MAPPING SPACES AND CS-NETWORKS

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In this paper the space of maps from an \( \mathfrak{N}_0 \)-space to a space \( Y \) is studied by means of convergent sequence-networks. The notion of a \( cs-\sigma \)-space, a simultaneous generalization of metric spaces and \( \mathfrak{N}_0 \)-spaces, is defined, and it is shown that if \( Y \) is a (paracompact) \( cs-\sigma \)-space then the mapping space from \( X \) to \( Y \) is a (paracompact) \( cs-\sigma \)-space when equipped with either the compact-open or the \( cs \)-open topology. It is proved that the compact sets are the same in the two topologies. The class of \( cs-\sigma \)-spaces and the class of \( \mathfrak{N} \)-spaces introduced by O'Meara are shown to be identical in the presence of paracompactness.

In this paper all maps are continuous and all spaces Hausdorff.

1. CS-networks. We shall call a collection \( \mathcal{P} \) of subsets of a space \( X \) a \( k \)-network for \( X \) if whenever \( C \subseteq U \), with \( C \) compact and \( U \) open in \( X \), there exist finitely many elements of \( \mathcal{P} \) whose union covers \( C \) and lies in \( U \). This is a slight modification of what E. Michael [2] called a pseudobase. We may define the \( \mathfrak{K}_0 \)-spaces of Michael as regular spaces with a countable \( k \)-network.

If \( X \) is a space with topology \( \mathcal{I} \) we shall denote by \( k(X) \) the \( k \)-space obtained by retopologizing \( X \) so that a set is closed if its intersection with every \( \mathcal{I} \)-compact set is \( \mathcal{I} \)-closed.

If \( \{z_1, z_2, \cdots\} \) is a sequence of points which converges to a point \( z \), then we call the set \( Z = \{z, z_1, z_2, \cdots\} \) a convergent sequence and denote by \( Z_n \) the convergent sequence \( \{z, z_n, z_{n+1}, \cdots\} \).

A collection \( \mathcal{P} \) of subsets of a space \( X \) is a convergent sequence-network or, more conveniently, a \( cs \)-network for \( X \) if whenever \( Z \subseteq U \), with \( Z \) a convergent sequence and \( U \) open in \( X \), then \( Z_n \subseteq P \subseteq U \) for some \( n \) and some \( P \in \mathcal{P} \). We call a collection \( \mathcal{P} \) of subsets of \( X \) a network for \( X \) if whenever \( x \in U \) with \( U \) open in \( X \), then \( x \in P \subseteq U \) for some \( P \in \mathcal{P} \).

The notion of \( cs \)-network was introduced in [1] where the following theorem was proved.

**Theorem 1.** For a topological space \( X \) the following are equivalent:

1. \( X \) is an \( \mathfrak{K}_0 \)-space.
2. \( X \) is a regular space with a countable \( cs \)-network.

We shall call a regular space with a \( \sigma \)-locally finite \( cs \)-network a \( cs-\sigma \)-space. It is clear from Theorem 1 that every \( \mathfrak{K}_0 \)-space is a \( cs-\sigma \)-space.
σ-space, and from the Nagata-Smirnov Metrization Theorem that all metric spaces are cs-σ-spaces.

2. Mapping spaces. We shall denote by \( C(X, Y) \) the space of all maps from \( X \) to \( Y \) with the compact-open topology, and by \( C_p(X, Y) \) the topology of pointwise convergence. The symbol \( C_{cs}(X, Y) \) will denote the space of maps from \( X \) to \( Y \) with the convergent sequence-open topology. This is the topology whose subbasic open sets are of the form \( (Z, U) = \{ f \mid f : X \to Y \text{ and } f(Z) \subset U \} \) where \( Z \) is a convergent sequence in \( X \) and \( U \) is open in \( Y \).

The fact that many of the desirable properties of the compact-open topology are also enjoyed by the cs-open topology was asserted in [1]. Proofs may be found in [7] where O. Wyler shows that a category in which the cs-open topology appears naturally is convenient (in the technical sense of Steenrod [6]) for algebraic topology.

The class of \( \mathfrak{K}_c \)-spaces appears to be especially suitable for the study of mapping spaces. For example, at the time he introduced \( \mathfrak{K}_c \)-spaces Michael [2] showed that if \( X \) and \( Y \) are \( \mathfrak{K}_c \)-spaces, so is \( C(X, Y) \). It is also true in this case [1] that \( C_{cs}(X, Y) \) is an \( \mathfrak{K}_o \)-space. These two results and an unsolved problem form the basis of the present investigation. The problem, also stated by Michael [3], asks whether \( X \) compact metric and \( Y \) a CW-complex implies that \( C(X, Y) \) is paracompact. More generally one can ask what properties added to the paracompactness of \( Y \) will insure the paracompactness of \( C(X, Y) \).

**Lemma 1.** If \( \mathcal{P} \) is a collection of subsets of a space \( X \), which is closed under finite intersections, then \( \mathcal{P} \) is a cs-network for \( X \) if whenever \( Z \subset S \), with \( Z \) a convergent sequence and \( S \) a subbasic open set in \( X \), then \( Z_n \subset P \subset S \) for some \( n \) and some \( P \in \mathcal{P} \).

**Proof.** Suppose \( Z \subset U \) with \( Z \) converging to \( z \) and \( U \) open in \( X \). Then there exists a basic open set \( B \) such that \( z \in B \subset U \). Now there exist finitely many subbasic open sets \( S_1, \ldots, S_k \) such that \( B = S_1 \cap \cdots \cap S_k \). Now \( z \in S_i \) for each \( i \), so there exist \( n(i) \) and \( P_i \in \mathcal{P} \) such that \( Z_{n(i)} \subset P_i \subset S_i \) for \( 1 \leq i \leq k \). Now let \( Z_n = Z_{n(1)} \cap \cdots \cap Z_{n(k)} \) and \( P = P_1 \cap \cdots \cap P_k \). Then \( Z_n \subset P \subset B \subset U \) and \( \mathcal{P} \) is a cs-network for \( X \).

**Theorem 2.** If \( X \) is an \( \mathfrak{K}_c \)-space and \( Y \) is a cs-σ-space, then \( C(X, Y) \) is a cs-σ-space.

**Proof.** By Theorem 11.4 (b) of [2] the \( \mathfrak{K}_c \)-space \( X \) is the image of a separable metric space \( S \) under a compact-covering map. Thus by Lemma 1 of [5] \( C(X, Y) \) is homeomorphic to a subspace of \( C(S, \)
Y). Since every subspace of a cs-σ-space is also a cs-σ-space, it will suffice to show that \( \mathcal{E}(S, Y) \) is a cs-σ-space.

Let \( \mathcal{P} = \{P_i\} \) be a countable open base for \( S \) which is closed under finite intersections, and let \( \mathcal{R} = \bigcup_{j=1}^{\infty} R_j \) be a σ-locally finite cs-network for \( Y \). Let \( \{P_i, R_i\} = \{(P_i, R) | R \in \mathcal{R}\} \), where \( (P_i, R) = \{f \in \mathcal{E}(S, Y) | f(P_i) \subset R\} \), and let \( [\mathcal{P}, \mathcal{R}] = \bigcup_{i=1}^{\infty} [P_i, \mathcal{R}] \).

We first show that \( [\mathcal{P}, \mathcal{R}] \) is σ-locally finite. Clearly \( [\mathcal{P}, \mathcal{R}] \) is the union of countably many \( [P_i, \mathcal{R}] \). To see that each \( [P_i, \mathcal{R}] \) is locally finite, let \( f \in \mathcal{E}(S, Y) \) and \( x \in P_i \). Then \( f(x) \in Y \), and there is a neighborhood \( V \) of \( f(x) \) which intersects at most finitely many members of \( \mathcal{R}_j \). Then \( (x, V) \) is a subbasic open neighborhood of \( f(x) \) which meets only those elements \( (P_i, 22) \) of \( [P_i, \mathcal{R}] \) for which \( 22 \) intersects \( V \).

It is the set of all finite intersections of elements of \( [\mathcal{P}, \mathcal{R}] \), which we will call \( [\mathcal{P}, \mathcal{R}]^* \), which is a σ-locally finite cs-network for \( \mathcal{E}(S, Y) \).

By Lemma 1 we need consider only subbasic open sets in showing that \( [\mathcal{P}, \mathcal{R}]^* \) is a cs-network for \( \mathcal{E}(S, Y) \). Let \( F = \{f_0, f_1, f_2, \ldots\} \) be a sequence of maps converging to \( f_0 \) in \( \mathcal{E}(S, Y) \). Let \( (C, U) \) be a subbasic open set containing \( f_0 \). Since \( F \) is compact, \( S \) is a k-space, and \( Y \) is regular, we may conclude by Lemma 9.2 of [2] that \( F^{-1}(U) = \{x \in S | f_i(x) \in U \text{ for some } f_i \in F\} \) is open in \( S \). Clearly \( F^{-1}(U) \supset C \). Let \( \mathcal{P}' = \{P \in \mathcal{P} | P \subset F^{-1}(U)\} \). For every \( x \in C \), let \( \mathcal{P}(x) = \{P \in \mathcal{P}' | x \in P \cap C\} \), and let \( \mathcal{P}'(x) = \{P_i | P_i = \bigcup_{j=1}^{\infty} P_j, P_j \in \mathcal{P}(x)\} \). Also let \( \mathcal{R}(x) = \{R \in \mathcal{R} | f_0(x) \in R \subset U\} \). Clearly \( \mathcal{R}(x) \) is countable.

There must exist integers \( N, i, \) and \( j \) such that \( F_N \subset (P_i, R_j) \subset (x, U) \). To see this, suppose not. Then since for every \( N, i, \) and \( j, x \in P_i \) and \( R_j \subset U \), we have \( (P_i, R_j) \subset (x, U) \). Therefore, it must be true for every \( N, i, \) and \( j \) that \( F_N \subset (P_i, R_j) \). That is, there is some \( n \geq N \) and some \( x_{ij} \in P_i \) such that \( f_n(x_{ij}) \in R_j \). We now extract a convergent subsequence of \( F \) using these results.

Choose \( f_{n(1)} \) such that \( f_{n(1)}(P_i) \not\subset R_i \). Then there is some \( n(2) \) such that \( f_{n(2)}(P_i) \not\subset R_i \). Similarly choose \( f_{n(3)} \) such that \( n(3) > n(2) \) and \( f_{n(3)}(P_i) \not\subset R_i \), and \( f_{n(4)} \) so that \( n(4) > n(3) \) and \( f_{n(4)}(P_i) \not\subset R_i \). Note that the \( P_i \) are being considered in order, but the \( R_i \) are being considered so that their subscripts form the sequence \( 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, \ldots \). That is, at any place in the sequence of \( R_i \), we proceed until we include the first \( R_j \) which had not been included before, and then start over with \( R_i \).

Set \( f_i' = f_{n(i)} \), and choose \( x_i \in P_i \) so that \( f_i'(x_i) \) is not an element of the \( R_j \) which corresponds to \( f_{n(i)} \) and \( P_i \). Now \( \{f_i'\} \) is a subsequence of \( F \), and hence it must converge to \( f_x \). The collection \( \mathcal{P}'(x) \) is a decreasing countable base for \( x \) in \( S \). Thus \( \{x_i\} \) converges to \( x \).

Since convergence in the compact-open topology implies continuous
convergence for sequences, \( \{ f'_i(x_i) \} \) converges to \( f_0(x) \). Thus all but finitely many elements of \( \{ f'_i(x_i) \} \) lie in \( U \). Therefore, there exist an integer \( N \) and an \( R_k \in \mathcal{R}(x) \) so that \( f'_i(x_i) \in R_k \) for all \( i \geq N \). But by the construction of the sequences \( \{ f'_i \} \) and \( \{ x_i \} \) there is some \( m > N \) such that \( f'_m(x_m) \in R_k \). This contradiction means that there do exist some \( N(x), i(x), j(x) \) such that \( i(x) \in (S, F) \) has a \( \sigma \)-locally finite \( cs \)-network. Since \( Y \) is regular, \( \mathcal{E}(S, Y) \) is regular, and hence is a \( cs-\sigma \)-space. Thus \( \mathcal{E}(X, Y) \) is also a \( cs-\sigma \)-space.

Now note that we could have obtained the collection of sets which forms the \( cs \)-network for \( \mathcal{E}(X, Y) \) in another way. Let \( f \) be the compact-covering map such that \( f(S) = X \). Then for every \( P \in \mathcal{P} \) and \( R \in \mathcal{R} \), \( (P, R) \cap \mathcal{E}(X, Y) = (f(P), R) \). Thus if we are interested in actually exhibiting a \( \sigma \)-locally finite \( cs \)-network for \( \mathcal{E}(X, Y) \) we may be assured one can be constructed from a countable \( k \)-network \( \mathcal{P} \) for \( X \) and a \( \sigma \)-locally finite \( cs \)-network \( \mathcal{R} \) for \( Y \) by forming \([\mathcal{P}, \mathcal{R}] \)' as above.

We now turn our attention to the \( cs \)-open topology. This topology is compared to the compact-open topology in the following.

**Lemma 2.** Let \( X \) be a space in which every compact set is sequentially compact. Then \( \mathcal{E}(X, Y) \) and \( \mathcal{E}_{cs}(X, Y) \) have the same convergent sequences.

**Proof.** Clearly any sequence converging in the compact-open topology converges in the coarser topology. Conversely, let \( \{ f_n \} \) be a sequence converging to \( f_0 \) in \( \mathcal{E}_{cs}(X, Y) \). We will show that every subbasic open set in \( \mathcal{E}(X, Y) \) which contains \( f_0 \) contains all but finitely many \( f_n \). Let \( f_0 \in (C, U) \). Suppose there are infinitely many \( f_{i(n)} \) for which \( f_{i(n)} \notin (C, U) \). Then for every \( n \) there exists \( x_n \in C \) such that \( f_{i(n)}(x_n) \notin U \). But \( C \) is sequentially compact, so \( \{ x_n \} \) has a convergent subsequence \( Z \subset C \). Now \( f_0 \in (Z, U) \), but for infinitely many \( f_n \), \( f_n(Z) \not\subset U \). Thus \( \{ f_n \} \) converges in \( \mathcal{E}(X, Y) \).

**Theorem 3.** If \( X \) is an \( \mathcal{K}_0 \)-space and \( Y \) is a \( cs-\sigma \)-space, \( \mathcal{E}_{cs}(X, Y) \) is a \( cs-\sigma \)-space.

**Proof.** By Theorem 2 \( \mathcal{E}(X, Y) \) has a \( \sigma \)-locally finite \( cs \)-network \( \mathcal{P} \). This same collection of sets forms a \( cs \)-network for \( \mathcal{E}_{cs}(X, Y) \) since \( \mathcal{E}(X, Y) \) and \( \mathcal{E}_{cs}(X, Y) \) have the same convergent sequences and \( \mathcal{E}(X, Y) \) has at least as many open sets as \( \mathcal{E}_{cs}(X, Y) \). The neighborhoods used in Theorem 2 to show that the \( cs \)-network for \( \mathcal{E}(S, Y) \)
was $\sigma$-locally finite were of the form $(x, U)$. Thus the restrictions of these open sets to the subspace $\mathcal{C}(X, Y)$ will illustrate the $\sigma$-locally finiteness of $\mathcal{R}$. Sets of the form $(x, U)$ are also open in $\mathcal{C}_s(X, Y)$. Thus $\mathcal{C}_s(X, Y)$ has a $\sigma$-locally finite $cs$-network, and since, by Proposition 1 of [1] $\mathcal{C}_s(X, Y)$ is regular, $\mathcal{C}_s(X, Y)$ is a $cs$-$\sigma$-space.

**Lemma 3.** If $X$ is separable and $Y$ has each point a $G_\delta$, then $\mathcal{C}_p(X, Y)$ has each point a $G_\delta$.

**Proof.** Let $\{x_i\}$ be a countable dense subset of $X$ and let $f \in \mathcal{C}_p(X, Y)$. For every $i$, let $\{U_{i,j}\}$ be a countable collection of open sets whose intersection is $f(x_i)$. Define $V_{i,j} = (x_i, U_{i,j})$. Clearly $f \in \bigcap_{i,j=1}^\infty V_{i,j}$. Conversely, suppose $g \neq f$. Then there is some $x_k$ such that $f(x_k) \neq g(x_k)$ and some $V_{k,i}$ such that $g(x_k) \in V_{k,i}$. Thus $g \in \bigcap_{i,j=1}^\infty V_{i,j}$ and $f$ is a $G_\delta$.

**Theorem 4.** If $X$ is a separable space in which every compact set is sequentially compact and $Y$ has each point a $G_\delta$, then $\mathcal{C}(X, Y)$ and $\mathcal{C}_s(X, Y)$ have the same compact sets.

**Proof.** $\mathcal{C}(X, Y)$ and $k(\mathcal{C}(X, Y))$ have the same compact subsets. Also $\mathcal{C}_s(X, Y)$ has the same compact subsets as $k(\mathcal{C}_s(X, Y))$. Now points are $G_\delta$-sets in $\mathcal{C}(X, Y)$ and $\mathcal{C}_s(X, Y)$ and hence points are $G_\delta$'s in the associated $k$-spaces. But a $k$-space in which every point is a $G_\delta$ is a sequential space [4]. Thus $k(\mathcal{C}(X, Y))$ and $k(\mathcal{C}_s(X, Y))$ are each sequential spaces, obtained by expanding the topologies of spaces which had the same convergent sequences. Thus $k(\mathcal{C}(X, Y))$ and $k(\mathcal{C}_s(X, Y))$ are homeomorphic under the identity map, and therefore have the same compact subsets. The conclusion of the theorem now follows.

**Corollary.** If $X$ is an $\aleph_0$-space and $Y$ is a $cs$-$\sigma$-space, then $\mathcal{C}(X, Y)$ and $\mathcal{C}_s(X, Y)$ have the same compact sets.

Another simultaneous generalization of $\aleph_0$-spaces and metric spaces has been introduced by P. O'Meara [5]. He calls a regular space an $\aleph$-space if it has a $\sigma$-locally finite $k$-network. Because of Theorem 1 it may be expected that there be some relation between $cs$-$\sigma$-spaces and $\aleph$-spaces. That this is, in fact, the case is established in the following two theorems.

**Theorem 5.** Every $cs$-$\sigma$-space is an $\aleph$-space.

**Proof.** A straightforward adaptation of the relevant part of the proof of Theorem 1 in [1] suffices.
THEOREM 6. In a paracompact space $X$ the following are equivalent:

1. $X$ is a cs-$\sigma$-space.
2. $X$ is an $\mathfrak{R}$-space.

Proof. In light of Theorem 5 we need to show only that (2) implies (1). Let $\mathcal{P} = \bigcup_{i=1}^{m} \mathcal{P}_i$ be a $\sigma$-locally finite $k$-network for $X$ such that $\mathcal{P}_i \subset \mathcal{P}_{i+1}$ and each $P \in \mathcal{P}$ is closed. For every natural number $i$ and every $x \in X$, let $V_{ix} = X \setminus \bigcup \{P \in \mathcal{P}_i \mid x \notin P\}$. Set $\mathcal{V}_i = \{V_{ix} \mid x \in X\}$. Then $\mathcal{V}_i$ is an open cover of $X$ for every $i$, and hence it has a precise locally finite open refinement $\mathcal{G}_i = \{G_{ix} \mid x \in X\}$ with $G_{ix} \subset V_{ix}$ for every $x$. Now for every $P \in \mathcal{P}_i$ such that $x \in P$, define $P_{ix} = P \cap G_{ix}$. For a fixed $i$ and $x$ there are at most finitely many $P_{ix}$. Denote the finite unions of these $P_{ix}$ by $R_{ix_1}, \ldots, R_{ix_h}$.

Now the collection $\mathcal{R}_i = \{R_{ixn} \mid x \in X, 1 \leq n < \infty\}$ is locally finite. For if $y \in X$ there exists an open neighborhood $N(y)$ which intersects at most finitely many $G_{ix} \in \mathcal{G}_i$. But each $G_{ix}$ intersects only those finitely many $R_{ixn}$ which it contains, and hence $N(y)$ intersects at most finitely many $R_{ixn}$ for each $i$.

It remains to be shown that $\mathcal{R} = \bigcup_{i=1}^{m} \mathcal{R}_i$ is a $\mathfrak{R}$-network for $X$. Suppose $Z$ is a sequence converging to $z$ and $U$ is an open set such that $Z \subset U$. Then since $Z$ is compact there exists a natural number $j$ and finitely many $P \in \mathcal{P}_j$, say $P_{j1}, \ldots, P_{jm}$, such that $Z \subset \bigcup_{i=1}^{m} P_{ji} \subset U$. We may assume that $z \in P_{ji}$ for $1 \leq i \leq m$.

Since $\mathcal{G}_j$ is an open cover of $X$ there is some $G_{jz} \in \mathcal{G}_j$ such that $z \in G_{jz}$. Each $P_{ji}$ must contain $x$, for if $x \in P_{ji}$ then $z \notin V_{jx} \supseteq G_{jx}$. Thus $\bigcup_{i=1}^{m} (P_{ji} \cap G_{jx}) \in \mathcal{R}_j$. But $G_{jx} \cap U$ is an open neighborhood of $z$ and hence there exists an $r$ such that $Z_r \subset G_{jx} \cap U$. Therefore, $Z_r \subset \bigcup_{i=1}^{m} (P_{ji} \cap G_{jx}) \subset U$, and $\mathcal{R}$ is a $\mathfrak{R}$-network for $X$.

The following lemma and theorem were obtained by O'Meara [5].

LEMMA 4. Let $X$ be a regular space with a $\sigma$-locally finite network $\mathcal{I} = \bigcup_{n=1}^{\infty} \mathcal{I}_n$. Suppose for every $n$ there is a locally finite family of neighborhoods $\{V_n(x) \mid x \in X\}$ such that $\text{Cl}(V_n(x))$ meets only finitely many $T \in \mathcal{I}_n$. Then $X$ is paracompact.

THEOREM 7. If $X$ is an $\mathfrak{R}$-space and $Y$ is a paracompact $\mathfrak{R}$-space, then $\mathcal{C}(X, Y)$ is a paracompact $\mathfrak{R}$-space.

We have a similar result if the mapping space is equipped with the $\mathfrak{R}$-open topology.

THEOREM 8. Let $X$ be an $\mathfrak{R}$-space and let $Y$ be a paracompact $\mathfrak{R}$-$\sigma$-space. Then $\mathcal{C}_{cs}(X, Y)$ is a paracompact $\mathfrak{R}$-$\sigma$-space.
Proof. Let $\mathcal{P} = \{P_i\}$ be a countable $k$-network for $X$, and let $\mathcal{R} = \bigcup_{i=1}^{\infty} \mathcal{R}_i$ be a $\sigma$-locally finite $cs$-network for $Y$. Let $[P_i, \mathcal{R}] = \{(P_i, R) | R \in \mathcal{R}_i\}$, and let $[\mathcal{P}, \mathcal{R}] = \bigcup_{i,j=1}^{\infty} [P_i, \mathcal{R}_j]$. By Theorem 3 and the remarks at the end of the proof of Theorem 2, it may be seen that the set of all finite intersections of $[\mathcal{P}, \mathcal{R}]$ forms a $\sigma$-locally finite $cs$-network for $\mathcal{E}_s(X, Y)$. We now show that Lemma 4 may be applied to this family.

For every $f \in \mathcal{E}_s(X, Y)$, choose $x \in P_i$ and let $V_{ij}(f)$ be an open neighborhood of $f(x)$ which intersects at most finitely many $R \in \mathcal{R}_j$. Consider the open cover $\{V_{ij}(f) | f \in \mathcal{E}_s(X, Y)\}$ of $Y$. By the paracompactness of $Y$ there exists a locally finite open refinement $W_{ij} = \{W_{ij}(f) | f \in \mathcal{E}_s(X, Y)\}$ such that $W_{ij}(f) \subset \text{Cl}(W_{ij}(f)) \subset V_{ij}(f)$ for every $f$. Then $\text{Cl}(x, W_{ij}(f)) \subset (x, \text{Cl}(W_{ij}(f)))$ which intersects at most finitely many $(P_i, R) \in [P_i, \mathcal{R}_j]$. Thus $\text{Cl}(x, W_{ij}(f))$ meets at most finitely many of the finite intersections of $[P_i, \mathcal{R}_j]$ and by Lemma 4, $\mathcal{E}_s(X, Y)$ is paracompact.

It can be seen from Example 1 of [1] that despite Theorem 4 the spaces $\mathcal{E}(X, Y)$ and $\mathcal{E}_s(X, Y)$ considered in Theorems 2, 3, 7, and 8 need not be homeomorphic even in the special case where both $X$ and $Y$ are separable metric spaces.

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