ADDENDUM TO: “SEQUENCES OF HOMEOMORPHISMS WHICH CONVERGE TO HOMEOMORPHISMS”

JEROME L. PAUL
ERRATA

Addendum to

SEQUENCES OF HOMEOMORPHISM WHICH CONVERGE TO HOMEOMORPHISMS

JEROME L. PAUL

Volume 24 (1968), 143–152

We observe that Theorem 1 remains valid when we omit the relative compactness requirement on the $U_i$, and add the hypothesis that $M$ be complete. The proof of the alternative theorem is the same as that of Theorem 1, except that some conclusions which were previously based on relative compactness arguments now follow from completeness.

The alternative Theorem 1 can be used to extend certain results of the paper to the Hilbert space $l_2$. In what follows, $M, N$ shall be locally convex topological (real) vector spaces, and $U$ a locally convex subset of $M$ having no isolated points. Given a continuous map $f: U \to N$, a point $x \in U$ is called a spiral point of $f$ if given any hyperplane (= translate of a codimension one linear subspace) $\Pi$ containing $f(x)$, and any point $y \neq x$ in $U$, then $f([x, y]) \cap \Pi$ is an infinite set, where $[x, y]$ denotes the (closed) line segment joining $x$ and $y$.

PROPOSITION. Given a continuous map $f: U \to N$, with $U, N$ as above, the set of nonspiral points of $f$ is dense in $U$.

Proof. Let $x$ and $y$ be any two points such that $[x, y] \subset U$. To prove the proposition, it suffices to show that there is a point $z$ in the open segment $(x, y)$ such that $z$ is not a spiral point of $f$. Note that if $f$ is constant on $[x, y]$, then $[x, y]$ consists entirely of nonspiral points of $f$. Hence, we can assume that there is a point $w \in (x, y)$ such that $f(x) \neq f(w)$. Let $\Pi$ be a hyperplane in $N$ which separates $f(x)$ and $f(w)$. Then there is a (unique) point $z \in (x, w)$ such that $f([x, z]) \cap \Pi = f(z)$, so that $z$ is not a spiral point of $f$, and the proposition is proved.

In spite of the existence of a dense set of nonspiral points for any continuous map $f: U \to N$, when $M = N = \text{the Hilbert space } l_2$, we have the following

THEOREM. Let $U$ be a locally convex subset, having no isolated points, of the Hilbert space $l_2$. Then those topological imbeddings of $U$ into $l_2$ which have a dense set (in $U$) of spiral points form
a dense subset, in the fine $C^0$ topology, of the set of topological im-
beddings of $U$ into $l_2$.

The proof of this theorem, which requires the alternative form
of Theorem 1, is similar to the proof of Theorem 2 and is therefore
omitted. The principal modification needed consists in allowing the
maps $F_{c,r,i,j,m}$, (which are now defined on $l_2$ in the obvious way using
(9)-(9)'), to act now on the left of the imbeddings via a suitably
defined infinite left composition, and where the positive integer $j$ is
not subject to the condition $j \leq n$ of Theorem 2.

Correction to

DIMENSION THEORY IN POWER SERIES RINGS

DAVID E. FIELDS

Volume 35 (1970), 601–611

While recently answering a letter of inquiry of T. Wilhelm, I
discovered an error in Corollary 4.6. The result, as originally stated,
clearly requires that $P \cdot V[[X]] \subset P[[X]]$. However, if $P$ is not branched,
it is possible that $P \cdot V[[X]] = P[[X]]$; a counterexample can be ob-
tained from Proposition A.

The following modification of Corollary 4.6 is sufficient for the
proof of Theorem 4.7.

**Corollary 4.6'.** Let $V$ be a valuation ring having a proper prime
ideal $P$ which is branched. If $P$ is idempotent, then there is a
prime ideal $Q$ of $V[[X]]$ which satisfies $P \cdot V[[X]] \subseteq Q \subset P[[X]]$.

**Proof.** Since $P$ is branched, there is a prime ideal $\bar{P}$ of $V$ with
$\bar{P} \subset P$ and such that there are no prime ideals of $V$ properly between
$\bar{P}$ and $P$ [1; 173]. By passing to $V[[X]]/\bar{P}[[X]] \cong (V/\bar{P})[[X]]$, we
may assume that $P$ is a minimal prime ideal of $V$.

Since $P$ is idempotent, $PV_r$ is idempotent by Lemma 4.1; hence
$V_r$ is a rank one nondiscrete valuation ring. By Theorem 3.4, there is
a prime ideal $Q$ of $V_r[[X]]$ such that $(PV_r) \cdot V_r[[X]] \subseteq Q \subset (PV_r)[[X]]$. But then we see that $Q \subset (PV_r)[[X]] = P[[X]] \subseteq V[[X]]$. Hence $Q \cap V[[X]] = Q$ and $Q$ is a prime ideal of $V[[X]]$ with $P \cdot V[[X]] \subseteq Q \subset P[[X]]$.

The following result is now of interest.
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