

# Pacific Journal of Mathematics

**$\sigma$  – 2-TRANSITIVE ORDERED PERMUTATION GROUPS**

STEPHEN H. MCCLEARY

## O-2-TRANSITIVE ORDERED PERMUTATION GROUPS

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The group of all automorphisms of a chain  $\Omega$  forms a lattice-ordered group  $A(\Omega)$  under the pointwise order. Let  $G$  be an  $l$ -subgroup of  $A(\Omega)$  which is  $o$ -2-transitive, i.e., for any  $\beta < \gamma$  and  $\sigma < \tau$ , there exists  $g \in G$  such that  $\beta g = \sigma$  and  $\gamma g = \tau$ . It is shown that  $G$  is a complete subgroup of  $A(\Omega)$  if and only if  $G$  is completely distributive if and only if  $G$  contains an element  $\neq 1$  of bounded support. There is a discussion of the pathological groups in which these conditions are absent.

1. The dichotomy among  $o$ -2-transitive groups. The group  $A(\Omega)$  or order-preserving permutations (automorphisms) of a chain  $\Omega$  becomes a lattice-ordered group ( $l$ -group) when ordered pointwise, i.e.,  $f \leq g$  if and only if and only if  $\beta f \leq \beta g$  for all  $\beta \in \Omega$ . We assume throughout this paper that  $(G, \Omega)$  is an  $l$ -permutation group, i.e., that  $G$  is an  $l$ -subgroup of  $A(\Omega)$  (simultaneously a subgroup and a sublattice).

Let  $\bar{\Omega}$  be the completion by Dedekind cuts (without end points) of  $\Omega$ . Each  $g \in G$  can be extended uniquely to an order-preserving permutation ( $o$ -permutation) of  $\bar{\Omega}$ , which will also be denoted by  $g$ . For  $\bar{\omega} \in \bar{\Omega}$ , let  $G_{\bar{\omega}}$  be the stabilizer subgroup  $\{g \in G \mid \bar{\omega}g = \bar{\omega}\}$ .  $G_{\bar{\omega}}$  is a prime subgroup of  $G$  (i.e., a convex  $l$ -subgroup of  $G$  such that  $g_1 \wedge g_2 = 1$ , with  $g_1, g_2 \in G$ , implies  $g_1 \in G_{\bar{\omega}}$  or  $g_2 \in G_{\bar{\omega}}$ ). If  $G$  is transitive on  $\Omega$ , then of course all  $G_{\alpha}$ 's ( $\alpha \in \Omega$ ) are conjugate in  $G$ .

The author showed in [5, Theorem 7] that for a transitive  $l$ -subgroup  $G$  of  $A(\Omega)$ , the following are equivalent:

(1)  $G_{\alpha}$  is a closed subgroup of  $G$  for one (hence every)  $\alpha \in \Omega$ , i.e., if  $g = \bigvee_{i \in I} g_i$  with each  $g_i \in G_{\alpha}$ , then  $g \in G_{\alpha}$ .

(2)  $G$  is a complete subgroup of  $A(\Omega)$ , i.e., if in  $G$ ,  $g = \bigvee_{i \in I} g_i$ , then also in  $A(\Omega)$ ,  $g = \bigvee_{i \in I} g_i$ .

(3) Sups in  $G$  are pointwise, i.e., if  $g = \bigvee_{i \in I} g_i$  with each  $g_i \in G$ , then for each  $\beta \in \Omega$ ,  $\beta g$  is the sup in  $\Omega$  of  $\{\beta g_i \mid i \in I\}$ .

Moreover, it was shown in [5, Corollary 15] that in the presence of these conditions, we have

(4)  $G$  is a completely distributive  $l$ -group, i.e.,  $\bigwedge_{i \in I} \bigvee_{k \in K} g_{ik} = \bigvee_{f \in KI} \bigwedge_{i \in I} g_{if(i)}$  for any collection  $\{g_{ik} \mid i \in I, k \in K\}$  of  $G$  for which the indicated sups and infs exist.

The distributive radical  $D(G)$  is the intersection of the closed prime subgroups of  $G$  [1, Theorem 3.4].  $D(G) = \{1\}$  iff  $G$  is completely distributive [1, Corollary 3.8]; and at the opposite extreme,  $D(G) = G$  iff  $G$  has no closed prime subgroups  $\neq G$ .

The *support*  $\text{Supp}(k)$  of  $k \in G$  means  $\{\omega \in \Omega \mid \omega k \neq \omega\}$ ; it is *bounded* if there exist  $\beta, \gamma \in \Omega$  such that  $\beta < \text{Supp}(k) < \gamma$  (i.e.,  $\beta < \sigma < \gamma$  for all  $\sigma \in \text{Supp}(k)$ ). It is well known (see, for example, the proof of [2, Theorem 6]) that the elements of bounded support in an  $o$ -2-transitive  $l$ -permutation group  $G$  form an  $l$ -ideal  $L$  of  $G$  which is contained in all  $l$ -ideals  $\neq \{1\}$ . If  $G$  contains no element  $\neq 1$  of bounded support, so that  $L = \{1\}$ , then with an eye on the next theorem, we shall say that  $G$  is a *pathologically*  $o$ -2-transitive group.

**MAIN THEOREM 1.** *Suppose that  $(G, \Omega)$  is an  $o$ -2-transitive  $l$ -permutation group. Then conditions (1), (2), (3), and (4) are all equivalent, and they fail if and only if  $G$  is pathological. Moreover, in the pathological case,  $G$  has no proper closed prime subgroups, so that the distributive radical  $D(G) = G$ .*

*Proof.* First, assume that  $G$  has an element  $\neq 1$  of bounded support. Then since  $G$  is  $o$ -2-transitive, given any nondegenerate interval  $\Delta$  of  $\Omega$ ,  $G$  has an element  $\neq 1$  with support a subset of  $\Delta$ . Now suppose  $g = \bigvee_{i \in I} g_i$ , with  $1 < g \in G \setminus G_\alpha$  and each  $g_i \in G_\alpha$ . Pick  $1 > h \in G$  such that  $\text{Supp}(h) \subseteq (\alpha g^{-1}, \alpha)$ , where the usual notation is used for intervals of  $\Omega$ . Then for each  $i \in I$ ,  $g_i \leq hg < g$  (since when  $\eta \in \text{Supp}(h)$ ,  $\eta g_i < \alpha < \alpha h g$ ), a contradiction. Therefore,  $G_\alpha$  is closed, and the other conditions follow.

Now assume that  $G$  lacks elements  $\neq 1$  of bounded support. We can express an arbitrary  $1 < g \in G$  as  $\bigvee_{i \in I} g_i$  with each  $g_i \in G_\alpha$ , as follows: For each  $\beta \notin [\alpha g^{-1}, \alpha]$ , we have either  $\alpha < \beta \leq \beta g$ , or else  $\beta \leq \beta g < \alpha$ , so we may use  $o$ -2-transitivity to pick  $g_\beta \in G_\alpha$  such that  $\beta g_\beta = \beta g$ . Now  $g = \bigvee (g_\beta \wedge g)$ , for if  $g_\beta \wedge g \leq h < g$  for each  $\beta$ , then  $\text{Supp}(h^{-1}g) \subseteq [\alpha g^{-1}, \alpha]$ , violating the hypothesis, since for  $\beta \notin [\alpha g^{-1}, \alpha]$ , we have  $\beta g = \beta(g_\beta \wedge g) \leq \beta h \leq \beta g$ . Since each  $g_\beta \wedge g \in G_\alpha$ ,  $G_\alpha$  is not closed in  $G$ .

It remains only to show that in the pathological case,  $G$  contains no proper closed prime subgroup, for then  $D(G) = G$  and  $G$  is not completely distributive. Suppose  $P$  is such a subgroup. In [6, Corollary 4], it is shown that every closed convex  $l$ -subgroup of an  $l$ -permutation group  $(G, \Omega)$  must be  $\bigcap \{G_{\bar{\omega}} \mid \bar{\omega} \in \bar{\Delta}\}$  for some  $\bar{\Delta} \subseteq \bar{\Omega}$ . (In [6], it is assumed that  $G$  is a complete subgroup of  $A(\Omega)$ , so that the  $G_{\bar{\omega}}$ 's will be closed, but no other use is made of completeness.) But in fact the  $G_{\bar{\omega}}$ 's are closed, for  $P$  is closed, and in any  $l$ -group, a prime subgroup containing a closed prime is itself closed [1, Lemma 3.3]. But it was shown above that no  $G_\alpha$ ,  $\alpha \in \Omega$ , is closed; and in view of the following lemma, the proof also applies to  $G_{\bar{\omega}}$ ,  $\bar{\omega} \in \Omega$ .

**LEMMA 2.** *Let  $(G, \Omega)$  be an  $o$ -2-transitive  $l$ -permutation group.*

Let  $\bar{\omega} \in \bar{\Omega}$ , and let  $\beta, \gamma \in \Omega$  with either  $\bar{\omega} < \beta < \gamma$ , or  $\beta < \gamma < \bar{\omega}$ . Then there exists  $g \in G_{\bar{\omega}}$  such that  $\beta g = \gamma$ .

*Proof.* Suppose that  $\bar{\omega} < \beta < \gamma$ , and pick  $\alpha \in \Omega$  such that  $\bar{\omega} < \alpha < \beta < \gamma$ . Use  $o$ -2-transitivity to pick  $k \in G$  such that  $\alpha k < \bar{\omega}$  and  $\beta k = \gamma$ , and take  $g$  to be  $k \vee 1$ . The other case is similar. This concludes the proofs of the lemma and the theorem.

Incidentally, conditions (1), (2), (3), and (4) still make sense when  $G$  is any subgroup (not necessarily an  $l$ -subgroup) of  $A(\Omega)$ ; and if  $G$  is  $o$ -2-transitive and contains an element  $\neq 1$  of bounded support, these conditions hold. (The first paragraph of the proof of Theorem 1 can easily be adapted to show that sups are pointwise. From this (2) and (1) follow as in [5], and (2) implies (4).)

If  $\omega g \neq \omega$ ,  $\{\gamma \in \Omega \mid \omega g^{-n} < \gamma < \omega g^n \text{ for some integer } n\}$  is called an *interval of support* of  $g$ ; and in [5],  $G$  is said to be *depressible* if for every  $g \in G$  and every interval of support  $\Delta$  of  $g$ , there exists  $k \in G$  such that  $\omega k = \omega g$  if  $\omega \in \Delta$ , but  $\omega k = \omega$  if  $\omega \notin \Delta$ . Convex  $l$ -subgroups of  $A(\Omega)$  are automatically depressible.

**PROPOSITION 3.** *Depressible  $o$ -2-transitive  $l$ -permutation groups are never pathological.*

*Proof.* The following lemma establishes the existence of an element having a bounded interval of support, and depressibility does the rest.

**LEMMA 4.** *Let  $(G, \Omega)$  be an  $o$ -2-transitive  $l$ -permutation group. Then, for every positive integer  $n$ ,  $(G, \Omega)$  is  $o$ - $n$ -transitive, i.e., if  $\beta_1 < \dots < \beta_n$  and  $\gamma_1 < \dots < \gamma_n$ , there exists  $g \in G$  such that  $\beta_i g = \gamma_i$ ,  $i = 1, \dots, n$ .*

*Proof.* Given  $\beta_1 < \dots < \beta_n$  and  $\gamma_1 < \dots < \gamma_n$ , we may suppose by induction that there exists  $h \in G$  such that  $\beta_i h = \gamma_i$ ,  $i = 1, \dots, n-1$ . If  $\beta_n h \geq \gamma_n$ , we use  $o$ -2-transitivity to pick  $k \in G$  such that  $\beta_1 k = \gamma_{n-1}$  and  $\beta_n h = \gamma_n$ . Now  $\beta_i(h \wedge k) = \gamma_i$ ,  $i = 1, \dots, n$ . If  $\beta_n h < \gamma_n$ , a similar argument works.

**2. Pathologically  $o$ -2-transitive groups.** The following example of a pathological group was given by Holland in [3, p. 433]. Let  $\Omega$  be the reals and let  $G$  be the  $l$ -subgroup of  $A(\Omega)$  consisting of those  $o$ -permutations  $g$  of  $\Omega$  for which there exists a positive integer  $n = n_g$  such that  $(\omega + n)g = \omega g + n$  for all  $\omega \in \Omega$ . Lloyd [4, p. 399] used very special properties of this example to show that  $G$  is not completely distributive (cf. Theorem 1), but is  $l$ -simple (has no proper  $l$ -ideals). Are all pathological groups  $l$ -simple? The author has been

unable to settle this question, but attempts to construct additional examples of pathological groups seem to lead inevitably to some sort of periodicity sufficient to guarantee  $l$ -simplicity, as in the following modification of Holland's example.<sup>1</sup>

As in that example, let  $\Omega$  be the reals. Now let  $H$  be the  $l$ -subgroup of  $A(\Omega)$  consisting of those  $o$ -permutations  $h$  of  $\Omega$  having the property that for all  $\omega \in \Omega$ , there exists a positive integer  $n = n_{h,\omega}$  such that  $(\omega + qn)h = \omega h + qn$  for all integers  $q$ . (For definiteness, let  $n_{h,\omega}$  be the least positive integer having this property.)  $H$  contains the previous group  $G$  and is also pathologically  $o$ -2-transitive.  $H$  and  $G$  are not isomorphic as  $l$ -groups, for there exists  $1 < z \in G$  (defined by  $\omega z = \omega + 1$ ) such that every  $g \in G$  commutes with some  $z^p$ ,  $p > 0$ , whereas it can be shown that  $H$  contains no such element  $z$ .

Of course, still other examples can be obtained by letting  $\Omega$  be the rationals (or other appropriate subgroup of the reals) and proceeding in either of the two ways already mentioned.

Lloyd's proof of the  $l$ -simplicity of the first example does not apply to the modification. However, all of the above examples can be shown to be  $l$ -simple by the following fairly similar argument, which will be phrased in terms of the modified group  $H$ . Let  $\{1\} \neq L$  be an  $l$ -ideal of  $H$ . Since for every  $1 < h \in H$ ,  $\{\omega h - \omega \mid \omega \in \Omega\}$  has an upper bound, namely  $0h - 0 + n_{h,0}$ , we shall have  $L = H$  provided there exists  $\varepsilon > 0$  such that the translation  $\omega \rightarrow \omega + \varepsilon$  is exceeded by some  $g \in L$ . Now pick  $1 < k \in L$  and  $\alpha \in \Omega$  such that  $\alpha k > \alpha$ . Let  $t$  be the translation  $\omega \rightarrow \omega + (\alpha k - \alpha)$ , and let  $p$  be an integer such that  $p(\alpha k - \alpha) > n_{k,\alpha}$ . Let  $k_1 = k$ , let  $k_i = t^{-1}k_{i-1}t$  ( $i = 2, \dots, p$ ), and let  $g = k_1 \dots k_p \in L$ . The reader can verify that  $(\alpha + qn_{k,\alpha})g = \alpha g + qn_{k,\alpha}$  for all integers  $q$ , so that  $n_{g,\alpha} \leq n_{k,\alpha}$ . Now  $\alpha + n_{g,\alpha} \leq \alpha + n_{k,\alpha} < \alpha g$ . Hence  $g$  exceeds the translation  $\omega \rightarrow \omega + \varepsilon$ , where  $\varepsilon = \alpha g - (\alpha + n_{g,\alpha})$ .

As mentioned above, it may be the case that all pathologically  $o$ -2-transitive groups are  $l$ -simple. At any rate, any proper  $l$ -ideal must itself be a pathologically  $o$ -2-transitive group, as we proceed to prove.

An  $o$ -block of a transitive  $l$ -permutation group  $(G, \Omega)$  is a nonempty convex subset  $\Delta$  of  $\Omega$  having the property that for any  $g \in G$ , either  $\Delta g = \Delta$  or  $\Delta g$  does not meet  $\Delta$ . The  $o$ -block system  $\tilde{\Delta}$  determined by  $\Delta$  is the set of translates  $\Delta g$  ( $g \in G$ ), a partition of  $\Omega$ . If  $\Delta$  contains more than one point and  $\Delta \neq \Omega$ ,  $\tilde{\Delta}$  is proper.  $G$  is  $o$ -primitive if it has no proper  $o$ -block systems.

LEMMA 5. *Let  $(G, \Omega)$  be a transitive  $l$ -permutation group, and let  $L$  be a proper  $l$ -ideal of  $G$  which is intransitive on  $\Omega$ . Then the*

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<sup>1</sup> Andrew Glass has recently shown that pathological groups need not be  $l$ -simple, and need not be periodic in any sense.

orbits of  $L$  form a proper  $o$ -block system of  $G$ .

*Proof.* The analogous statement for nonordered permutation groups is precisely Proposition 7.1 of [7]. Here the fact that  $L$  is an  $l$ -ideal forces the blocks to be convex.

**THEOREM 6.** *Let  $(G, \Omega)$  be a (pathologically)  $o$ -2-transitive  $l$ -permutation group, and let  $\{1\} \neq L$  be an  $l$ -ideal of  $G$ . Then  $(L, \Omega)$  is also (pathologically)  $o$ -2-transitive.*

*Proof.*  $o$ -2-transitive groups are certainly  $o$ -primitive, so by the lemma,  $L$  must be transitive on  $\Omega$ . Let  $\alpha < \beta < \gamma$ , all in  $\Omega$ . Pick  $1 \leq g \in G_\alpha$  such that  $\beta g = \gamma$ , and pick  $1 \leq k \in L$  such that  $\beta k \geq \gamma$ . Then  $1 \leq k \wedge g \leq k \in L$ , so  $k \wedge g \in L_\alpha$ , and  $\beta(k \wedge g) = \gamma$ . Hence  $\{\beta \in \Omega \mid \beta > \alpha\}$  is all one orbit of  $G_\alpha$ , from which it follows easily that  $L$  is  $o$ -2-transitive. Certainly if  $G$  contains no element  $\neq 1$  of bounded support, neither does  $L$ .

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