THE LOCAL COMPACTNESS OF $vX$

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Necessary and sufficient conditions are given for the local compactness of the Hewitt realcompactification $\nu X$ of a completely regular Hausdorff space $X$; the conditions are expressed in terms of the space $X$ alone. In addition, the local compactness of other extensions is considered.

Introduction. There has been much recent interest in determining conditions on a completely regular Hausdorff space $X$ that are equivalent to the local compactness of its Hewitt realcompactification $\nu X$. This interest stems primarily from the fact that the seemingly artificial hypothesis "$\nu X$ is locally compact" enters quite naturally into the examination of the relation $\nu X \times \nu Y = \nu(X \times Y)$. Apparently the only known condition equivalent to the local compactness of $\nu X$ is one discussed by Comfort in [1] and [2]. As remarked by Comfort, the condition is not on $X$ alone, but involves $\nu X$ essentially in its statement.

In the present paper a condition on $X$ is given which is equivalent to the local compactness of $\nu X$ (Theorem 2.7) and a number of known results are obtained as corollaries of this characterization theorem. Another characterization (Theorem 2.3) is given of the local compactness of $\nu X$ in terms of real maximal ideals.

It was shown by Comfort in [1] and [2] that the local pseudo-compactness of $X$ plays an important role in connection with the local compactness of $\nu X$. The precise role is established below, where it is shown that the local pseudocompactness of $X$ is equivalent to the local compactness of the extension $\eta X$ of $X$ constructed by Johnson and Mandelker in [9]. In addition a characterization is given of those spaces for which the extension $\nu X$ constructed by Johnson and Mandelker is locally compact.

Our attention will be restricted entirely to completely regular Hausdorff spaces. The terminology and notation of [4] will be used without further comment.

Given $f \in C(X)$ the symbols $N(f)$ and $S(f)$ represent respectively $\{x \in X : f(x) \neq 0\}$ and $\text{cl}_X \{x \in X : f(x) \neq 0\}$; these sets are called the cozero set and the support of $f$. If $A$ and $B$ are subsets of $X$, write $A \ll B$ if $A$ is completely separated from $X - B$. We shall frequently apply [4, 1.15] to construct additional separating zero sets when $A \ll B$.

The symbol $M^p$ will denote the maximal ideal in $C(X)$ which corresponds to the point $p$ of $\beta X$, and $\mathcal{M}^p$ will denote the corresponding $\sigma$-ultrafilter (written $A^p$ in [4]). Similarly $O^p$ represents the ideal defined in [4, 7.12] and $O^p$ the corresponding $\sigma$-filter.
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2. The local compactness of $\nu X$. The family $C_\psi(X)$ of functions with pseudocompact support, discussed at length in [9] and [11], plays the major role in our condition for local compactness of $\nu X$. This is to be expected since by 2.1(d) below the isomorphism $f \rightarrow f^\circ$ is an isomorphism of $C_\psi(X)$ with $C_\kappa(\nu X)$. We write $C_\psi(X)$ for the corresponding collection of zero sets.

The following results are either found in [4] or may be established using results from [4].

2.1. (a) $Z(f^\circ) = \text{cl}_{\nu X} Z(f)$.
(b) $N(f^\circ) = \text{int}_{\nu X} (\nu X - Z(f))$.
(c) $S(f^\circ) = \text{cl}_{\nu X} S(f) = \text{cl}_{\nu X} N(f)$.
(d) $S(f^\circ)$ is compact if and only if $S(f)$ is pseudocompact.

The following result from [11] is frequently useful.

2.2. Every support in a pseudocompact space is pseudocompact.

Since the isomorphism $f \rightarrow f^\circ$ induces a bijection between real maximal ideals in $C(X)$ and fixed ideals in $C(\nu X)$, the following result is an immediate consequence of [4, 4D3].

**Theorem 2.3.** The space $\nu X$ is locally compact if and only if $C_\psi(X)$ is not contained in any real maximal ideal.

We turn now toward a condition expressible in terms of $X$ alone. For each $\varepsilon > 0$ and each $f \in C(X)$, define $U_\varepsilon(f) = \{x \in X: |f(x)| \leq \varepsilon\}$; this is a zero set in $X$. The following results are essential for our characterization theorem.

2.4. (a) If $\varepsilon > \delta > 0$ then $N(f) \supseteq U_\varepsilon(f) \supseteq U_\delta(f)$.
(b) If $p \in \beta X$ and $f \in C^*(X)$ then $f^\delta(p) = 0$ if and only if $U_\varepsilon(f) \in \mathcal{M}^p$ for each $\varepsilon > 0$. (Also $\mathcal{M}^p$ may be replaced by $\mathcal{O}^p$ in this condition).
(c) For each $f \in C(X)$, $U_\varepsilon(f^\circ) = \text{cl}_{\nu X} U_\varepsilon(f)$.

**Proof.** The proofs of (a) and (c) are straightforward.

(b) For any $\varepsilon > 0$, if $U_\varepsilon(f) \in \mathcal{M}^p$, then $p \in \text{cl}_{\beta X} U_\varepsilon(f)$ and $f(p) \geq \varepsilon$, a contradiction. Hence $U_\varepsilon(f) \in \mathcal{M}^p$.

Conversely let $U_\varepsilon(f) \in \mathcal{O}^p$ for each $\varepsilon > 0$. For every $\varepsilon > 0$ we have $\{x \in X: |f(x)| \leq \varepsilon\} \in \mathcal{M}^p$ by [4, 7.12(b)]; hence $f^\delta(p) \leq \varepsilon$. Thus $f^\delta(p) = 0$.

Let $U(X)$ represent the set of units in $C(X)$; by [4, 1.12], these are the functions with empty zero set. Clearly the image of $U(X)$
under the isomorphism \( f \to f^* \) is \( U(\nu X) \).

A scale on \( X \) is a function \( \varepsilon: f \to \varepsilon(f) \) from \( U(X) \) to the positive real numbers. If \( \varepsilon \) is a scale, put \( \mathcal{E}(\varepsilon) = \{ U_{\varepsilon(f)}(f): f \in U(X) \} \).

**Theorem 2.5.** A \( \nu \)-ultrafilter is real if and only if it contains \( \mathcal{E}(\varepsilon) \) for some scale \( \varepsilon \).

**Proof.** Let \( \varepsilon \) be a scale on \( X \) and let \( \mathcal{M}^p \) be a hyper-real \( \nu \)-ultrafilter. By [4, 8.8] there is a bounded unit \( f \) in \( C(X) \) with \( f^*(p) = 0 \); hence \( U_{\varepsilon(f)}(f) \in \mathcal{M}^p \). Thus \( \mathcal{M}^p \) does not contain \( \mathcal{E}(\varepsilon) \).

Let \( p \in \nu X \). For any \( f \in U(X) \) put \( \varepsilon(f) = |f^*(p)| \). Since \( f^* \) is a unit of \( C(\nu X) \), \( \varepsilon(f) > 0 \), and thus \( \varepsilon \) is a scale on \( X \). Since \( p \in U_{\varepsilon(f)}(f^*) = \text{cl}_X U_{\varepsilon(f)}(f) \subset \text{cl}_X U_{\varepsilon(f)}(f) \), it follows that \( U_{\varepsilon(f)}(f) \in \mathcal{M}^p \). Thus \( \mathcal{E}(\varepsilon) \subseteq \mathcal{M}^p \).

**Corollary 2.6.** A filter \( \mathcal{F} \) on \( X \) is contained in a real \( \nu \)-ultrafilter if and only if every member of \( \mathcal{F} \) meets every member of \( \mathcal{E}(\varepsilon) \) for some scale \( \varepsilon \).

**Theorem 2.7.** The space \( \nu X \) is locally compact if and only if \( X \) satisfies the following condition: \( (RL) \). For every scale \( \varepsilon \) there are \( f_1, \cdots, f_k \in U(X) \) and \( g \in C(\nu X) \) such that \( Z(g) \cap (\bigcap_{i=1}^k U_{\varepsilon(f_i)}(f_i)) = \emptyset \).

**Proof.** Let \( X \) satisfy \( (RL) \). For any \( p \in \nu X \), by 2.5 there is a scale \( \varepsilon \) on \( X \) such that \( \mathcal{E}(\varepsilon) \subseteq \mathcal{M}^p \). By \( (RL) \), \( \mathcal{E}_\nu(X) \not\subseteq \mathcal{M}^p \); thus \( \nu X \) is locally compact by 2.3.

Suppose \( \nu X \) is locally compact and \( \varepsilon \) is a scale on \( X \). By 2.3, \( \mathcal{E}_\nu(X) \not\subseteq \mathcal{M}^p \) for any \( p \in \nu X \). Thus, by 2.5, \( \mathcal{E}_\nu(X) \cup \mathcal{E}(\varepsilon) \) lacks the finite intersection property; that is, condition \( RL \) is satisfied.

**Remark 2.8.** It is clear that we need consider in condition \( RL \) only those scales for which the family \( \mathcal{E}(\varepsilon) \) has the finite intersection property, since the condition is trivially fulfilled when some finite subfamily of \( \mathcal{E}(\varepsilon) \) has empty intersection. The condition is also fulfilled trivially when \( \mathcal{E}_\nu(X) \) contains a unit of \( C(X) \), and this occurs precisely when \( X \) is pseudocompact.

Certainly if \( \mathcal{E}_\nu(X) \) lacks the countable intersection property then it is not contained in a real \( \nu \)-ultrafilter. It will now be shown that \( \mathcal{E}_\nu(X) \) lacks the property precisely when \( \nu X \) is locally compact and \( \sigma \)-compact. Our condition is shown to be related to one given by Hager in [7].

**Theorem 2.9.** The following are equivalent for a space \( X \).

(a) \( \nu X \) is locally compact and \( \sigma \)-compact.
(b) \( \mathcal{E}_\Psi(X) \) lacks the countable intersection property.
(c) (Hager) \( X = \bigcup_{n=1}^\infty A_n \), where each \( A_n \) is pseudocompact and \( A_n \ll A_{n+1} \) for each \( n \).

**Proof.** (a) implies (c). If \( \nu X \) is locally compact and \( \sigma \)-compact then \([3, XI, 7.2]\) \( \nu X = \bigcup_{n=1}^\infty U_n \), where each \( U_n \) is open and has compact closure and \( \text{cl}_{\nu X} U_n \subset U_{n+1} \) for each \( n \). By \([4, 3.11(a)]\), \( U_n \ll U_{n+1} \). Setting \( A_n = \text{cl}_{X} (U_n \cap X) \) it follows from \([2, 4.1]\) that each \( A_n \) is pseudocompact.

(c) implies (b). Let \( X = \bigcup_{n=1}^\infty A_n \), with \( A_n \) pseudocompact and \( A_n \ll A_{n+1} \) for each \( n \). Choose for each \( n \) a function \( f_n \) such that \( A_n \subset N(f_n) \subset S(f_n) \subset A_{n+1} \). Then, by \([2.2]\) each \( f_n \in C_\Psi(X) \), and clearly \( \bigcap_{n=1}^\infty Z(f_n) = \phi \).

(b) implies (a). Let \( \bigcap_{n=1}^\infty Z(f_n) = \phi \), where \( f_n \in C_\Psi(X) \) for each \( n \). Then, by \([2.1(a)]\) and \([4, 8.7]\), \( \bigcap_{n=1}^\infty Z(f_n) = \phi \), and thus \( \bigcup_{n=1}^\infty S(f_n) = \nu X \). By \([2.1(d)]\) each \( S(f_n) \) is compact, thus \( \nu X \) is \( \sigma \)-compact. By \([2.3]\), \( \nu X \) is locally compact.

Comfort \([2, 4.6]\) gives another condition (C) which is equivalent to the local compactness of \( \nu X \). A direct proof of the equivalence of (C) with the condition of Theorem 2.3 will now be given.

2.10. \( \mathcal{E}_\Psi(X) \) is not contained in any real \( z \)-ultrafilter if and only if: (C) For each \( p \in \nu X \) there exist pseudocompact subsets \( A \) and \( B \) of \( X \) such that \( p \in \text{cl}_{\nu X} A \) and \( A \ll B \).

**Proof.** If \( X \) satisfies condition (C) and \( \mathcal{M}^p \) is a real maximal ideal then there are pseudocompact sets \( A \) and \( B \) and functions \( f, g \in C(X) \) such that \( p \in \text{cl}_{\nu X} A \) and \( A \subset Z(f) \subset N(g) \subset B \). It follows from \([2.2]\) that \( g \in C_\Psi(X) \). Since \( p \in \text{cl}_{\nu X} A \) then \( f \in M^p \), and thus \( g \in M^p \). Thus \( C_\Psi(X) \not\subset M^p \).

Conversely, for any \( p \in \nu X \) there is \( f \in C_\Psi(X) \) and \( g \in \mathcal{M}^p \) such that \( Z(f) \cap Z(g) = \phi \); thus \( Z(g) \ll N(f) \) and there exist \( h, k \in C(X) \) such that \( Z(g) \subset N(k) \subset Z(h) \subset N(f) \). Put \( A = S(k) \) and \( B = S(f) \). Since \( A \subset S(f) \), it follows from \([2.2]\) that \( A \) is pseudocompact. Also \( p \in \text{cl}_{\nu X} Z(g) \), since \( g \in \mathcal{M}^p \), so \( p \in \text{cl}_{\nu X} A \). Finally, \( A \subset Z(h) \) and \( X - B \subset Z(f) \), with \( Z(h) \cap Z(f) = \phi \), so \( A \ll B \). Thus condition (C) is satisfied.

3. The local pseudocompactness of \( X \). The space \( X \) is **locally pseudocompact** if every point has a pseudocompact neighborhood. Locally pseudocompact spaces are discussed in \([1]\) and \([2]\). The results in this section clarify the relationship between the local pseudocompactness of \( X \) and the local compactness of \( \nu X \).
3.1. The space $X$ is locally pseudocompact if and only if $C_\psi(X)$ is not contained in any fixed maximal ideal.

Proof. Let $X$ be locally pseudocompact. Then any point $x$ in $X$ has a pseudocompact neighborhood $A$. Therefore, there is $f \in C(X)$ with $x \in N(f) \subseteq A$. Thus $x \in Z(f)$ and by 2.2, the set $S(f)$ is pseudocompact, so $f \in C_\psi(X)$. Therefore, $C_\psi(X)$ is not contained in any fixed maximal ideal. Conversely suppose $C_\psi(X)$ is contained in no fixed maximal ideal. Then for each $x \in X$ there is $f \in C_\psi(X)$ with $x \in N(f)$, and thus $S(f)$ is a pseudocompact neighborhood of $x$.

For any space $Y$ denote by $L(Y)$ the set of all points of $Y$ that have a compact neighborhood in $Y$; i.e., $L(Y) = Y - R(Y)$, where $R(Y)$ is as defined in [8, p. 87]. Clearly $L(Y)$ is locally compact. For any space $X$ define $\kappa X = \{ p \in \beta X: C_\psi(X) \nsubseteq \mathcal{M} \}$; equivalently, $\kappa X = \beta X - \theta(C_\psi(X))$, where $\theta(C_\psi(X))$ is as defined in [4, 70].

Theorem 3.2. For each space $X$, $\kappa X = L(\nu X) = \text{int}_{\bar{\beta}X} \nu X$, and thus $\kappa X$ is locally compact.

Proof. The relation $L(\nu X) = \text{int}_{\bar{\beta}X} \nu X$ follows from [4, 3.15(b)]. By [9, 3.1], $\beta X - \kappa X = \theta(C_\psi(X)) = \text{cl}_{\bar{\beta}X} (\beta X - \nu X)$, so $\kappa X = \text{int}_{\bar{\beta}X} \nu X$.

Corollary 3.3. The space $X$ is locally pseudocompact if and only if $X \subseteq \kappa X$. In this case $\kappa X$ is the largest locally compact space between $X$ and $\nu X$.

The following result is due to Comfort ([1] and [2]).

Corollary 3.4. The space $X$ is locally pseudocompact if and only if there is a locally compact space $Y$ between $X$ and $\nu X$.

4. Functions with small support. Another ideal in $C(X)$ plays an important role in connection with local compactness. Before discussing this ideal, the class of small sets will be examined, where a set $A \subseteq X$ is small if any zero set contained in $A$ is compact.

4.1. The set $A$ is small if and only if every zero set that intersects $X - A$ in a compact set is compact.

Proof. Certainly in the latter condition holds then $A$ is small. Now suppose $A$ is small and $Z$ is a zero set such that $Z \cap (X - A)$ is compact. If $\alpha$ is a cover of $Z$ by cozero sets then finitely many of the cozero sets cover $Z \cap (X - A)$. Their union is a cozero set $N(g)$ and $Z(g) \cap Z$ is compact, since it is a zero set. Thus, finitely many
additional members of $\alpha$ can be chosen to complete the choice of a finite subcover of $Z$.

4.2. The finite union of small cozero sets is small.

Proof. Let $N(f)$ and $N(g)$ be small, and let $Z(h) \subset N(f) \cup N(g)$. Then $Z(h) \cap (X - N(g)) = Z(h) \cap Z(g) \subset N(f)$. Since $N(f)$ and $N(g)$ are small it follows from 4.1 that $Z(h)$ is compact.

A function $f \in C(X)$ has small support if and only if $N(f)$ is small. Equivalently, according to [4, 4E2], the function $f$ belongs to every free maximal ideal in $C(X)$. It is clear from this latter characterization that the collection $C_s(X)$ of functions with small support is an ideal; this can also be shown directly from 4.2.

Remark 4.3. The term small support may be misleading; the condition applies to $N(f)$ and not $S(f)$. The ideal $C_s(X)$ contains the ideal $C_K(X)$ [4, 4D5 and 4E2]. Spaces for which $C_K(X) = C_s(X)$ are called $\mu$-compact and are fully discussed in [9] and [11]; in [9] the ideal $C_s(X)$ is called $I(X)$.

The following result should be compared with [4, 4D1 and 4D3], as well as with Theorem 2.3.

Theorem 4.4. The space $X$ is locally compact if and only if $C_s(X)$ is not contained in any fixed maximal ideal.

Proof. If $X$ is locally compact then $C_K(X)$ is not contained in any fixed maximal ideal; since $C_K(X) \subset C_s(X)$ then $C_s(X)$ is not contained in any fixed maximal ideal.

Now if $C_s(X)$ is not contained in any fixed maximal ideal then for each $x \in X$ there is $f \in C_s(X)$ such that $x \in N(f)$. Thus, there is a zero set neighborhood of $x$ such that $Z \subset N(f)$, and it follows that $Z$ is compact. Thus $X$ is locally compact.

Remark 4.5. One sense in which Theorem 4.4 is more appropriate than the characterization [4, 4D3] of local compactness is when the generalization to $T_1$ spaces and $T_1$ compactifications is considered. In [5] the compact small sets of a space $X$ are defined as those sets such that any closed set contained in $A$ is compact. It is shown there that the spaces for which each point has a compact-small neighborhood are appropriate generalizations of locally compact completely regular spaces. It is shown in [6] that results analogous to Theorem 4.4 hold for locally compact-small spaces.

5. The local compactness of $\gamma X$ and $\psi X$. Two additional
subspaces of $\nu X$ are of special interest in connection with local compactness. Mandelker defines (in [11]) a space $X$ to be $\psi$-compact if $C_\kappa(X) = C_\psi(X)$, and Mandelker and Johnson define (in [9]) a space $X$ to be $\eta$-compact if $C_\epsilon(X) = C_\psi(X)$; they construct extensions $\eta X$ and $\psi X$ as the intersections respectively of the $\eta$-compact and the $\psi$-compact subspaces of $\beta X$.

The following results are shown in [9].

5.1. (a) $\eta X = X \cup \text{int}_\beta \nu X$.
(b) $\psi X - X = \bigcup_{f \in C_\psi(X)} [S(f^*) - S(f)]$.

The next results are immediate from 5.1(a) and Theorem 3.2.

5.2. (a) $\kappa X = \text{int}_\beta \eta X = \text{int}_\beta \psi X$.
(b) $\eta X = X \cup \kappa X$

The next theorem characterizes the local compactness of $\eta X$. The proof is immediate from 5.2 and Corollary 3.3.

**Theorem 5.3.** The space $\eta X$ is locally compact if and only if $X$ is locally pseudocompact.

**Theorem 5.4.** The space $\psi X$ is locally compact if and only if $X$ is locally pseudocompact and $C_\psi(X)$ is round.

**Proof.** Let $\psi X$ be locally compact. Then $X$ is locally pseudocompact by Corollary 3.4. Also $\psi X$ is open in $\beta X$, so $\beta X - \psi X$ is closed. By [9, 5.3], $C_\psi(X) = M_{\beta X - \psi X}$ and thus $\beta X - \psi X$ is round; hence by [10, 4.2] $C_\psi(X)$ is round.

Let $X$ be locally pseudocompact and let $C_\psi(X)$ be round. By 3.3 and 5.2(a), $X \subset \kappa X \subset \psi X$. Let $p \in \nu X - X$. Using 5.1(b) choose $f \in C_\psi(X)$ so that $p \in S(f^*)$; since $C_\psi(X)$ is round there is $g \in C_\psi(X)$ with $Zg \ll Zf$. By [4, 7.14], $\text{cl}_\beta Z(f)$ is a neighborhood of $\text{cl}_\beta Z(g)$, and thus there is a compact set $F$ with $\beta X - \text{cl}_\beta Z(f) \subset F \subset \beta X - \text{cl}_\beta Z(g)$. Since $N(f) \subset \beta X - \text{cl}_\beta Z(f)$ and (by [9, 3.1]) $\beta X - \nu X \subset \text{cl}_\beta Z(g)$, it follows that $p \in S(f^*) \subset F \subset \beta X - \text{cl}_\beta Z(g) \subset \nu X$, and hence

$$p \in \text{int}_\beta \nu X = \kappa X.$$ 

Thus $\psi X = \kappa X$ and so $\psi X$ is locally compact.

It is instructive in the use of scales to deduce directly from Condition ($RL$) that $X$ is locally pseudocompact and $C_\psi(X)$ is round.

5.5. If $X$ satisfies Condition ($RL$) then $X$ is locally pseudocompact and $C_\psi(X)$ is round.

**Proof.** The first paragraph of the proof of Theorem 2.7 shows that $X$ will be locally pseudocompact. Now suppose $f \in C_\psi(X)$. Then
$S(f)$ is pseudocompact, and it follows that every $h \in U(X)$ is bounded away from zero on $N(f)$. Choose a scale $\varepsilon$ so that $|h| \leq \varepsilon(h)$ on $N(f)$, for each $h \in U(X)$. Since (RL) is satisfied, there are $h_1, \ldots, h_k \in U(X)$ and $g \in C_\Phi(X)$ such that $Zg \cap (\bigcap U_{\varepsilon(h_i)}(h_i)) = \emptyset$. Clearly

$$Z(g) \subseteq \bigcup_{i=1}^k \{ x \in X : |h_i(x)| < \varepsilon(h_i) \} \subseteq Z(f).$$

It follows that $g \ll f$.

**References**


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