that $F^{-1}(F(A^2)) \cap (B \times \{w\})$ is $0$-dimensional. Once again $h(Y) \cap (B \times \{w\})$ is nowhere dense in $B \times \{w\}$, which means that $(E^n - B) \times \{w\}$ is $1$-$ULC$ in $((E^n) \times \{w\}) - h(Y)$. Cover $F^{-1}(F(A^2)) \cap (B \times \{w\})$ by finitely many pairwise disjoint $2$-cells $D_1, \ldots, D_t$ in $\text{Int} \; A^2$ such that $F|\partial D_i$ can be extended to a map $G_i$ of $D_i$ into $(U \times \{w\}) - h(Y)$. By redefining $F$ as $G_i$ on $D_i(i = 1, \ldots, t)$ one can easily see that $f|\partial A^2: \partial A^2 \to (U \times V) - h(Y)$ is homotopic to a constant map.

Because $h^{-1}$ is an $(\varepsilon/6)$-homeomorphism and diam $U < \varepsilon/2$, diam $h^{-1}(U \times V) < \varepsilon$. In addition, $h^{-1}(U' \times V)$ is a neighborhood of $c$ such that any map $g: \partial A^2 \to h^{-1}(U' \times V) - C$ can be extended to a map $G: A^2 \to h^{-1}(U \times V) - Y$. This completes the proof.

**COROLLARY 10.** Let $B$ denote an $m$-cell in $E^n(m < n)$ such that, for each $(m - 1)$-dimensional compactum $X \subset B$, $E^n - B$ is $1$-$ULC$ in $E^n - X$. Then each $p$-dimensional polyhedron $P$ in $B \times I^k \subset E^n \times E^k$ ($p + 3 \leq n + k, p < m + k$) is tame.

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Received January 5, 1972. Partially supported by NSF Grant GP-19966.

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<table>
<thead>
<tr>
<th>Authors</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allan Francis Abrahamse</td>
<td>Uniform integrability of derivatives on $\sigma$-lattices</td>
<td>1</td>
</tr>
<tr>
<td>Ronald Alter and K. K. Kubota</td>
<td>The diophantine equation $x^2 + D = p^n$</td>
<td>11</td>
</tr>
<tr>
<td>Grahame Bennett</td>
<td>Some inclusion theorems for sequence spaces</td>
<td>17</td>
</tr>
<tr>
<td>William Cutler</td>
<td>On extending isotopies</td>
<td>31</td>
</tr>
<tr>
<td>Robert Jay Daverman</td>
<td>Factored codimension one cells in Euclidean $n$-space</td>
<td>37</td>
</tr>
<tr>
<td>Patrick Barry Eberlein and Barrett O’Neill</td>
<td>Visibility manifolds</td>
<td>45</td>
</tr>
<tr>
<td>M. Edelstein</td>
<td>Concerning dentability</td>
<td>111</td>
</tr>
<tr>
<td>Edward Graham Evans, Jr.</td>
<td>Krull-Schmidt and cancellation over local rings</td>
<td>115</td>
</tr>
<tr>
<td>C. D. Feustel</td>
<td>A generalization of Kneser’s conjecture</td>
<td>123</td>
</tr>
<tr>
<td>Avner Friedman</td>
<td>Uniqueness for the Cauchy problem for degenerate parabolic equations</td>
<td>131</td>
</tr>
<tr>
<td>David Golber</td>
<td>The cohomological description of a torus action</td>
<td>149</td>
</tr>
<tr>
<td>Alain Goullet de Rugy</td>
<td>Un théorème du genre “Andô-Edwards” pour les Fréchet ordonnés normaux</td>
<td>155</td>
</tr>
<tr>
<td>Louise Hay</td>
<td>The class of recursively enumerable subsets of a recursively enumerable set</td>
<td>167</td>
</tr>
<tr>
<td>John Paul Helm, Albert Ronald da Silva Meyer</td>
<td>On orders of translations and enumerations</td>
<td>185</td>
</tr>
<tr>
<td>Julien O. Hennefeld</td>
<td>A decomposition for $B(X)^*$ and unique Hahn-Banach extensions</td>
<td>197</td>
</tr>
<tr>
<td>Gordon G. Johnson</td>
<td>Moment sequences in Hilbert space</td>
<td>201</td>
</tr>
<tr>
<td>Thomas Rollin Kramer</td>
<td>A note on countably subparacompact spaces</td>
<td>209</td>
</tr>
<tr>
<td>Yves A. Lequain</td>
<td>Differential simplicity and extensions of a derivation</td>
<td>215</td>
</tr>
<tr>
<td>Peter Lorimer</td>
<td>A property of the groups $\text{Aut PU}(3, \mathbb{Q}^2)$</td>
<td>225</td>
</tr>
<tr>
<td>Yasou Matsugu</td>
<td>The Levi problem for a product manifold</td>
<td>231</td>
</tr>
<tr>
<td>John M.F. O’Connell</td>
<td>Real parts of uniform algebras</td>
<td>235</td>
</tr>
<tr>
<td>William Lindall Paschke</td>
<td>A factorable Banach algebra without bounded approximate unit</td>
<td>249</td>
</tr>
<tr>
<td>Ronald Joel Rudman</td>
<td>On the fundamental unit of a purely cubic field</td>
<td>253</td>
</tr>
<tr>
<td>Tsuan Wu Ting</td>
<td>Torsional rigidities in the elastic-plastic torsion of simply connected cylindrical bars</td>
<td>257</td>
</tr>
<tr>
<td>Philip C. Tonne</td>
<td>Matrix representations for linear transformations on analytic sequences</td>
<td>269</td>
</tr>
<tr>
<td>Jung-Hsien Tsai</td>
<td>On $E$-compact spaces and generalizations of perfect mappings</td>
<td>275</td>
</tr>
<tr>
<td>Alfons Van Daele</td>
<td>The upper envelope of invariant functionals majorized by an invariant weight</td>
<td>283</td>
</tr>
<tr>
<td>Giulio Varsi</td>
<td>The multidimensional content of the frustum of the simplex</td>
<td>303</td>
</tr>
</tbody>
</table>