Pacific Journal of Mathematics

CORRECTION TO: "COMMUTANTS OF SOME HAUSDORFF MATRICES"

Billy E. Rhoades

Vol. 49, No. 2 June 1973

PROPOSITION A. *Let V be a valuation ring having a proper prime ideal P which is not branched; then $P = \bigcup_{\lambda \in \Lambda} M_\lambda$, where $\{M_\lambda\}_{\lambda \in \Lambda}$ is the collection of prime ideals of V which are properly contained in P. In this case, $P \cdot V[[X]] = P[[X]]$ if and only if* (*) *given any countable subcollection $\{M_{\lambda_i}\}$ of $\{M_\lambda\}$, $\bigcup_{i=1}^{\infty} M_{\lambda_i} \subset P$.*

Proof. Assuming (*), let $f(X) = \sum_{i=0}^{\infty} f_i X^i \in P[[X]]$. For each i, $f_i \in M_{\bar{\lambda}_i}$ for some $\bar{\lambda}_i \in \Lambda$. Let $p \in P$, $p \notin \bigcup_{i=0}^{\infty} M_{\bar{\lambda}_i}$; since $p \notin M_{\bar{\lambda}_i}$, it follows that $f_i \in M_{\bar{\lambda}_i} \subseteqq (p)V$ for each i and $f(X) \in (p)V[[X]] \subseteqq P \cdot V[[X]]$.

Conversely, assuming that (*) fails, let $\{M_{\lambda_i}\}_{i=1}^{\infty}$ be a countable subcollection of $\{M_\lambda\}_{\lambda \in \Lambda}$ such that $\bigcup_{i=1}^{\infty} M_{\lambda_i} = P$. By extracting a subsequence of $\{M_{\lambda_i}\}$, we may assume that $M_{\lambda_i} \subset M_{\lambda_{i+1}}$ for each i. Let $f_i \in M_{\lambda_{i+1}}$, $f_i \notin M_{\lambda_i}$ and let $f(X) = \sum_{i=1}^{\infty} f_i X^i$; then $f(X) \in P[[X]]$ but $f(X) \notin P \cdot V[[X]]$.

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Correction to

COHOMOLOGY OF FINITELY PRESENTED GROUPS

P. M. CURRAN

Volume 42 (1972), 615–620

In the second paragraph of the abstract, p. 615, the first sentence, "If G is generated by n elements, $\cdots$" should read "If G is a residually finite group generated by n elements, $\cdots$".

Correction to

COMMUTANTS OF SOME HAUSDORFF MATRICES

B. E. RHOADES

Volume 42 (1973), 715–719

In [2] it is shown that, for A a conservative triangle, B a matrix with finite norm commuting with A, B is triangular if and only if

(1) for each $t \in l$ and each n, $t(A - a_{nn}I) = 0$ implies t belongs to the linear span of $(e_0, e_1, \cdots, e_n)$. On page 716 of [2] it is asserted that

(2) $(U^*)^{n+1}(A - a_{nn}I)U^{n+1}$ of type M for each n is equivalent to

(1). This assertion is false. Condition (2) is sufficient for (1) but, as the following example shows, it is not necessary.

EXAMPLE 1. Let A be a triangle with entries $a_{nk} = p_{n-k}$, $n, k = 0, 1, \cdots$, where $p_k = 2^{-k}$. Then, for each n, $A - a_{nn}I = (U^*)^{n+1}(A - a_{nn}I)U^{n+1} = B$, where $b_{nn} = 0$ for each n, and $b_{nk} = a_{nk}$ otherwise. The only solutions of $tB = 0$ for $t \in l$ lie in the linear span of e_0, so that A satisfies (1). However, B is not of type M.

Since (2) is not equivalent to (1), some of the material on pages 717 and 718 of [2] must now be reworked.

We establish the following facts:

1. If a factorable triangular matrix A contains at least two zeros on the main diagonal, then $\operatorname{Com}(A)$ in $\Delta \neq \operatorname{Com} A$ in Γ.

2. If A is not factorable, then the number of zeros on the main diagonal gives no information about the size of $\operatorname{Com}(A)$ in Γ.

3. Having distinct diagonal entries is necessary but not sufficient for a conservative Hausdorff matrix H to satisfy $\operatorname{Com}(H)$ in $\Delta = \operatorname{Com}(H)$ in Γ.

Proof of 1. Let n and k denote the smallest integers for which $a_{kk} = a_{nn} = 0$, $n > k$. Then the system $t(A - a_{kk}I) = 0$ clearly has a solution in the space spanned by $(e_0, e_1, \cdots, e_n)$. It remains to show that there is a solution not in the subspace spanned by $(e_0, e_1, \cdots, e_k)$. Since A is factorable, either the kth row or the kth column of A is zero. In either case we can obtain a solution of the system using $t_n = 1$, $t_k = 0$ for $k > n$, which can be used to construct a non-triangular conservative matrix B which commutes with A.

Proof of 2. Define $D = (d_{nk})$ by $d_{n0} = 1$, $d_{nk} = 0$ otherwise. Then $D \leftrightarrow B$ implies that $(DB)_{nk} = b_{0k}$, whereas $(BD)_{n0} = \sum_j b_{nj}$, and $(BD)_{nk} = 0$ for $k > 0$. Thus, if B is any matrix satisfying (i) $b_{0k} = 0$ for all $k > 0$ and (ii) $\sum_{j=0}^{\infty} b_{nj} = b_{00}$ for all $n > 0$, then $B \leftrightarrow D$. For example, if $b_{00} = 1$, $b_{nk} = 2^{n-k-1}$, $k \geqq n > 0$ $b_{nk} = 0$ otherwise, then B is row infinite and commutes with D. D is factorable, but $A = D + I$ is not. Moreover, since $\operatorname{Com}(I)$ in $\Gamma = \Gamma$, $\operatorname{Com}(A)$ in $\Gamma = \operatorname{Com}(D)$ in $\Gamma \neq \operatorname{Com}(D)$ in Δ.

Example 1 with $p_0 = 0$ is a nonfactorable matrix with an infinite number of zeros on the main diagonal, and yet $\operatorname{Com}(A)$ in $\Gamma = \operatorname{Com}(A)$ in Δ.

The following examples establish 3.

EXAMPLE 2. Let H be the Hausdorff matrix generated by $\mu_n = -2n(n-1)/(n+1)(n+2)$, $B = (b_{nk})$ with $b_{0k} = b_{1k} = 1$ for all k, $b_{nk} = 0$ otherwise. Then $B \leftrightarrow H$, but $B \notin \Delta$ since $b_{01} \neq 0$.

EXAMPLE 3. Let H be generated by $\mu_n = n(n-1/2)/(n+1)(n+2)$. We can regard H as the product of two Hausdorff matrices H_α and H_β, with generating sequences $\alpha_n = (n-1/2)/(n+1)$ and $\beta_n = n/(n+2)$, respectively. From Theorem 1 of [1], the sequence $t = \{t_n\}$, with $t_0 = 1$, $t_n = (-1)^n(1/2)(-3/2)\cdots(-n+3/2)/n!$, $n > 0$ satisfies $tH_\alpha = 0$. Therefore $tH = 0$. Let B be the matrix with the sequence t as each row. Then

$$(HB)_{nk} = \sum_{j=0}^{n} h_{nj}b_{jk} = t_k \sum_{j=0}^{n} h_{nj} = t_k\mu_0 = 0 \ ,$$

and

$$(BH)_{nk} = \sum_{j=k}^{\infty} b_{nj}h_{jk} = \sum_{j=k}^{\infty} t_j h_{jk} = 0, \quad \text{so that} \quad B \longleftrightarrow H \, .$$

REFERENCES

1. B. E. Rhoades, *Some Hausdorff matrices not of type M*, Proc. Amer. Math., Soc., **15** (1964), 361-365.

2. ———, *Commutants of some Hausdorff matrices*, Pacific J. Math., **42** (1972), 715-719.

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Corrections to

VERSUM SEQUENCES IN THE BINARY SYSTEM

CHARLES W. TRIGG

Volume 47 (1973), 263-275

Line 12 should read "the universal verity of the conjecture [5, 6]". Instead of the universal verity of the conjecture [1, 2].

The first page should be 263 instead of 163.

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The *Pacific Journal of Mathematics* is issued monthly as of January 1966. Regular subscription rate: $48.00 a year (6 Vols., 12 issues). Special rate: $24.00 a year to individual members of supporting institutions.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley, California, 94708.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), 270, 3-chome Totsuka-cho, Shinjuku-ku, Tokyo 160, Japan.

* C. R. DePrima California Institute of Technology, Pasadena, CA 91109, will replace J. Dugundji until August 1974.

Pacific Journal of Mathematics

Vol. 49, No. 2 June, 1973