

Pacific Journal of Mathematics

**THE NUMER OF MULTINOMIAL COEFFICIENTS DIVISIBLE
BY A FIXED POWER OF A PRIME**

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THE NUMBER OF MULTINOMIAL COEFFICIENTS DIVISIBLE BY A FIXED POWER OF A PRIME

F. T. Howard

In this paper some results of L. Carlitz and the writer concerning the number of binomial coefficients divisible by p^j but not by p^{j+1} are generalized to multinomial coefficients. In particular $\theta_j(k; n)$ is defined to be the number of multinomial coefficients $n!/n_1! \cdots n_k!$ divisible by exactly p^j , and formulas are found for $\theta_j(k; n)$ for certain values of j and n . Also the generating function technique used by Carlitz for binomial coefficients is generalized to multinomial coefficients.

1. Introduction. Let p be a fixed prime and let n and j be nonnegative integers. L. Carlitz [2], [3] has defined $\theta_j(n)$ as the number of binomial coefficients

$$\binom{n}{r} \quad (r = 0, 1, \dots, n)$$

divisible by exactly p^j and he has found formulas for $\theta_j(n)$ for certain values of j and n . In particular, if we write

$$(1.1) \quad n = a_0 + a_1p + \cdots + a_s p^s \quad (0 \leq a_i < p)$$

then

$$\begin{aligned} \theta_0(n) &= (a_0 + 1)(a_1 + 1) \cdots (a_s + 1) \\ \theta_1(n) &= \sum_{i=0}^{s-1} (a_0 + 1) \cdots (a_{i-1} + 1)(p - a_i - 1)a_{i+1}(a_{i+2} + 1) \cdots (a_s + 1). \end{aligned}$$

The writer [5], [6] has also considered the problem of evaluating $\theta_j(n)$.

The purpose of this paper is to consider the analogous problem for multinomial coefficients and to generalize some of the formulas developed by Carlitz and the writer. Thus we define $\theta_j(k; n)$ as the number of multinomial coefficients

$$\binom{n}{n_1, \dots, n_k} = \frac{n!}{n_1! \cdots n_k!} \quad (n_1 + \cdots + n_k = n)$$

divisible by exactly p^j . In this definition the order of the terms n_1, \dots, n_k is important. We are distinguishing, for example, between (1, 2, 3) and (2, 1, 3). Clearly $\theta_j(2; n) = \theta_j(n)$.

In this paper we find formulas for $\theta_0(k; n)$, $\theta_1(k; n)$, and $\theta_2(k; n)$. We also show how the generating function method used by Carlitz

can be generalized to multinomial coefficients, and we evaluate $\theta_j(k; n)$ for special values of j and n .

Throughout this paper we assume p is a fixed prime number and k is a fixed positive integer, $k > 1$.

2. Preliminaries. Let $E(n_1, \dots, n_k)$ denote the largest value of w such that p^w divides (n_1, \dots, n_k) . To determine $E(n_1, \dots, n_k)$ we shall make use of an analogue [4] of Kummer's famous theorem for binomial coefficients:

LEMMA 2.1. *Let n have expansion (1.1), let $n = n_1 + \dots + n_k$ and let*

$$(2.1) \quad n_i = a_{i,0} + a_{i,1}p + \dots + 2_{i,s}p^s \quad (0 \leq a_{i,r} < p)$$

for $i = 1, \dots, k$. If

$$\begin{aligned} a_{1,0} + \dots + a_{k,0} &= \varepsilon_0 p + a_0 \\ \varepsilon_0 + a_{1,1} + \dots + a_{k,1} &= \varepsilon_1 p + a_1 \\ &\dots\dots\dots \\ \varepsilon_{s-1} + a_{1,s} + \dots + a_{k,s} &= a_s \end{aligned}$$

where each $\varepsilon_i = 0, 1, \dots$, or $k - 1$, then

$$E(n_1, \dots, n_k) = \varepsilon_0 + \varepsilon_1 + \dots + \varepsilon_{s-1}.$$

If n has expansion (1.1) and if $\nu(n)$ is the largest value of w such that p^w divides $n!$, then it is familiar [1, p. 55] that

$$\nu(n) = \frac{n - S(n)}{p - 1}$$

where $S(n) = a_0 + a_1 + \dots + a_s$. Thus we have

LEMMA 2.2. *If $n = n_1 + \dots + n_k$ then*

$$E(n_1, \dots, n_k) = \frac{S(n_1) + \dots + S(n_k) - S(n)}{p - 1}.$$

Furthermore, if $E_i(n_1, \dots, n_k)$ is the largest value of w such that p^w divides

$$(n + t) \dots (n + 1) \quad (n_1, \dots, n_k),$$

then

$$E_i(n_1, \dots, n_k) = \frac{S(n_1) + \dots + S(n_k) - S(n + t) + t}{p - 1}.$$

Compositions, or ordered partitions, are important in evaluating $\theta_j(k; n)$. We define a composition of a nonnegative integer u into r parts to be an ordered sequence of r nonnegative integers whose sum is u . This is more general than the usual definition of composition in that we allow 0 to be one or more of the parts. See [7, pp. 124-125] for example.

Throughout this paper we shall let $C(u)$ denote the number of compositions of u into exactly k parts, with no part larger than $p - 1$. We define $C(u) = 0$ if $u < 0$.

LEMMA 2.3. $C(u)$ is the coefficient of x^u in the expansion of

$$(1 + x + x^2 + \dots + x^{p-1})^k = \left[\sum_{i=0}^{\infty} \binom{k+i-1}{i} x^i \right] (1 - x^p)^k.$$

It is clear from Lemma 2.3 that if $0 \leq a < p$ and if $0 \leq b$, then

$$(2.2) \quad C(a + bp) = \sum_{i=0}^b (-1)^i \binom{k}{i} \binom{k-1+a+(b-i)p}{k-1}.$$

In particular, for $0 \leq a < p$,

$$\begin{aligned} C(a) &= \binom{k-1+a}{k-1}, \\ C(a+p) &= \binom{k-1+a+p}{k-1} - k \binom{k-1+a}{k-1}, \\ C(a+2p) &= \binom{k-1+a+2p}{k-1} - k \binom{k-1+a+p}{k-1} + \binom{k}{2} \binom{k-1+a}{k-1}. \end{aligned}$$

3. Evaluation of $\theta_0(k; n)$, $\theta_1(k; n)$, $\theta_2(k; n)$.

THEOREM 3.1. If n has expansion (1.1) then

$$\theta_0(k; n) = C(a_0)C(a_1) \dots C(a_s).$$

Proof. We use Lemma 2.1. If $E(n_1, \dots, n_k) = 0$ then we must have

$$\sum_{i=1}^k a_{i,r} = a_r \quad (r = 0, \dots, s).$$

For a given r , the total number of ways we can have this equality is equal to $C(a_r)$.

Note that by Lemma 2.3 we have

$$C(a_r) = \binom{a_r + k - 1}{k - 1} \quad (r = 0, \dots, s).$$

THEOREM 3.2. *If n has expansion (1.1) then*

$$\theta_1(k; n) = \sum_{i=0}^{s-1} C(a_0) \cdots C(a_{i-1}) C(a_i + p) C(a_{i+1} - 1) C(a_{i+2}) \cdots C(a_s).$$

Proof. Using Lemma 2.1, we see that if $E(n_1, \dots, n_k) = 1$ then we must have exactly one $\varepsilon_i = 1$, $0 \leq i < s$. So for some i we have

$$\begin{aligned} a_{1,i} + \cdots + a_{k,i} &= a_i + p, \\ a_{1,i+1} + \cdots + a_{k,i+1} &= a_{i+1} - 1, \\ a_{1,r} + \cdots + a_{k,r} &= a_r \end{aligned} \quad (r \neq i, i+1).$$

Clearly the total number of ways we can have these equalities is

$$C(a_0) \cdots C(a_{i-1}) C(a_i + p) C(a_{i+1} - 1) C(a_{i+2}) \cdots C(a_s).$$

To simplify the formula for $\theta_2(k; n)$ we introduce the following notation. Let

$$\begin{aligned} A_i &= \left[\prod_{t=0}^s C(a_t) \right] / \left[C(a_i) C(a_{i+1}) C(a_{i+2}) \right], \\ B_i &= \left[\prod_{t=0}^s C(a_t) \right] / \left[C(a_i) C(a_{i+1}) \right], \\ H_{i,r} &= \left[\prod_{t=0}^s C(a_t) \right] / \left[C(a_i) C(a_{i+1}) C(a_r) C(a_{r+1}) \right]. \end{aligned}$$

THEOREM 3.3. *If n has expansion (1.1) then*

$$\begin{aligned} \theta_2(k; n) &= \sum_{i=0}^{s-2} C(p + a_i) C(p + a_{i+1} - 1) C(a_{i+2} - 1) A_i \\ &\quad + \sum_{i=0}^{s-1} C(2p + a_i) C(a_{i+1} - 2) B_i \\ &\quad + \sum_{r=i+2}^{s-1} \sum_{i=0}^{s-3} C(p + a_i) C(a_{i+1} - 1) C(p + a_r) C(a_{r+1} - 1) H_{i,r}. \end{aligned}$$

Proof. The proof is similar to the proof of Theorem 3.2. We determine the number of ways we can have exactly two of the ε 's equal to 1 or exactly one ε equal to 2, and all other ε 's equal to 0.

For example, let $p = 5$, $k = 3$, and $n = 278 = 3 + 5^2 + 2 \cdot 5^3$. We have

$$\begin{aligned} \theta_0(3; 278) &= C(3)C(0)C(1)C(2) = 180; \\ \theta_1(3; 278) &= C(3)C(5)C(0)C(2) + C(3)C(0)C(6)C(1) = 1650, \\ \theta_2(3; 278) &= C(8)C(4)C(0)C(2) + C(3)C(5)C(5)C(1) \\ &\quad + C(3)C(0)C(11)C(0) = 11,100. \end{aligned}$$

In each example we have used (2.2) to evaluate $C(u)$.

4. Generating functions for $\theta_j(k; n)$. Let $\psi_{t,j}(k; n)$ denote the

number of products $(n + t) \cdots (n + 1)(n_1, \dots, n_k)$, $n_1 + \cdots + n_k = n$, divisible by exactly p^j . Clearly

$$(4.1) \quad \psi_{t,j}(k; n) = \theta_{j-r}(k; n)$$

if p^r is the highest power of p dividing $(n + t) \cdots (n + 1)$.

Also

$$\psi_{t,j}(k; n) = 0$$

if p^{j+1} divides $(n + t) \cdots (n + 1)$. We introduce the following generating functions:

$$F_0(x, y) = \sum_{n=0}^{\infty} \sum_{j=0}^{\infty} \theta_j(k; n) x^n y^j,$$

$$F_t(x, y) = \sum_{n=0}^{\infty} \sum_{j=0}^{\infty} \psi_{t,j}(k; n) x^n y^j \quad (t > 0).$$

Using an argument analogous to that of Carlitz [3], we obtain

$$(4.2) \quad F_0(x, y) = \sum_{t=0}^m y^t f_t(x) F_t(x^p, y)$$

where m is the integer such that

$$(4.3) \quad mp \leq k(p - 1) < (m + 1)p$$

and

$$f_t(x) = \sum_{a=tp}^{tp+p-1} C(a) x^a \quad (0 \leq t < m),$$

$$f_m(x) = \sum_{a=mp}^{kp-k} C(a) x^a.$$

Comparing coefficients of $x^n y^j$ on both sides of (4.2), we have, for $0 \leq a < p$,

$$(4.4) \quad \theta_j(k; a + bp) = C(a) \theta_j(k; b) + \sum_{t=1}^m C(a + tp) \psi_{t,j-t}(k; b - t).$$

In (4.4) it is understood that $\psi_{t,j}(k; u) = 0$ if $u < 0$ and $\psi_{t,-1}(k; u) = 0$.

Also, for $t < p$,

$$F_t(x, y) = \sum_{r=1}^h y^r g_r(x) F_r(x^p, y)$$

where h is the integer such that

$$(4.5) \quad hp - t \leq k(p - 1) < (h + 1)p - t,$$

and

$$\begin{aligned} g_0(x) &= \sum_{a=0}^{p-t-1} C(a)x^a, \\ g_r(x) &= \sum_{a=r}^{(r+1)p-t-1} C(a)x^a \quad (r = 1, \dots, h-1), \\ g_h(x) &= \sum_{a=h}^{kp-k} C(a)x^a. \end{aligned}$$

Thus for $0 \leq a < p-t$, $hp+a \leq kp-k$, we have

$$(4.6) \quad \psi_{t,j}(k; a+bp) = C(a)\theta_j(k; b) + \sum_{r=1}^h C(a+rp)\psi_{r,j-r}(k; b-r).$$

For $0 \leq a < p-t$, $hp+a > kp-k$, we have

$$(4.7) \quad \psi_{t,j}(k; a+bp) = C(a)\theta_j(k; b) + \sum_{r=1}^{h-1} C(a+rp)\psi_{r,j-r}(k; b-r).$$

For $p-t \leq a < p$, we have

$$(4.8) \quad \psi_{t,j}(k; a+bp) = \sum_{r=1}^h C(a+(r-1)p)\psi_{r,j-r}(k; a-r+1).$$

Here again it is understood that $\psi_{r,j}(k; u) = 0$ if $u < 0$. We remark that in all of these formulas specific values for $C(u)$ can be found from formula (2.2).

Using (4.4) we can compute $\theta_j(k; n)$ for special values of n . By (4.4) and (4.1) we have, for $0 \leq a < p$, $0 \leq b < p$,

$$\begin{aligned} \theta_j(k; a+bp) &= C(a+jp)\theta_0(k; b-j) \\ &= C(a+jp)C(b-j) \quad \text{if } j \leq m, \\ &= 0 \quad \text{if } j > m \end{aligned}$$

where m is defined by (4.3).

Also, if $0 \leq a < p$,

$$\begin{aligned} \theta_j(k; a+p^2) &= C(a)C(1) \quad \text{if } j = 0, \\ &= C(a+(j-1)p)C(p-j+1) \quad \text{if } 1 \leq j \leq m+1, \\ &= 0 \quad \text{if } j > m+1. \end{aligned}$$

If $0 \leq a < p$, $p > 2$,

$$\begin{aligned} \theta_j(k; a+2p^2) &= C(a+(j-2)p)\theta_1(k; 2p-j+2) \\ &\quad + C(a+(j-1)p)\theta_0(k; 2p-j+1) \quad (1 < j \leq p+1, j \leq m+1), \\ &= C(a+(j-2)p)\theta_1(k; 2p-j+2) \quad (j = m+2 \leq p+1), \\ &= C(a+(j-2)p)\theta_1(k; p) \quad (j = p+2 \leq m+2), \\ &= C(a+(j-2)p)\theta_0(k; p-r+2) \quad (j = p+r \leq m+2, 2 < r \leq p+2), \\ &= 0 \quad \text{if } j > m+2. \end{aligned}$$

If $0 \leq a < p, 0 \leq b < p,$

$$\begin{aligned} \theta_j(k; a + bp + p^2) &= C(a + (j - 1)p)\theta_1(k; p + b - j + 1) \\ &\quad + C(a + jp)\theta_0(k; p + b - j) \qquad (b \geq j; m + 1 > j) \\ &= C(a + (j - 1)p)\theta_0(k; p + b - j + 1) \quad (b < j \leq p + b, m + 1 > j), \\ &= C(a + mp)\theta_1(k; p + b - m) \qquad (j = m + 1, b \geq m), \\ &= C(a + mp)\theta_0(k; p + b - m) \qquad (j = m + 1, b < m), \\ &= 0 \quad \text{if } j > m + 1. \end{aligned}$$

Some of the results in [2] can also be generalized. We use the symbols $E(n_1, \dots, n_k)$ and $E_t(n_1, \dots, n_k)$ as they are used in Lemma 2.2.

Let

$$\begin{aligned} F_j(n; x_1, \dots, x_k) &= \sum_{\substack{a_1 + \dots + a_k = n \\ E(a_1, \dots, a_k) = j}} x_1^{a_1} \dots x_k^{a_k}, \\ G_{t,j}(n; x_1, \dots, x_k) &= \sum_{\substack{a_1 + \dots + a_k = n \\ E_t(a_1, \dots, a_k) = j}} x_1^{a_1} \dots x_k^{a_k} \qquad (t > 0), \\ G_{0,j}(n; x_1, \dots, x_k) &= F_j(n; x_1, \dots, x_k). \end{aligned}$$

Note that

$$\begin{aligned} F_j(n; x, \dots, x) &= x^n \theta_j(k; n), \\ G_{t,j}(n; x, \dots, x) &= x^n \psi_{t,j}(k; n). \end{aligned}$$

By generalizing Carlitz's work in [2] in the natural way, we obtain

$$(4.9) \quad \begin{aligned} &F_j(a + bp; x_1, \dots, x_k) \\ &= \sum_{s=0}^m c_{sp+a}(x_1, \dots, x_k) G_{s,j-s}(b - s; x_1^p, \dots, x_k^p) \end{aligned}$$

where $0 \leq a < p, m$ is defined by (4.3), and

$$c_r(x_1, \dots, x_k) = \sum_{s_1 + \dots + s_k = r} x_1^{s_1} \dots x_k^{s_k}.$$

Also, if h is defined by (4.5),

$$(4.10) \quad \begin{aligned} &G_{t,j}(a + bp; x_1, \dots, x_k) \\ &= \sum_{s=0}^h c_{sp+a}(x_1, \dots, x_k) G_{s,j-s}(b - s; x_1^p, \dots, x_k^p) \\ &\qquad (hp + a \leq kp - k, 0 \leq a < p - t), \\ &= \sum_{s=0}^{h-1} c_{sp+a}(x_1, \dots, x_k) G_{s,j-s}(b - s; x_1^p, \dots, x_k^p) \\ &\qquad (hp + a > kp - k, 0 \leq a < p - t), \\ &= \sum_{s=1}^h c_{(s-1)p+a}(x_1, \dots, x_k) G_{s,j-s}(a - s + 1; x_1^p, \dots, x_k^p) \\ &\qquad (p - t \leq a < p - 1). \end{aligned}$$

5. **Some special evaluations.** If $j > \nu(n)$, where $\nu(n)$ is the exponent of the highest power of p that divides $n!$, then it is clear that $\theta_j(k; n) = 0$. For example, if $0 \leq a < p$, $0 \leq b < p$ then

$$\theta_j(k; a + bp) = 0 \quad (j > b).$$

Let n have expansion (1.1). By Lemma 2.1 it is clear that $\theta_j(k; n) = 0$ for $j > M$, where

$$\begin{aligned} M &= s(k - 1) \quad \text{if } k \leq a_s + 1 \\ &= (s - 1)(k - 1) + a_s \quad \text{if } k > a_s + 1. \end{aligned}$$

Also,

$$\begin{aligned} \theta_M(k; n) &= C(a_0 + (k - 1)p)C(a_s - k + 1) \prod_{i=1}^{s-1} C(a_i - k + 1 + (k - 1)p) \\ &\hspace{20em} (k \leq a_s + 1), \\ &= C(a_0 + (k - 1)p)C(a_{s-1} - k + 1 + a_s p) \prod_{i=1}^{s-2} C(a_i - k + 1 + (k - 1)p) \\ &\hspace{15em} (k > a_s + 1, s > 1), \\ &= C(a_0 + a_1 p) \hspace{15em} (k > a_s + 1, s = 1). \end{aligned}$$

For example, if $k = 2$ and $a_s \neq 0$ then $M = s$. This is the case for ordinary binomial coefficients. We have in this case

$$\theta_s(2; n) = (p - a_0 - 1)(p - a_1) \cdots (p - a_{s-1})a_s.$$

For $p = 2$ we can generalize the method used in [6]. Let

$$(5.1) \quad n = 2^{e_1} + \cdots + 2^{e_r}, \quad 0 \leq e_1 < \cdots < e_r,$$

$$(5.2) \quad n_i = 2^{e_{i,1}} + \cdots + 2^{e_{i,S(i)}}, \quad 0 \leq e_{i,1} < \cdots < e_{i,S(i)}.$$

Consider all the different compositions $n = n_1 + \cdots + n_k$ such that (5.1) and (5.2) hold, such that

$$S(n_1) + \cdots + S(n_k) = r + j,$$

and such that there are a total of $r + j - t$ $e_{i,w}$'s having the property that $e_{i,w} \neq e_{x,y}$ for all x, y (except for the one case $i = x, w = y$). Let $b_{j,t}$ be the sum over all these compositions of the number of different ways of distributing the remaining t $e_{i,w}$'s into k distinct cells with no two identical objects in the same cell. Then for $p = 2, j > 0$,

$$(5.3) \quad \theta_j(k; n) = b_{j,2}k^{m+j-2} + b_{j,3}k^{m+j-3} + \cdots + b_{j,m+j}.$$

Using the convention that $e_1 - e_0 = t$ means $e_1 = t - 1$ and that $e_1 - e_0 > t$ means $e_1 > t - 1$, let

$$\begin{aligned}
 e_i - e_{i-1} &> 1 \text{ for } q_1 \text{ terms } e_i, \\
 &> 2 \text{ for } q_2 \text{ terms } e_i, \\
 &= 1, e_{i-1} - e_{i-2} = 1 \text{ for } q_3 \text{ terms } e_i \quad (i \neq 2), \\
 &= 1, e_{i-1} - e_{i-2} > 1 \text{ for } q_4 \text{ terms } e_i, \\
 &= 2 \text{ for } q_5 \text{ terms } e_i \quad (i \neq 1), \\
 &= 1 \text{ for } q_6 \text{ terms } e_i \quad (i \neq 1).
 \end{aligned}$$

Then, by (5.3), for $p = 2$,

$$\theta_0(k; n) = k^r,$$

$$\theta_1(k; n) = q_1 \binom{k}{2} k^{r-1} + q_6 \binom{k}{3} k^{r-2},$$

$$\begin{aligned}
 \theta_2(k; n) &= q_2 \binom{k}{2} k^r + q_5 \binom{k}{3} k^{r-1} \\
 &+ \left[\binom{q_1}{2} + q_4 \right] \binom{k}{2}^2 k^{r-2} \\
 &+ \left[q_4(q_1 - 1) + q_3 \right] \binom{k}{3} \binom{k}{2} k^{r-3} \\
 &+ \left[\binom{q_4}{2} + q_3(q_3 - 1) + q_4 q_3 \right] \binom{k}{3}^2 k^{r-4}.
 \end{aligned}$$

For example, let $n = 2^4 + 2^5 + 2^{20} + 2^{26} + 2^{28}$. Then $q_1 = 4, q_2 = 3, q_3 = 0, q_4 = 1, q_5 = 1$ and $q_6 = 1$. Thus

$$\theta_0(k; n) = k^5$$

$$\theta_1(k; n) = 4 \binom{k}{2} k^4 + \binom{k}{3} k^3,$$

$$\theta_2(k; n) = 3 \binom{k}{2} k^5 + \binom{k}{3} k^4 + 7 \binom{k}{2}^2 k^3 + 3 \binom{k}{3} \binom{k}{2} k^2.$$

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Received September 11, 1972.

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Pacific Journal of Mathematics

Vol. 50, No. 1

September, 1974

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