CORRECTION TO: “COMMUTANTS OF SOME HAUSDORFF MATRICES”

Billy E. Rhoades
PROPOSITION A. Let $V$ be a valuation ring having a proper prime ideal $P$ which is not branched; then $P = \bigcup_{\lambda \in A} M_{\lambda}$, where $\{M_{\lambda}\}_{\lambda \in A}$ is the collection of prime ideals of $V$ which are properly contained in $P$. In this case, $P \cdot V[[X]] = P[[X]]$ if and only if (*) given any countable subcollection $\{M_{i}\}$ of $\{M_{\lambda}\}$, $\bigcup_{i=1}^{\infty} M_{i} \subset P$.

Proof. Assuming (*), let $f(X) = \sum_{i=0}^{\infty} f_{i}X^{i} \in P[[X]]$. For each $i$, $f_{i} \in M_{i}$ for some $\lambda_{i} \in A$. Let $p \in P$, $p \notin \bigcup_{i=0}^{\infty} M_{i}$; since $p \notin M_{i}$, it follows that $f_{i} \notin M_{i} \subseteq (p)V$ for each $i$ and $f(X) \in (p)V[[X]] \subseteq P \cdot V[[X]]$.

Conversely, assuming that (*) fails, let $\{M_{i}\}_{i=1}^{\infty}$ be a countable subcollection of $\{M_{\lambda}\}_{\lambda \in A}$ such that $\bigcup_{i=1}^{\infty} M_{i} = P$. By extracting a subsequence of $\{M_{i}\}_{i=1}^{\infty}$, we may assume that $M_{2i} \subset M_{i+1}$ for each $i$. Let $f_{i} \in M_{i+1}$, $f_{i} \notin M_{i}$ and let $f(X) = \sum_{i=1}^{\infty} f_{i}X^{i}$; then $f(X) \in P[[X]]$ but $f(X) \notin P \cdot V[[X]]$. 

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Correction to

COHOMOLOGY OF FINITELY PRESENTED GROUPS

P. M. CURRAN

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In the second paragraph of the abstract, p. 615, the first sentence, "If $G$ is generated by $n$ elements, ..." should read "If $G$ is a residually finite group generated by $n$ elements, ...".

Correction to

COMMUTANTS OF SOME HAUSDORFF MATRICES

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In [2] it is shown that, for $A$ a conservative triangle, $B$ a matrix with finite norm commuting with $A$, $B$ is triangular if and only if

1. for each $t \in I$ and each $n$, $t(A - a_{nn}I) = 0$ implies $t$ belongs to the linear span of $(e_{o}, e_{n}, \ldots, e_{n})$. On page 716 of [2] it is asserted that

2. $(U^{*})^{n+1}(A - a_{nn}I)U^{n+1}$ of type $M$ for each $n$ is equivalent to
(1). This assertion is false. Condition (2) is sufficient for (1) but, as the following example shows, it is not necessary.

**Example 1.** Let $A$ be a triangle with entries $a_{nk} = p_{n-k}$, $n, k = 0, 1, \ldots$, where $p_k = 2^{-k}$. Then, for each $n$, $A - a_{nn}I = (U^*)^{n+1}(A - a_{nn}I)U^{n+1} = B$, where $b_{nn} = 0$ for each $n$, and $b_{nk} = a_{nk}$ otherwise. The only solutions of $tB = 0$ for $t \in \ell$ lie in the linear span of $e_0$, so that $A$ satisfies (1). However, $B$ is not of type $M$.

Since (2) is not equivalent to (1), some of the material on pages 717 and 718 of [2] must now be reworked.

We establish the following facts:

1. If a factorable triangular matrix $A$ contains at least two zeros on the main diagonal, then $\text{Com}(A)$ in $\Delta \neq \text{Com} A$ in $\Gamma$.

2. If $A$ is not factorable, then the number of zeros on the main diagonal gives no information about the size of $\text{Com}(A)$ in $\Gamma$.

3. Having distinct diagonal entries is necessary but not sufficient for a conservative Hausdorff matrix $H$ to satisfy $\text{Com}(H)$ in $\Delta = \text{Com}(H)$ in $\Gamma$.

**Proof of 1.** Let $n$ and $k$ denote the smallest integers for which $a_{kk} = a_{nn} = 0$, $n > k$. Then the system $t(A - a_{kk}I) = 0$ clearly has a solution in the space spanned by $(e_0, e_1, \ldots, e_n)$. It remains to show that there is a solution not in the subspace spanned by $(e_0, e_1, \ldots, e_k)$. Since $A$ is factorable, either the $k$th row or the $k$th column of $A$ is zero. In either case we can obtain a solution of the system using $t_n = 1$, $t_k = 0$ for $k > n$, which can be used to construct a non-triangular conservative matrix $B$ which commutes with $A$.

**Proof of 2.** Define $D = (d_{nk})$ by $d_{n0} = 1$, $d_{nk} = 0$ otherwise. Then $D \leftrightarrow B$ implies that $(DB)_{nk} = b_{nk}$, whereas $(BD)_{n0} = \sum_j b_{nj}$, and $(BD)_{nk} = 0$ for $k > 0$. Thus, if $B$ is any matrix satisfying (i) $b_{nk} = 0$ for all $k > 0$ and (ii) $\sum_{j=0}^\infty b_{nj} = b_{00}$ for all $n > 0$, then $B \leftrightarrow D$. For example, if $b_{00} = 1$, $b_{nk} = 2^{n-k-1}$, $k \geq n > 0$ $b_{nk} = 0$ otherwise, then $B$ is row infinite and commutes with $D$. $D$ is factorable, but $A = D + I$ is not. Moreover, since $\text{Com}(I)$ in $\Gamma = \Gamma$, $\text{Com}(A)$ in $\Gamma = \text{Com}(D)$ in $\Gamma \neq \text{Com}(D)$ in $\Delta$.

Example 1 with $p_0 = 0$ is a nonfactorable matrix with an infinite number of zeros on the main diagonal, and yet $\text{Com}(A)$ in $\Gamma = \text{Com}(A)$ in $\Delta$.

The following examples establish 3.

**Example 2.** Let $H$ be the Hausdorff matrix generated by $\mu_n = -2n(n-1)/(n+1)(n+2)$, $B = (b_{nk})$ with $b_{0k} = b_{1k} = 1$ for all $k$, $b_{nk} = 0$ otherwise. Then $B \leftrightarrow H$, but $B \not\in \Delta$ since $b_{01} \neq 0$. 

EXAMPLE 3. Let \( H \) be generated by \( \mu_n = \frac{n(n - 1/2)}{(n + 1)(n + 2)} \). We can regard \( H \) as the product of two Hausdorff matrices \( H_a \) and \( H_b \), with generating sequences \( \alpha_n = \frac{(n - 1/2)}{(n + 1)} \) and \( \beta_n = \frac{n}{(n + 2)} \), respectively. From Theorem 1 of [1], the sequence \( t = \{t_n\} \), with \( t_0 = 1, t_n = (-1)^n(1/2)(-3/2) \cdots (-n + 3/2)/n! \), \( n > 0 \) satisfies \( tH_a = 0 \). Therefore \( tH = 0 \). Let \( B \) be the matrix with the sequence \( t \) as each row. Then

\[
(HB)_{nk} = \sum_{j=0}^{m} h_{nj}b_{jk} = t_k \sum_{j=0}^{m} h_{nj} = t_k \mu_0 = 0 ,
\]

and

\[
(BH)_{nk} = \sum_{j=k}^{\infty} b_{nj}h_{jk} = \sum_{j=k}^{\infty} t_jh_{jk} = 0, \quad \text{so that } B \leftrightarrow H .
\]

REFERENCES


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Corrections to

VERSUM SEQUENCES IN THE BINARY SYSTEM

CHARLES W. TRIGG

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Line 12 should read “the universal verity of the conjecture [5, 6]”. Instead of the universal verity of the conjecture [1, 2].

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