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**The concepts of formations, $\mathscr{F}$-projectors and $\mathscr{F}$-normalizers have all been developed for solvable Lie algebras. In this note, for each saturated formation $\mathscr{F}$ of solvable Lie algebras, the class $\mathscr{T}(\mathscr{F})$ of solvable Lie algebras $L$ in which each $\mathscr{F}$-normalizer of $L$ is an $\mathscr{F}$-projector is considered. This is the natural generalization of the Lie algebra analogue to SC groups which were first investigated by R. Carter. It is shown that $\mathscr{T}(\mathscr{F})$ is a formation. Then some properties of $\mathscr{F}$-normalizers of $L \in \mathscr{T}(\mathscr{F})$ are considered.**

All Lie algebras considered here are solvable and finite dimensional over a field $F$. $\mathscr{F}$ will always denote a saturated formation of solvable Lie algebras and $L$ will be a solvable Lie algebra. $N(L)$ is the nil-radical of $L$ and $\Phi(L)$ is the Frattini subalgebra of $L$. For definitions and properties of all these concepts see [3], [4], and [9]. For SC groups see [6].

We begin with a general lemma.

LEMMA 1. *Let $N$ be an ideal of $L$ and $D/N$ be an $\mathscr{F}$-normalizer of $L/N$. Then there exists an $\mathscr{F}$-normalizer $E$ of $L$ such that $E + N = D$.*

*Proof.* Let $L$ be a minimal counterexample and we may assume that $N$ is a minimal ideal of $L$. If $D/N = L/N$, then any $\mathscr{F}$-normalizer of $L$ has the desired property, hence we may suppose that $D/N \subset L/N$. Suppose first that $N$ is $\mathscr{F}$-central in $L$. Let $N^*/N = N(L/N)$ and $C = C_L(N)$. Then $N(L) = N^* \cap C$. Let $M/N$ be a maximal $\mathscr{F}$-critical subalgebra of $L/N$ such that $D/N$ is an $\mathscr{F}$-normalizer of $M/N$. Now either $M$ is $\mathscr{F}$-critical in $L$ or $M$ complements a chief factor of $L$ between $N^*$ and $N(L)$. In the first case, by induction, there exists an $\mathscr{F}$-normalizer $E$ of $M$ such that $E + N = D$ and $E$ is also an $\mathscr{F}$-normalizer in $L$. In the second case, $L/C \in \mathscr{F}$ and $C + N^*/C$ is operator isomorphic to $N^*/N^* \cap C = N^*/N(L)$. Hence each chief factor of $L$ between $N^*$ and $N(L)$ is $\mathscr{F}$-central which contradicts $M$ being $\mathscr{F}$-abnormal.

Now suppose that $N$ is $\mathscr{F}$-eccentric and assume $N \subseteqq \Phi(L)$. Let $M/N$ be as in the above paragraph. Again, by induction, there exists an $\mathscr{F}$-normalizer $E$ of $M$ such that $E + N = D$. But $N \subseteqq \Phi(L)$ yields that $M$ is $\mathscr{F}$-critical in $L$ using Theorem 2.5 of [4]. Hence $E$ is an

$\mathscr{F}$-normalizer of $L$ and this case is completed.

Finally suppose that $N$ is $\mathscr{F}$-eccentric and assume $N \nsubseteq \Phi(L)$. Then $N$ is complemented by a maximal subalgebra $M$ which must be $\mathscr{F}$-critical in $L$. Now there must exist an $\mathscr{F}$-normalizer $E$ of $M$ such that $E + N = D$. Again $E$ must be an $\mathscr{F}$-normalizer of $L$ and the result is shown.

COROLLARY. *$\mathscr{T}(\mathscr{F})$ is closed under homomorphisms.*

*Proof.* Let $N$ be a minimal ideal of $L$, $L \in \mathscr{T}(\mathscr{F})$. Let $D/N$ be an $\mathscr{F}$-normalizer of $L/N$. Then $D = E + N$ for some $\mathscr{F}$-normalizer of $L$. Now $E$ is an $\mathscr{F}$-projector of $L$ and $E + N/N = D/N$ is an $\mathscr{F}$-projector of $L/N$.

LEMMA 2. *If $L \in \mathscr{T}(\mathscr{F})$ and $C$ is an $\mathscr{F}$-projector of $L$, then $C$ is an $\mathscr{F}$-normalizer of $L$.*

*Proof.* Let $N$ be a minimal ideal of $L$. $L/N \in \mathscr{T}(\mathscr{F})$ hence $C + N/N$ is an $\mathscr{F}$-normalizer of $L/N$ by induction. Hence $C + N = D + N$ for some $\mathscr{F}$-normalizer $D$ of $L$. Now $D$ is also an $\mathscr{F}$-projector of $L$ and both $C$ and $D$ are $\mathscr{F}$-projectors of $C + N$. Then $C$ and $D$ are conjugate in $C + N$ by an inner automorphism of $C + N$ induced by an element of $N$ by Lemma 1.11 of [3]. Hence $D$ and $C$ are conjugate in $L$ and the result holds.

Note that $\mathscr{T}(\mathscr{F})$ contains a large class of Lie algebras. In fact by Theorem 3 of [9] we have

LEMMA 3. $\mathscr{N}\mathscr{F} \subseteqq \mathscr{T}(\mathscr{F})$.

In order to obtain that $\mathscr{T}(\mathscr{F})$ is a formation, we record a characterization of $\mathscr{F}$-projectors which is completely analogous to a result in group theory due to Bauman [5]. Since the proofs carry over virtually unchanged, we omit them.

DEFINITION. If $M$ is a subalgebra of $L$, then a series $0 = L_0 \subset \cdots \subset L_n = L$ is called an $M$-series if $L_i$ is an ideal in $L_{i+1}$, if $M \subseteqq N_L(L_i)$ and if each $L_{i+1}/L_i$ is a nontrivial, irreducible $M$-factor of $L$.

THEOREM 1. *If $C$ is an $\mathscr{F}$-projector of $L$ and $\{L_i\}$, $0 \leqq i \leqq n$, is any $C$-series of $L$, then $C$ covers $L_i/L_{i-1}$ if and only if $C + L_i/L_{i-1} \in \mathscr{F}$.*

*Proof.* See proof of Theorem 1 of [5].

THEOREM 2. *If $\{L_i\}$ is a C-series of $L$ such that $C$ covers $L_i/L_{i-1}$ if and only if $C + L_i/L_{i-1} \in \mathscr{F}$, then $C$ is an $\mathscr{F}$-projector of $L$.*

*Proof.* See proof of Theorem 2 of [5].

We intend to use these results in a slightly different form by means of

LEMMA 4. *Let $M$ be a subalgebra of $L$, $M \in \mathscr{F}$ and $H/K$ be a nontrivial, irreducible M-factor of $L$. Then $M + H/K \in \mathscr{F}$ if and only if the split extension of $H/K$ by $M/C_M(H/K)$ is in $\mathscr{F}$.*

*Proof.* Since $M + H/H \in \mathscr{F}$, $M + H/K$ will be in $\mathscr{F}$ if and only if the minimal ideal $H/K$ of $M + H/K$ is $\mathscr{F}$-central in $M + H/K$; that is, if and only if the split extension of $H/K$ by $M + H/C_{M+H}(H/K)$ is in $\mathscr{F}$. But

$$\begin{aligned} M/C_M(H/K) &= M/M \cap C_{M+H}(H/K) \cong M + C_{M+H}(H/K)/C_{M+H}(H/K) \\ &= M + H/C_{M+H}(H/K) \; . \end{aligned}$$

Now the corresponding split extensions of $H/K$ by $M + H/C_{M+H}(H/K)$ and $H/K$ by $M/C_M(H/K)$ are isomorphic and the result holds.

THEOREM 3. *$\mathscr{T}(\mathscr{F})$ is a formation.*

*Proof.* $\mathscr{T}(\mathscr{F})$ is closed under homomorphisms has been noted already. Hence let $N_1$ and $N_2$ be ideals of $L$ such that $L/N_1, L/N_2 \in \mathscr{T}(\mathscr{F})$. We may assume $N_1 \cap N_2 = 0$ and show that $L \in \mathscr{T}(\mathscr{F})$. Let $D$ be an $\mathscr{F}$-normalizer of $L$. Then $D + N_1/N_1$ is an $\mathscr{F}$-normalizer of $L/N_1$, hence is an $\mathscr{F}$-projector of $L/N_1$ and the corresponding statement holds for $D + N_2/N_2$. Consider a $D$-series of $L$ which passes through $N_1$ and $N_1 + N_2$. There is a $D$-series of $L$ which passes through $N_2$ and $N_1 + N_2$ which is the same as the original $D$-series above $N_1 + N_2$ and corresponds to the original $D$-series below $N_1 + N_2$ in the natural way. In particular, a factor $H/K$ in the new $D$-series which is between $N_2$ and $N_1 + N_2$ corresponds to $H \cap N_1/K \cap N_1$ in the original $D$-series and we claim that $D$ covers (avoids) $H/K$ if and only if $D$ covers (avoids) $H \cap N_1/K \cap N_1$. For if $D$ avoids $H/K$, then $D \cap H \subseteqq K$, hence $D \cap H \cap N_1 \subseteqq K \cap N_1$ and $D$ avoids $H \cap N_1/K \cap N_1$. Suppose that $D$ covers $H/K$. Then $H \subseteqq K + D$. In order to show that $D$ covers $H \cap N_1/K \cap N_1$ it is sufficient to show that $D + (K \cap N_1) \supseteqq H \cap N_1$. Since $H \subseteqq K + D$, $D \subseteqq N_L(K)$ and $H \subseteqq N_1 + N_2$, it follows that $H \subseteqq K + (D \cap (N_1 + N_2))$. Using the corollary on p. 241 of [9], $H \subseteqq K + ((D \cap N_1) + (D \cap N_2)) = K + (D \cap N_1)$. Then, since $D \cap N_1 \subset N_L(K)$ it follows that $H \cap N_1 \subseteqq (K + (D \cap N_1)) \cap N_1 \subseteqq (K \cap N_1) + (D \cap N_1) \subseteqq (K \cap N_1) + D$, hence $D$ covers $H \cap N_1/K \cap N_1$.

By Theorem 1 and Lemma 4, a factor $H/K$ above $N_1$ in the original $D$-series is covered by $D + N_1/N_1$ (hence $D$) if and only if the split extension of $H/K$ by $D + N_1/C_{D+N_1}(H/K)$ is in $\mathscr{F}$. That is, $H/K$ is covered by $D$ if and only if the split extension of $H/K$ by $D/C_D(H/K)$ is in $\mathscr{F}$. A similar statement holds above $N_2$. Every $D$-factor in the original series is operator isomorphic to a $D$-factor above $N_1$ or above $N_2$ and, using the result of the above paragraph, in the original $D$-series a factor $H/K$ is covered by $D$ if and only if the split extension of $H/K$ by $D/C_D(H/K)$ is in $\mathscr{F}$. Now by Lemma 4 and Theorem 2, $D$ is an $\mathscr{F}$-projector of $L$ and $\mathscr{T}(\mathscr{F})$ is a formation.

The following example shows that $\mathscr{N}\mathscr{N} \subset \mathscr{T}(\mathscr{N})$ and that $\mathscr{T}(\mathscr{N})$ is not closed under taking ideals. It is a variant of an example on p. 52 of [7].

EXAMPLE. Let $F$ be a field of characteristic $p \geqq 2$ and let $A$ be a vector space over $F$ with basis $e_0, \cdots, e_{p-1}$. Define linear transformations $x, y, z$ on $A$ by

$$x(e_i) = ie_i$$
$$y(e_i) = e_{i+1}$$

and

$$z(e_i) = e_i$$

(subscripts mod $p$). Then $[x, y] = xy - yx = y$ and $[x, z] = [y, z] = 0$. Let $B$ be the three dimensional Lie algebra generated by $x, y, z$. Let $L$ be the semi-direct sum of $A$ and $B$ with the natural product. As on p. 53 of [7], $B$ acts irreducibly on $A$ so that $A$ is a minimal ideal of $L$. Evidently $A$ is self-centralizing in $L$, hence $A$ is the unique minimal ideal of $L$ and $N(L) = A$. Hence each $\mathscr{N}$-critical maximal subalgebra of $L$ complements $A$. Furthermore, $L$ is clearly of nilpotent length three.

Consider first any $\mathscr{N}$-normalizer $E$ of $L$ which is also an $\mathscr{N}$-normalizer of $B$. Such $\mathscr{N}$-normalizer exists since $B$ is a maximal $\mathscr{N}$-critical subalgebra of $L$. By the covering-avoidance property of $\mathscr{N}$-normalizers of $B$, $E = ((z, x + \alpha y))$ where $\alpha \in F$. Now $B$ is of nilpotent length 2, hence $E$ is a Cartan subalgebra of $B$. Now since $z \in E$, it is easily verified that $E$ is a Cartan subalgebra of $L$.

Now in general, each $\mathscr{N}$-normalizer of $L$ is an $\mathscr{N}$-normalizer of some $\mathscr{N}$-critical maximal subalgebra $M$ of $L$ and $M$ must complement $A$. But $L$ is of nilpotent length 3 and $L/A$ is of nilpotent length 2, hence $M$ must be conjugate to $B$ by Theorem 8 of [8].

Consequently, any $\mathscr{N}$-normalizer of $L$ is a Cartan subalgebra of $L$ and $L \in \mathscr{T}(\mathscr{N})$.

Now the ideal $P = A + ((x, y))$ of $L$ is not in $\mathscr{T}(\mathscr{N})$. For $((x)) \subset ((x, y)) \subset P$ is a maximal $\mathscr{N}$-critical chain of $P$, hence $((x))$ is an $\mathscr{N}$-normalizer of $P$. However, the normalizer of $((x))$ in $P$ is $((x, e_0))$. Hence $L \notin \mathscr{T}(\mathscr{N})$.

We recall that each $\mathscr{F}$-normalizer is contained in an $\mathscr{F}$-projector (Theorem 6 of [9]). However, the usual converse result, namely each $\mathscr{F}$-projector contains an $\mathscr{F}$-normalizer has not been obtained, even for $\mathscr{N}\mathscr{N}\mathscr{F}$-Lie algebras. We now show that this result holds if $L \in \mathscr{N}\mathscr{T}(\mathscr{F})$. First we record the following result which is needed.

THEOREM 4. *Let $L \in \mathscr{N}\mathscr{T}(\mathscr{F})$. Then each $\mathscr{F}$-normalizer of $L$ is contained in a unique $\mathscr{F}$-projector of $L$.*

*Proof.* Same as the proof of Theorem 9 of [9].

THEOREM 5. *Let $L \in \mathscr{N}\mathscr{T}(\mathscr{F})$. Then each $\mathscr{F}$-projector of $L$ contains an $\mathscr{F}$-normalizer of $L$.*

*Proof.* Let $N$ be a minimal ideal of $L$ and let $C$ be an $\mathscr{F}$-projector of $L$. Then $C + N/N$ is an $\mathscr{F}$-projector of $L/N$ and $C + N/N$ contains an $\mathscr{F}$-normalizer $D/N$ of $L/N$ by induction. Let $T = C + N$ and let $F$ be an $\mathscr{F}$-normalizer of $L$ such that $F + N = D \subseteqq T$. Then $F$ is contained in an $\mathscr{F}$-projector $G$ of $L$ and $D/N \subseteqq G + N/N$. Hence $G + N = C + N$ by Theorem 4 and $G$ and $C$ are $\mathscr{F}$-projectors of $T$. By Lemma 1.11 of [3], $G$ and $C$ are conjugate in $T$ by an inner automorphism of $T$ induced by an element of $N$. Hence $G$ and $C$ are conjugate in $L$ and the result holds.

$\mathscr{F}$-normalizers have the covering-avoidance property but the converse is not true in general. However, if $L \in \mathscr{T}(\mathscr{F})$, then the converse is true.

THEOREM 6. *Let $L \in \mathscr{T}(\mathscr{F})$. If $D$ is a subalgebra of $L$ which covers the $\mathscr{F}$-central chief factors of $L$ and avoids the $\mathscr{F}$-eccentric chief factors of $L$, then $D$ is an $\mathscr{F}$-normalizer of $L$.*

*Proof.* Let $N$ be a minimal ideal of $L$. Then $D + N/N$ has the covering-avoidance property in $L/N \in \mathscr{T}(\mathscr{F})$. By induction, $D + N/N$ is an $\mathscr{F}$-normalizer of $L/N$ and $D + N = E + N = T$ for some $\mathscr{F}$-normalizer $E$ of $L$. Since $L \in \mathscr{T}(\mathscr{F})$, $E$ is an $\mathscr{F}$-projector of $L$ and then also of $T$. If $N$ is $\mathscr{F}$-central in $L$, then $N \subseteqq D$ and $N \subseteqq E$, hence $D = E$. Suppose $N$ is $\mathscr{F}$-eccentric. Then $D \cap N = 0 = E \cap N$. Now $T \in \mathscr{N}\mathscr{F}$, hence $E$ is an $\mathscr{F}$-normalizer of $T$ by Theorem 3 of [9]. Furthermore, in a given chief series of $T$ passing through $N$, $E$

covers all chief factors above $N$ and avoids all chief factors below $N$ and the same is true for $D$. Since $E$ is an $\mathscr{F}$-normalizer of $T$, each chief factor below $N$ must be $\mathscr{F}$-eccentric and each chief factor above $N$ must be $\mathscr{F}$-central. Hence, by Theorem 4 of [9], $D$ must be an $\mathscr{F}$-normalizer of $T$. By Theorem 3 of [9], $D$ must also be an $\mathscr{F}$-projector of $T$. Now $D$ and $E$ are conjugate in $T$ (hence in $L$) by an inner automorphism induced by an element of $N$. Hence $D$ is an $\mathscr{F}$-normalizer of $L$.

Henceforth we shall be concerned with the case $\mathscr{F} = \mathscr{N}$. Here we have the following stronger form of Theorem 4.

THEOREM 7. *Let $L \in \mathscr{N}\mathscr{T}(\mathscr{N})$ and $D$ be an $\mathscr{N}$-normalizer of $L$. Then there exists a Cartan subalgebra $C$ of $L$ which contains every subalgebra $H$ of $L$ in which $D$ is subinvariant. In particular, $D$ is contained in a unique Cartan subalgebra of $L$. $C$ is the Fitting null component of $D$ acting on $L$.*

*Proof.* $D + N(L)/N(L)$ is subinvariant in $H + N(L)/N(L)$ and $D + N(L)/N(L)$ is an $\mathscr{N}$-normalizer of $L/N(L) \in \mathscr{T}(\mathscr{N})$. Hence $D + N(L)/N(L) = H + N(L)/N(L)$ is a Cartan subalgebra of $L/N(L)$. Let $T = D + N(L) = H + N(L)$ and let $S$ be the Fitting null component of $D$ acting on $T$. Evidently $N_T(S) = S$ and $H \subseteqq S$. Furthermore, $S = S \cap T = S \cap (D + N(L)) = D + (S \cap N(L))$. Each element of $D$ induces a nilpotent derivation on $S$ and $S \cap N(L)$ is a nilpotent ideal of $S$. Then, using Engel's theorem, $S$ is nilpotent. Hence $S$ is a Cartan subalgebra of $T$ and also of $L$ by Lemma 1.8 of [3]. If $K$ is another Cartan subalgebra of $L$ containing $D$, then $D$ is subinvariant in $K$, hence $K = S$. The last past of the theorem follows from the next lemma.

LEMMA 5. *Let $L$ be a solvable Lie algebra and $D$ be a nilpotent subalgebra of $L$. Let $F$ be the Fitting null component of $D$ acting on $L$. Then $D$ is subinvariant in $F$.*

*Proof.* We may suppose that $F = L$. Let $A$ be a minimal ideal of $L$. Now in $D + A$, $A$ is an abelian ideal and each element of $D$ induces a nilpotent derivation of $D + A$. Hence, using Engel's theorem, $D + A$ is nilpotent and $D$ is subinvariant in $D + A$. But $D + A/A$ satisfies the conditions in $L/A$, hence $D + A/A$ is subinvariant in $L/A$ by induction. Therefore, $D$ is subinvariant in $L$.

For Lie algebras of nilpotent length three, a result somewhat stronger than Theorem 7 holds. The proof is the same as the proof of Theorem 7, using Theorem 1 of [8] instead of the defining property of $\mathscr{T}(\mathscr{N})$, and may be omitted.

THEOREM 8. *Let $L$ be of nilpotent length three (or less) and let $D$ be a nilpotent subalgebra of $L$ which can be joined to $L$ by a maximal chain of subalgebras, each self-normalizing in the next. Then there exists a Cartan subalgebra $C$ of $L$ which contains every subalgebra $H$ of $L$ in which $D$ is subinvariant. In particular, $D$ is contained in a unique Cartan subalgebra $C$ of $L$ and $C$ is the Fitting null component of $D$ acting on $L$.*

We may use this to find a Lie algebra analogue to Theorem 10 of [2].

THEOREM 9. *Let $M$ be a self-normalizing maximal subalgebra of $L$. Suppose that $L$ is of nilpotent length three. Then each Cartan subalgebra of $M$ is of the form $M \cap C$ for some Cartan subalgebra $C$ of $L$.*

*Proof.* Let $D$ be a Cartan subalgebra of $M$. Then $D$ is contained in a Cartan subalgebra $C$ of $L$ by Theorem 8 and Lemma 1 of [8]. Now $M \cap C$ is nilpotent and $D$ is a Cartan subalgebra of $M \cap C$. Hence $D = M \cap C$.

The final result is of a slightly different nature. We consider the following: If an $\mathscr{N}$-normalizer $D$ of $L$ is contained in the self-normalizing maximal subalgebra $M$ of $L$, then is $D$ contained in an $\mathscr{N}$-normalizer of $M$. The analogous question for finite groups is answered negatively in [1]. The Lie algebra case also has a negative answer as in shown in the following result. The second part of this example is also an analogue to the example of [1].

THEOREM 10. *There exists a solvable Lie algebra $L \in \mathscr{N}\mathscr{N}\mathscr{N}$ which has an $\mathscr{N}$-normalizer $D$, ideal $A$ and maximal subalgebra $M$ containing $D$ such that*

(1) *$D$ is not contained in an $\mathscr{N}$-normalizer of $M$*

(2) $N_{L/A}(D + A/A) \supset N_L(D) + A/A$.

*Proof.* This example is also a variant of an example found on p. 52 of [7]. Let $F$ be a field of characteristic $p > 2$. Let $A$ be the Lie algebra over $F$ with basis $a_0, a_1, \cdots, a_{p-1}, b, c_0, c_1, \cdots, c_{p-1}$ and products $[a_i, b] = c_i$ for $i = 0, \cdots, p-1$ and all other products of basis elements equal to 0. Define linear transformations $x, y$ on $A$ such that

$$\begin{aligned} x(a_i) &= a_{i+1} & y(a_i) &= ia_i \\ x(b) &= 0 & y(b) &= 0 \\ x(c_i) &= c_{i+1} & y(c_i) &= ic_i \end{aligned}$$

(everything mod $p$). Then $x$ and $y$ are derivations of $A$ and $[y, x] = x$. Let $B$ be the 2-dimensional Lie algebra generated by $x$ and $y$ and let $L$ be the semi-direct sum of $A$ and $B$ with the natural product.

Let $R = ((c_0, \cdots, c_{p-1}))$ and $S = ((c_0, \cdots, c_{p-1}, b))$. The same argument used in [7] shows that $R$ and $A/S$ are $\mathscr{N}$-eccentric chief factors of $L$ and $S/R$ is clearly and $\mathscr{N}$-central chief factor of $L$. Let $M = ((x, y, b, c_0, \cdots, c_{p-1}))$, $M_1 = ((x, y, b))$ and $M_2 = ((y, b))$. Each of these is a maximal $\mathscr{N}$-critical subalgebra of the preceding and $M$ is maximal, $\mathscr{N}$-critical in $L$. Now $\exp a_0$ is an automorphism of $L$ since char $F \neq 2$. Then $C = M_2^{\exp a_0} = ((y, b + c_0)) \subseteqq M$ and $D$ is an $\mathscr{N}$-normalizer of $L$.

Now the $\mathscr{N}$-normalizers of $M$ have dimension 2 by the covering-avoidance property of $\mathscr{N}$-normalizers, hence, if $D$ is contained in an $\mathscr{N}$-normalizer of $M$, then it is one of them. If this is the case, then, since $b \in Z(M)$, $b \in D$ and $\dim D > 2$, a contradiction.

For the second part, note that

$$N_{L/R}(M_2 + R/R) = ((y + R, b + R, a_0 + R)) .$$

However, an element of the form $\alpha a_0 + t, \alpha \in F, t \in R$ is not in $N_L(M_2)$ unless $\alpha = 0$, since $[b, \alpha a_0 + t] = -\alpha c_0$. Hence

$$N_L(M_2) + R/R \subset N_{L/R}(M_2 + R/R) .$$

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