

Pacific Journal of Mathematics

**CORRECTION TO: "COMMUTANTS OF SOME HAUSDORFF
MATRICES"**

BILLY E. RHOADES

PROPOSITION A. *Let V be a valuation ring having a proper prime ideal P which is not branched; then $P = \bigcup_{\lambda \in \Lambda} M_\lambda$, where $\{M_\lambda\}_{\lambda \in \Lambda}$ is the collection of prime ideals of V which are properly contained in P . In this case, $P \cdot V[[X]] = P[[X]]$ if and only if (*) given any countable subcollection $\{M_{\lambda_i}\}$ of $\{M_\lambda\}$, $\bigcup_{i=1}^{\infty} M_{\lambda_i} \subset P$.*

Proof. Assuming (*), let $f(X) = \sum_{i=0}^{\infty} f_i X^i \in P[[X]]$. For each i , $f_i \in M_{\bar{\lambda}_i}$ for some $\bar{\lambda}_i \in \Lambda$. Let $p \in P$, $p \notin \bigcup_{i=0}^{\infty} M_{\bar{\lambda}_i}$; since $p \notin M_{\bar{\lambda}_i}$, it follows that $f_i \in M_{\bar{\lambda}_i} \not\subseteq (p)V$ for each i and $f(X) \in (p)V[[X]] \not\subseteq P \cdot V[[X]]$.

Conversely, assuming that (*) fails, let $\{M_{\lambda_i}\}_{i=1}^{\infty}$ be a countable subcollection of $\{M_\lambda\}_{\lambda \in \Lambda}$ such that $\bigcup_{i=1}^{\infty} M_{\lambda_i} = P$. By extracting a subsequence of $\{M_{\lambda_i}\}$, we may assume that $M_{\lambda_i} \subset M_{\lambda_{i+1}}$ for each i . Let $f_i \in M_{\lambda_{i+1}}$, $f_i \notin M_{\lambda_i}$ and let $f(X) = \sum_{i=1}^{\infty} f_i X^i$; then $f(X) \in P[[X]]$ but $f(X) \notin P \cdot V[[X]]$.

MARSHALL UNIVERSITY

Correction to

COHOMOLOGY OF FINITELY PRESENTED GROUPS

P. M. CURRAN

Volume 42 (1972), 615-620

In the second paragraph of the abstract, p. 615, the first sentence, "If G is generated by n elements, \dots " should read "If G is a residually finite group generated by n elements, \dots ".

Correction to

COMMUTANTS OF SOME HAUSDORFF MATRICES

B. E. RHOADES

Volume 42 (1973), 715-719

In [2] it is shown that, for A a conservative triangle, B a matrix with finite norm commuting with A , B is triangular if and only if

(1) for each $t \in l$ and each n , $t(A - a_{nn}I) = 0$ implies t belongs to the linear span of (e_0, e_1, \dots, e_n) . On page 716 of [2] it is asserted that

(2) $(U^*)^{n+1}(A - a_{nn}I)U^{n+1}$ of type M for each n is equivalent to

(1). This assertion is false. Condition (2) is sufficient for (1) but, as the following example shows, it is not necessary.

EXAMPLE 1. Let A be a triangle with entries $a_{nk} = p_{n-k}$, $n, k = 0, 1, \dots$, where $p_k = 2^{-k}$. Then, for each n , $A - a_{nn}I = (U^*)^{n+1}(A - a_{nn}I)U^{n+1} = B$, where $b_{nn} = 0$ for each n , and $b_{nk} = a_{nk}$ otherwise. The only solutions of $tB = 0$ for $t \in l$ lie in the linear span of e_0 , so that A satisfies (1). However, B is not of type M .

Since (2) is not equivalent to (1), some of the material on pages 717 and 718 of [2] must now be reworked.

We establish the following facts:

1. If a factorable triangular matrix A contains at least two zeros on the main diagonal, then $\text{Com}(A)$ in $\Delta \neq \text{Com } A$ in Γ .
2. If A is not factorable, then the number of zeros on the main diagonal gives no information about the size of $\text{Com}(A)$ in Γ .
3. Having distinct diagonal entries is necessary but not sufficient for a conservative Hausdorff matrix H to satisfy $\text{Com}(H)$ in $\Delta = \text{Com}(H)$ in Γ .

Proof of 1. Let n and k denote the smallest integers for which $a_{kk} = a_{nn} = 0$, $n > k$. Then the system $t(A - a_{kk}I) = 0$ clearly has a solution in the space spanned by (e_0, e_1, \dots, e_n) . It remains to show that there is a solution not in the subspace spanned by (e_0, e_1, \dots, e_k) . Since A is factorable, either the k th row or the k th column of A is zero. In either case we can obtain a solution of the system using $t_n = 1$, $t_k = 0$ for $k > n$, which can be used to construct a non-triangular conservative matrix B which commutes with A .

Proof of 2. Define $D = (d_{nk})$ by $d_{n0} = 1$, $d_{nk} = 0$ otherwise. Then $D \leftrightarrow B$ implies that $(DB)_{nk} = b_{0k}$, whereas $(BD)_{n0} = \sum_j b_{nj}$, and $(BD)_{nk} = 0$ for $k > 0$. Thus, if B is any matrix satisfying (i) $b_{0k} = 0$ for all $k > 0$ and (ii) $\sum_{j=0}^{\infty} b_{nj} = b_{n0}$ for all $n > 0$, then $B \leftrightarrow D$. For example, if $b_{00} = 1$, $b_{nk} = 2^{n-k-1}$, $k \geq n > 0$, $b_{nk} = 0$ otherwise, then B is row infinite and commutes with D . D is factorable, but $A = D + I$ is not. Moreover, since $\text{Com}(I)$ in $\Gamma = \Gamma$, $\text{Com}(A)$ in $\Gamma = \text{Com}(D)$ in $\Gamma \neq \text{Com}(D)$ in Δ .

Example 1 with $p_0 = 0$ is a nonfactorable matrix with an infinite number of zeros on the main diagonal, and yet $\text{Com}(A)$ in $\Gamma = \text{Com}(A)$ in Δ .

The following examples establish 3.

EXAMPLE 2. Let H be the Hausdorff matrix generated by $\mu_n = -2n(n-1)/(n+1)(n+2)$, $B = (b_{nk})$ with $b_{0k} = b_{1k} = 1$ for all k , $b_{nk} = 0$ otherwise. Then $B \leftrightarrow H$, but $B \notin \Delta$ since $b_{01} \neq 0$.

EXAMPLE 3. Let H be generated by $\mu_n = n(n - 1/2)/(n + 1)(n + 2)$. We can regard H as the product of two Hausdorff matrices H_α and H_β , with generating sequences $\alpha_n = (n - 1/2)/(n + 1)$ and $\beta_n = n/(n + 2)$, respectively. From Theorem 1 of [1], the sequence $t = \{t_n\}$, with $t_0 = 1$, $t_n = (-1)^n(1/2)(-3/2) \cdots (-n + 3/2)/n!$, $n > 0$ satisfies $tH_\alpha = 0$. Therefore $tH = 0$. Let B be the matrix with the sequence t as each row. Then

$$(HB)_{nk} = \sum_{j=0}^n h_{nj} b_{jk} = t_k \sum_{j=0}^n h_{nj} = t_k \mu_0 = 0,$$

and

$$(BH)_{nk} = \sum_{j=k}^{\infty} b_{nj} h_{jk} = \sum_{j=k}^{\infty} t_j h_{jk} = 0, \text{ so that } B \longleftrightarrow H.$$

REFERENCES

1. B. E. Rhoades, *Some Hausdorff matrices not of type M*, Proc. Amer. Math. Soc., **15** (1964), 361-365.
2. ———, *Commutants of some Hausdorff matrices*, Pacific J. Math., **42** (1972), 715-719.

INDIANA UNIVERSITY

Corrections to

VERSUM SEQUENCES IN THE BINARY SYSTEM

CHARLES W. TRIGG

Volume 47 (1973), 263-275

Line 12 should read "the universal verity of the conjecture [5, 6]".
Instead of the universal verity of the conjecture [1, 2].

The first page should be 263 instead of 163.

PACIFIC JOURNAL OF MATHEMATICS

EDITORS

RICHARD ARENS (Managing Editor)

University of California
Los Angeles, California 90024

J. DUGUNDJI*

Department of Mathematics
University of Southern California
Los Angeles, California 90007

R. A. BEAUMONT

University of Washington
Seattle, Washington 98105

D. GILBARG AND J. MILGRAM

Stanford University
Stanford, California 94305

ASSOCIATE EDITORS

E. F. BECKENBACH

B. H. NEUMANN

F. WOLF

K. YOSHIDA

SUPPORTING INSTITUTIONS

UNIVERSITY OF BRITISH COLUMBIA
CALIFORNIA INSTITUTE OF TECHNOLOGY
UNIVERSITY OF CALIFORNIA
MONTANA STATE UNIVERSITY
UNIVERSITY OF NEVADA
NEW MEXICO STATE UNIVERSITY
OREGON STATE UNIVERSITY
UNIVERSITY OF OREGON
OSAKA UNIVERSITY

UNIVERSITY OF SOUTHERN CALIFORNIA
STANFORD UNIVERSITY
UNIVERSITY OF TOKYO
UNIVERSITY OF UTAH
WASHINGTON STATE UNIVERSITY
UNIVERSITY OF WASHINGTON
* * *
AMERICAN MATHEMATICAL SOCIETY
NAVAL WEAPONS CENTER

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies.

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be in typed form or offset-reproduced, (not dittoed), double spaced with large margins. Underline Greek letters in red, German in green, and script in blue. The first paragraph or two must be capable of being used separately as a synopsis of the entire paper. Items of the bibliography should not be cited there unless absolutely necessary, in which case they must be identified by author and Journal, rather than by item number. Manuscripts, in duplicate if possible, may be sent to any one of the four editors. Please classify according to the scheme of Math. Rev. Index to Vol. 39. All other communications to the editors should be addressed to the managing editor, or Elaine Barth, University of California, Los Angeles, California, 90024.

50 reprints are provided free for each article; additional copies may be obtained at cost in multiples of 50.

The *Pacific Journal of Mathematics* is issued monthly as of January 1966. Regular subscription rate: \$48.00 a year (6 Vols., 12 issues). Special rate: \$24.00 a year to individual members of supporting institutions.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley, California, 94708.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), 270, 3-chome Totsuka-cho, Shinjuku-ku, Tokyo 160, Japan.

* C. R. DePrima California Institute of Technology, Pasadena, CA 91109, will replace J. Dugundji until August 1974.

Copyright © 1973 by
Pacific Journal of Mathematics
All Rights Reserved

Pacific Journal of Mathematics

Vol. 49, No. 2

June, 1973

Wm. R. Allaway, <i>On finding the distribution function for an orthogonal polynomial set</i>	305
Eric Amar, <i>Sur un théorème de Mooney relatif aux fonctions analytiques bornées</i>	311
Robert Morgan Brooks, <i>Analytic structure in the spectrum of a natural system</i>	315
Bahattin Cengiz, <i>On extremely regular function spaces</i>	335
Kwang-nan Chow and Moses Glasner, <i>Atoms on the Royden boundary</i>	339
Paul Frazier Duvall, Jr. and Jim Maxwell, <i>Tame Z^2-actions on E^n</i>	349
Allen Roy Freedman, <i>On the additivity theorem for n-dimensional asymptotic density</i>	357
John Griffin and Kelly Denis McKennon, <i>Multipliers and the group L_p-algebras</i>	365
Charles Lemuel Hagopian, <i>Characterizations of λ connected plane continua</i>	371
Jon Craig Helton, <i>Bounds for products of interval functions</i>	377
Ikuko Kayashima, <i>On relations between Nörlund and Riesz means</i>	391
Everett Lee Lady, <i>Slender rings and modules</i>	397
Shozo Matsuura, <i>On the Lu Qi-Keng conjecture and the Bergman representative domains</i>	407
Stephen H. McCleary, <i>The lattice-ordered group of automorphisms of an α-set</i>	417
Stephen H. McCleary, <i>o-2-transitive ordered permutation groups</i>	425
Stephen H. McCleary, <i>o-primitive ordered permutation groups. II</i>	431
Richard Rochberg, <i>Almost isometries of Banach spaces and moduli of planar domains</i>	445
R. F. Rossa, <i>Radical properties involving one-sided ideals</i>	467
Robert A. Rubin, <i>On exact localization</i>	473
S. Sribala, <i>On Σ-inverse semigroups</i>	483
H. M. (Hari Mohan) Srivastava, <i>On the Konhauser sets of biorthogonal polynomials suggested by the Laguerre polynomials</i>	489
Stuart A. Steinberg, <i>Rings of quotients of rings without nilpotent elements</i>	493
Daniel Mullane Sunday, <i>The self-equivalences of an H-space</i>	507
W. J. Thron and Richard Hawks Warren, <i>On the lattice of proximities of Čech compatible with a given closure space</i>	519
Frank Uhlig, <i>The number of vectors jointly annihilated by two real quadratic forms determines the inertia of matrices in the associated pencil</i>	537
Frank Uhlig, <i>On the maximal number of linearly independent real vectors annihilated simultaneously by two real quadratic forms</i>	543
Frank Uhlig, <i>Definite and semidefinite matrices in a real symmetric matrix pencil</i> ...	561
Arnold Lewis Villone, <i>Self-adjoint extensions of symmetric differential operators</i>	569
Cary Webb, <i>Tensor and direct products</i>	579
James Victor Whittaker, <i>On normal subgroups of differentiable homeomorphisms</i>	595
Jerome L. Paul, <i>Addendum to: "Sequences of homeomorphisms which converge to homeomorphisms"</i>	615
David E. Fields, <i>Correction to: "Dimension theory in power series rings"</i>	616
Peter Michael Curran, <i>Correction to: "Cohomology of finitely presented groups"</i>	617
Billy E. Rhoades, <i>Correction to: "Commutants of some Hausdorff matrices"</i>	617
Charles W. Trigg, <i>Corrections to: "Versum sequences in the binary system"</i>	619