FACTORED CODIMENSION ONE CELLS IN EUCLIDEAN $n$-SPACE

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Seebeck has proved that if the $m$-cell $C$ in Euclidean $n$-space $E^n$ factors $k$ times, where $m \leq n - 2$ and $n \geq 5$, then every embedding of a compact $k$-dimensional polyhedron in $C$ is tame relative to $E^n$. In this note we prove the analogous result for the case $m + 1 = n \geq 5$ and $n - k \geq 3$. In addition we show that if $C$ factors 1 time, then each $(n - 3)$-dimensional polyhedron properly embedded in $C$ can be homeomorphically approximated by polyhedra in $C$ that are tame relative to $E^n$.

Following Seebeck [8] we say that an $m$-cell $C$ in $E^n$ factors $k$ times if for some homeomorphism $h$ of $E^n$ onto itself and some $(m - k)$-cell $B$ in $E^{n-k}$, $h(C) = B \times I^k$, where $I^k$ denotes the $k$-fold product of the interval $I$ naturally embedded in $E^k$ and where

$$B \times I^k \subset E^{n-k} \times E^k = E^n$$

is the product embedding.

In another paper [6] the author has studied results comparable to Seebeck’s for factored cells in $E^n$, but the techniques employed here differ slightly from those used in [6] and [8]. The main result generalizes work of Bryant [2], and the final section here expands on his methods to obtain a strong conclusion about tameness of all subpolyhedra in certain factored cells.

1. Definitions and Notation. For any point $p$ in a metric space $S$ and any positive number $\delta$, $N_{\delta}(p)$ denotes the set of points in $S$ whose distance from $p$ is less than $\delta$.

The symbol $\Delta^2$ denotes a 2-simplex fixed throughout this paper, $\partial \Delta^2$ its boundary, and $\text{Int} \, \Delta^2$ its interior.

Let $A$ denote a subset of a metric space $X$ and $p$ a limit point of $A$. We say that $A$ is locally simply connected at $p$, written $1-LC$ at $p$, if for each $\varepsilon > 0$ there is a $\delta > 0$ such that each map of $\partial \Delta^2$ into $A \cap N_{\delta}(p)$ can be extended to a map of $\Delta^2$ into $A \cap N_{\delta}(p)$. Furthermore, we say that $A$ is uniformly locally simply connected, written $1-ULC$, if for each $\varepsilon > 0$ there is a $\delta > 0$ such that each map of $\partial \Delta^2$ into a $\delta$-subset of $A$ can be extended to a map of $\Delta^2$ into an $\varepsilon$-subset of $A$. Similarly, we say that $A$ is locally simply connected in $X$ at $p$, written $1-LC$ in $X$ at $p$, if for each $\varepsilon > 0$ there is a $\delta > 0$ such that each map of $\partial \Delta^2$ into $A \cap N_{\delta}(p)$ extends to a map of $\Delta^2$ into $N_{\delta}(p)$, and we say that $A$ is uniformly locally simply connected in $X$ (1-ULC in $X$) if the corresponding uniform property is satisfied.
Suppose \( f \) and \( g \) are maps of a space \( X \) into a space \( Y \) that has a metric \( \rho \). The symbol \( \rho(f, g) < \varepsilon \) means that \( \rho(f(x), g(x)) < \varepsilon \) for each \( x \) in \( X \).

A subset \( S \) of a metric space is called an \( \varepsilon \)-subset if the diameter of \( S \), written \( \text{diam} \ S \), is less than \( \varepsilon \).

A compact 0-dimensional subset \( X \) of a cell \( C \) is said to be tame (relative to \( C \)) if \( X \cap \partial C \) is tame relative to \( \partial C \) and \( X \cap \text{Int} \ C \) is tame relative to \( \text{Int} \ C \). In addition, a 0-dimensional \( F_\varepsilon \) set \( F \) in \( C \) is said to be tame (relative to \( C \)) if \( F \) can be expressed as a countable union of tame (relative to \( C \)) compact subsets.

For definitions of other terms used here the reader is referred to such papers as [3, 8].

2. Tame polyhedra in factored cells. The goal of this section is to show that for any \( k \)-dimensional polyhedron \( P \) in a cell \( C \) that factors \( k \) times, \( E^n - P \) is 1-ULC. However, instead of arguing this directly, we prove first that \( E^n - C \) is 1-ULC in \( E^n - P \).

**Proposition 1.** If \( C \) is an \((n-1)\)-cell in \( E^n \) that factors \( k \) times \((k \leq n - 3)\) and \( P \) a \( k \)-dimensional polyhedron (topologically) embedded in \( C \), then \( E^n - C \) is 1-ULC in \( E^n - P \).

**Proof.** Suppose \( C = B \times I^k \subset E^{n-k} \times E^k \). Define a subset \( Z \) of \( P \) as the set of all points \( p \) of \( P \) for which there exist a neighborhood \( N_p \) of \( p \) (relative to \( P \)) and a point \( b \) in \( B \) such that \( N_p \subset (b) \times I^k \), and define \( Q = P - Z \). We prove first that, for each point \( c \) in \( C \), \( E^n - C \) is 1-LC in \( E^n - Q \) at \( c \).

Consider \( c \) to be of the form \((b, y)\), where \( b \in B \) and \( y \in \text{Int} \ I^k \) (the case \( y \in \partial I^k \) is similar and easier). Suppose \( N \) is a neighborhood of \((b, y)\) such that \( N \cap (B \times \partial I^k) = \emptyset \). There exist an open subset \( U \) of \( E^{n-k} \) and a contractible open subset \( V \) of \( I^k \) such that \((b, y) \in U \times V \subset N \). By the construction of \( Q \) there exists a point \( y' \in V \) such that \((b, y') \in Q \). Let \( U' \) be an open subset of \( E^{n-k} \) such that

\[
\begin{align*}
\text{b} & \in U' \subset U \\
(U' \times \{y'\}) & \cap Q = \emptyset.
\end{align*}
\]

Now we obtain an open subset \( W \) of \( E^{n-k} \) such that \( b \in W \subset U' \) and the inclusion map \( i: W \to U' \) is homotopic to a constant map.

Let \( L \) be a loop in \((W \times V) - C \). Since \( V \) is contractible to \( y' \), \( L \) is homotopic in \((W \times V) - C \) to a loop \( L' \) in \( W \times \{y'\} \). But \( L' \) is contractible in

\[
U' \times \{y'\} \subset N - Q.
\]

Thus, \( E^n - C \) is 1-LC in \( E^n - Q \) at \( c \).
The definition of $Z$ implies that $P$ is locally tame at each point of $Z$. Hence, if $f: \Delta^2 \to E^n - Q$ is a map such that $f(\partial \Delta^2) \subset E^n - P$, then $f$ can be approximated arbitrarily closely by maps $g: \Delta^2 \to E^n$ such that $g|\partial \Delta^2 = f|\partial \Delta^2$ and $g(\Delta^2) \subset E^n - P$. Thus, $E^n - C$ is $1$-LC in $E^n - P$ at each point $c$ of $C$. Since $C$ is compact, the corresponding uniform property holds as well.

There may be some value in observing that this argument also gives the following result.

**Proposition 2.** Let $B \times I^k \subset E^{n-k} \times E^k = E^n$ be an $m$-cell ($m < n$, $k \leq n - 3$) and $X$ a compactum in $B \times I^k$ such that $\dim(X \cap (\{b\} \times I^k)) < k$ for each $b$ in $B$. Then $E^n - (B \times I^k)$ is $1$-ULC in $E^n - X$.

**Theorem 3.** If $C$ is an $(n-1)$-cell in $E^n$ that factors $k$ times $(k \leq n - 3)$ and $X$ is either a $k$-dimensional polyhedron or a $(k-1)$-dimensional compactum in $C$, then $E^n - X$ is $1$-ULC.

This theorem follows immediately from [1, Prop. 1] and either Proposition 1 or Proposition 2.

**Corollary 4.** If $C$ is an $(n-1)$-cell in $E^n(n \geq 5)$ that factors $k$ times $(k \leq n - 3)$, then each $k$-dimensional polyhedron $P$ in $C$ is tame.

The corollary is a straightforward application of the Bryant-Seebeck characterization of tameness [3] for codimension $3$ polyhedra in terms of the $1$-ULC property.

3. Approximations in cells that factor $1$ time. This section contains a proof of the analogue of Seebeck's Corollary 5.1 [8] for codimension one cells.

**Proposition 5.** If $C$ is an $(n-1)$-cell in $E^n$ that factors $1$ time, then there exists a tame $0$-dimensional $F$, set $F$ in $\text{Int } C$ such that, for each point $c$ of $\text{Int } C$, $E^n - C$ is $1$-LC in $(E^n - C) \cup F$ at $c$.

**Proof.** Assume $C = B \times I \subset E^{n-1} \times E^1 = E^n$. Let $c = (b, t)$ be a point of $\text{Int } C$ and $U$ a neighborhood of $c$ such that $U \cap C \subset \text{Int } C$. We assume further that $U$ is a product neighborhood $U = U' \times J$, where $U' \subset E^{n-1}$ and $J \subset E^1$. Corresponding to $U$ is a neighborhood $V$ of $c$ such that any map $f': \partial \Delta^2 \to V - C$ extends to a map $f: \Delta^2 \to U$ such that $f^{-1}(f(\Delta^2) \cap C)$ is $0$-dimensional (\[4, \text{Cor. 2C, 2.1}\] or [5, Th. 3.2]). We can change this map $f$ near $C$, altering only the $E^1$ coordi-
nates of points in the range, so that in addition $f(A^2) \cap C \subset B \times \{t\}$. We shall obtain a map $g: \mathcal{A} \rightarrow \mathbb{R}$ satisfying

(i) $g|_{\partial A^2} = f|_{\partial A^2} = f'$,

(ii) $g(A^2) \cap C$ is a tame (relative to $C$) 0-dimensional subset of Int $C$.

Let $\varepsilon$ be a positive number such that if $g: \mathcal{A} \rightarrow E^n$ and $\rho(f, g) < \varepsilon$, then $g(A^2) \subset U$.

Cover $f^{-1}(f(A^2) \cap C)$ by the interiors of a collection of small, pairwise disjoint 2-cells $1D_1, 1D_2, \ldots, 1D_{k(1)}$ in Int $\mathcal{A}$. Slide the sets $f(1D_i)$ vertically to define a map $g_i: \mathcal{A} \rightarrow E^n$ satisfying

(A) $g_i|_{\mathcal{A}} = f|_{\mathcal{A}} - \bigcup_{1D_i}$,

(B) $\rho(g_i, f) < \varepsilon/2$,

(C) $g_i(1D_i) \cap C \subset B \times \{t_i\}$, where $t_i \neq t_j$ whenever $i \neq j$,

(D) $g_i^{-1}(g_i(A^2) \cap C)$ is 0-dimensional.

The $1D_i$'s must be chosen with sufficiently small diameters that each set $g_i(1D_i) \cap C$ is contained in the interior of a small $(n - 2)$-cell in $B \times \{t\}$. Thus,

(E) there exist pairwise disjoint $(n - 1)$-cells $1K_1, 1K_2, \ldots, 1K_{k(1)}$ in Int $\mathcal{A}$, each of diameter $< \varepsilon/2$, such that $\bigcup \text{Int} \ K_i \supset g_i(A^2) \cap C$.

The remaining approximations $g_j$ will be so close to $g_i$ that $\bigcup \text{Int} \ K_i \supset g_j(A^2) \cap C$.

Let $\varepsilon_2 = \min \left\{ \varepsilon/4, 1/2\rho(g_1(A^2) \cap C, C - \bigcup_1K_i) \right\}$. To repeat this process, cover $g_i^{-1}(g_i(A^2) \cap C)$ by the interiors of a collection of a very small, pairwise disjoint 2-cells $2D_1, 2D_2, \ldots, 2D_{k(2)}$ in $\bigcup \text{Int} \ 1D_i \subset \mathcal{A}$. Slide the sets $g_i(2D_i)$ vertically to define a map $g_2: \mathcal{A} \rightarrow E^n$ satisfying

(A) $g_2|_{\mathcal{A}} = g_i|_{\mathcal{A}} - \bigcup_{2D_i}$,

(B) $\rho(g_2, g_i) < \varepsilon_2$,

(C) $g_2(2D_i) \cap C \subset B \times \{t_i\}$, where $t_i \neq t_j$ whenever $i \neq j$,

(D) $g_2^{-1}(g_2(A^2) \cap C)$ is 0-dimensional.

The $2D_i$'s must be chosen with sufficiently small diameters that each set $g_2(2D_i) \subset C$ in a small $(n - 2)$-cell in some $B \times \{t_i\} \cap (\bigcup \text{Int} \ K_i)$. Thus,

(E) there exist pairwise disjoint $(n - 1)$-cells $2K_1, 2K_2, \ldots, 2K_{k(2)}$ in $\bigcup \text{Int} \ K_i$, each of diameter $< \varepsilon_2$, such that $\bigcup \text{Int} \ K_i \supset g_2(A^2) \cap C$.

By continuing in this manner we construct a sequence of maps $g_n: \mathcal{A} \rightarrow E^n$ satisfying analogous conditions $(A_n) - (B_n)$ and an associated sequence of collections $\{K_i\}$ of $n - 1$ cells in $C$ satisfying an analogous condition $(E_n)$. The restrictions of condition $(B_n)$ guarantee that $g = \lim g_n$ is a continuous function of $\mathcal{A}$ into $U$, and the restrictions of $(E_n)$ guarantee that

$$g(A^2) \cap C \subset \bigcap_{n=1}^{\infty} \left( \bigcup_{i=1}^{k(n)} \text{Int} K_i \right).$$

Thus, $g(A^2) \cap C$ is a tame (relative to $C$) 0-dimensional subset of $C$ [7, Lemma 2].
To prove the theorem from this fact, observe that for each $\varepsilon > 0$ there exists a countable collection $\{V_i\}$ of open sets covering $\text{Int} C$ such that any map $f^* : \partial D^r \to V_i - C$ extends to a map $g$ of $D^r$ into an $\varepsilon$-subset of $E^n$ such that $g(D^r) \cap C$ is a tame 0-dimensional subset of $\text{Int} C$. Since there are only countably many homotopy classes of maps of $\partial D^r$ into $V_i - C$, the desired set $F$ can be defined as the countable union of sets $g(D^r) \cap C$.

**Theorem 6.** Suppose $C$ is an $(n - 1)$-dimensional cell in $E^n$ that factors 1 time, $P$ is an $(n - 3)$-dimensional polyhedron properly embedded in $C$, and $\varepsilon > 0$. There exists an $\varepsilon$-push $h$ of $(C, P)$ such that $h(P)$ is tame relative to $E^n$.

**Proof.** The case $n = 4$ is trivial, and no push is needed [6]; hence, we assume $n \geq 5$. By [8, Cor. 5.1] there exists an $\varepsilon/2$ push $h_1$ of $(C, P)$ such that $h_1(P \cap \partial C)$ is tame. Let $F$ denote the 0-dimensional $F_\sigma$ set of Proposition 5. There exists an $\varepsilon/2$ push $h_2$ of $(C, h_1(P))$ such that $h_2h_1(P) \cap F = \emptyset$ and $h_2| \partial C = 1$. Let $h$ denote the $\varepsilon$-push $h_2h_1$. It follows that $E^n - C$ is 1-LC in $E^n - h(P)$ at each point of $\text{Int} C$, and in stronger form, as shown in § 2, that $E^n - h(P)$ is 1-LC at each point of $\text{Int} C$. The tameness of $h(P) \cap \partial C$ then implies that $E^n - h(P)$ is 1-LC at every point of $h(P)$. Thus, $h(P)$ is tame [3].

**Corollary 7.** Let $S$ denote an $(n - 2)$ sphere in $S^{n-1}$, the $(n - 1)$-sphere, and $\Sigma$ the suspension of $S$ in $S^n$, the suspension of $S^{n-1}$. Then there exists a tame (relative to $\Sigma$) 0-dimensional $F_\sigma$ set $F$ in $\Sigma$ such that $S^n - \Sigma$ is 1-ULC in $(S^n - \Sigma) \cup F$. Furthermore, if $P$ is an $(n - 3)$-dimensional polyhedron in $\Sigma$ and $\varepsilon > 0$, there exists an $\varepsilon$-push $h$ of $(\Sigma, P)$ such that $h(P)$ is tame relative to $S^n$.

4. Factored cells in which all lower dimensional compacta are locally nice. Let $C = B \times I^k \subset E^{n-k} \times E^k = E^n$ be an $r$-cell ($r < n$). Although the low dimensional polyhedra in $C$ are nicely embedded, some $(k + 1)$-cell in $C$ may be wild. In this section we mention a property of certain cells $B$ that implies every $(r - 1)$-dimensional polyhedron in $C$ is nicely embedded.

**Theorem 8.** Let $B$ denote an $m$-cell in $E^n(m \leq n - 2)$ such that, for each $(m - 1)$-dimensional compactum $X \subset B$, $E^n - X$ is 1-ULC, and let $C$ denote $B \times I^k$, contained in $E^n \times E^k = E^{n+k}$. Then, for each $(m + k - 1)$-dimensional compactum $Y \subset C$, $E^{n+k} - Y$ is 1-ULC.

**Proof.** It suffices to consider only the case $k = 1$. Let $\varepsilon > 0$ and $w \in \text{Int} I$. We shall construct an $\varepsilon$-push $h$ of $(E^{n+1}, Y)$ such that
\((E^n \times \{w\}) - h(Y)\) is 1\-ULC. Let \(V\) denote the \(\varepsilon\)-neighborhood of \(Y\), \(\{b_i\}\) a countable dense subset of \(B\), and \(\pi\) the natural projection of \(E^{n+1} = E^n \times E^1\) onto the first factor. For any open subset \(N\) of \(E^{n+1}\) containing \((b, w) \in B \times I\) there exists a point \((b', w') \in N \cap (B \times I - Y)\). If \(N\) is a connected open set of the form \(N = W \times J\), then there exists a homeomorphism \(g\) of \(E^{n+1}\) onto \(E^{n+1}\) such that (a) \(g|E^{n+1} - N = \text{identity}\), (b) \(g((b', w')) = (b', w)\), (c) \(g(C) = C\) and (d) \(\pi g = g\). Consequently, there exist a sequence \(\{h_i\}\) of homeomorphisms of \(E^{n+1}\) onto itself and a sequence of points \(\{b'_i\}\) in \(B\) such that for \(i = 1, 2, \ldots\)

\[
\begin{align*}
0 & \quad \rho(x, h_i(x)) < \varepsilon/2^i \quad \text{for all } x \in E^n, \\
1 & \quad \rho(b_i, b'_i) < 1/i, \\
2 & \quad (b'_i, w) \in h_\varepsilon \circ h_{\varepsilon^{-1}} \circ \cdots \circ h_i(Y), \\
3 & \quad h_{\varepsilon^{-k}}((b'_i, w)) = (b'_i, w) \quad \text{for all } k > 0, \\
4 & \quad h_i(C) = C, \\
5 & \quad \pi h_i = h_i, \\
6 & \quad h_i|E^{n+1} - V = \text{identity}.
\end{align*}
\]

Furthermore, using Condition (a) and careful epsilomics we can construct the sequence \(\{h_i\}\) so that the function \(h = \lim_{n \to \infty} h_n \circ \cdots \circ h_i\) is an \(\varepsilon\)-homeomorphism of \(E^{n+1}\) onto itself. Then Condition (6) implies that \(h\) is an \(\varepsilon\)-push of \((E^{n+1}, Y)\).

Condition (1) implies that \(\{b'_i\}\) is a dense subset of \(B\), and Conditions (2) and (3) yield that \((b'_i, w) \in h(Y)\) \((i = 1, 2, \ldots)\). Thus, \(h(Y) \cap (B \times \{w\})\) is nowhere dense in \(B \times \{w\}\). Consequently, \(E^n \times \{w\} - h(Y)\) is 1\-ULC by hypothesis (since \(h(Y) \subset B \times I\)), and we obtain the desired conclusion by appealing to Theorem 1 of [1].

We exploit the construction of the push \(h\) a second time in proving the following:

**Theorem 9.** Let \(B\) denote an \((n - 1)\)-cell in \(E^n\) such that, for each \((n - 2)\)-dimensional compactum \(X \subset B\), \(E^n - B\) is 1\-ULC in \(E^n - X\), and let \(C\) denote \(B \times I^k\), contained in \(E^n \times E^k\). Then for each \((n + k - 2)\)-dimensional compactum \(Y \subset C\), \(E^{n+k} - C\) is 1\-ULC in \(E^{n+k} - Y\).

**Proof.** Simplifying as before, we consider \(k = 1\) and \(c \in C\) a point of the form \((b, w)\), where \(b \in B\) and \(w \in \text{Int} I\), and we shall show that \(E^{n+1} - C\) is 1\-LC in \(E^{n+1} - Y\) at \(c\).

Let \(\varepsilon > 0\). Choose a countable dense subset \(\{b_i\}\) of \(B\). Then reapplying the techniques found in the proof of Theorem 8, we find an \((\varepsilon/6)\)-homeomorphism \(h\) of of \(E^{n+1}\) onto itself and a sequence \(\{b'_i\}\) of points in \(B\) satisfying Conditions (0)-(6) stated there. Let \(U\) denote the \(\varepsilon/6\)-neighborhood of \(b\) in \(E^n\) and \(V\) the \((\varepsilon/3)\)-neighborhood of \(w\) in
Int \( I \). Then both \((b, w)\) and \(h((b, w))\) are contained in \(U \times V\), and \(\text{diam } (U \times V) < \varepsilon/2\). Since \(B\) is an \((n - 1)\)-cell there exists a neighborhood \(U'\) of \(b\) in \(E^n\) such that \(b \in U' \subset U\) and each map \(f\) of \(\partial \Delta^2\) into \(U' - B\) can be extended to a map \(F\) of \(\Delta^2\) into \(U\) such that \(F^{-1}(F(\Delta^2) \cap B)\) is 0-dimensional.

In this paragraph we prove that \(U' \times V\) is a neighborhood of \(h(c)\) such that any loop in \((U' \times V) - C\) is contractible in an \(\varepsilon/2\)-subset of \(E^{n+1} - h(Y)\). If \(f: \partial \Delta^2 \to (U' \times V) - C\), \(f\) is homotopic in \((U' \times V) - C\) to a map \(f': \partial \Delta^2 \to U' \times \{w\}\). Let \(F: \Delta^2 \to U \times \{w\}\) be an extension of \(f'\) such that \(F^{-1}(F(\Delta^2) \cap (B \times \{w\}))\) is 0-dimensional. Once again \(h(Y) \cap (B \times \{w\})\) is nowhere dense in \(B \times \{w\}\), which means that \((E^n - B) \times \{w\}\) is 1-ULC in \(((E^n) \times \{w\}) - h(Y)\). Cover \(F^{-1}(F(\Delta^2) \cap (B \times \{w\}))\) by finitely many pairwise disjoint 2-cells \(D_1, \ldots, D_t\) in \(\text{Int} \Delta^2\) such that \(F|\partial D_i\) can be extended to a map \(G_i\) of \(D_i\) into \((U \times \{w\}) - h(Y)\). By redefining \(F\) as \(G_i\) on \(D_i\) for \(i = 1, \ldots, t\) one can easily see that \(f|\partial \Delta^2: \partial \Delta^2 \to (U \times V) - h(Y)\) is homotopic to a constant map.

Because \(h^{-1}\) is an \((\varepsilon/6)\)-homeomorphism and \(\text{diam } (U \times V) < \varepsilon/2\), \(\text{diam } h^{-1}(U \times V) < \varepsilon\). In addition, \(h^{-1}(U' \times V)\) is a neighborhood of \(c\) such that any map \(g: \partial \Delta^2 \to h^{-1}(U' \times V) - C\) can be extended to a map \(G: \Delta^2 \to h^{-1}(U \times V) - Y\). This completes the proof.

**Corollary 10.** Let \(B\) denote an \(m\)-cell in \(E^n(m < n)\) such that, for each \((m - 1)\)-dimensional compactum \(X \subset B, E^n - B\) is 1-ULC in \(E^n - X\). Then each \(p\)-dimensional polyhedron \(P\) in \(B \times I^k \subset E^n \times E^k\) \((p + 3 \leq n + k, p < m + k)\) is tame.

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