Pacific Journal of Mathematics

 π -HOMOGENEITY AND π '-CLOSURE OF FINITE GROUPS

ZVI ARAD

Vol. 51, No. 1

November 1974

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ZVI ARAD

The purpose of this paper is to present a proof, under additional conditions, of the following conjecture: Let π be a set of primes, and let all π -subgroups of G be 2-closed. (If $2 \notin \pi$, this condition is satisfied.) If G is π -homogeneous, then G is π '-closed.

All groups considered here are finite. If π is a set of prime numbers, we say that the element x of a group G is a π -element if |x| is divisible only by primes in π . In particular, one may speak of a *p*-element, p a prime. Similarly, a group G is called a π -group if |G| is divisible only by primes in π . In addition, $\pi(G)$ will denote the set of primes dividing |G|. The set of primes not in π will be denoted by π' . A group G is termed π -closed, if the subset of Gconsisting of π -elements is a subgroup of G. We say that a group G is π -homogeneous if $N_G(H)/C_G(H)$ is a π -group for every nonidentity π -subgroup H of G.

It is well known that π' -closed groups are π -homogeneous. The converse, in general, does not hold. For instance, A_5 is not 5-closed, but it is 5'-homogeneous.

For $\pi = \{p\}$, p a prime, the conjecture reduces to Frobenius' theorem ([11], Theorem 7.4.5).

The conjecture is closely connected to other well known problems in group theory. The proof of the conjecture would imply the solution of Baer's problem [3] (see also [5], p. 117), the answer to which is not known.

Baer's Problem. Let $\pi \subseteq \pi(G)$. Suppose that G is π and π' -homogeneous. Is G a direct product of a π -group and a π' -group?

In order to show the connection with Frobenius' problem, we need some additional notation. For any prime p, we denote by $|G|_p$ the highest power of the prime p that divides |G|. Define G to be weakly π -closed if for every subgroup U of G the number of π -elements of U is exactly $\prod_{p \in \pi} |U|_p$.

Baer proved that if G is weakly π -closed then G is π '-homogeneous ([2], Lemma 2). Therefore, in the case that $2 \in \pi$, the proof of the above conjecture would imply also a solution of Frobenius' problem ([2], p. 325).

ZVI ARAD

Frobenius' Problem. Let G be a weakly π -closed group. Is G π -closed?

Our first result is that the conjecture holds if $2 \in \pi$.

THEOREM A. Let π be a set of primes which includes 2. Assume that all π -subgroups of G are 2-closed. Then G is π '-closed if and only if G is π -homogeneous. (Compare with [2], Satze A, A^{*}.)

In the next omnibus theorem, $2 \notin \pi$. The proofs of Theorems B and C, as well as the proof of Corollary B, rely on the recent classification of simple 3'-groups by J. Thompson.

THEOREM B. Let π be a set of odd primes. Then G is π' -closed if G is π -homogeneous and any one of the following conditions holds: (i) $3 \notin \pi(G)$.

(ii) The π' -subgroups of G are solvable (hence if $N_G(H)$ is π' closed for every nonidentity π -subgroup of G and the π' -subgroups of G are solvable, then G is π' -closed).

(iii) G has dihedral or abelian S_2 -subgroups.

(iv) Every chain of subgroups has length at most 7.

A similar result holds if every 3rd maximal subgroup is nilpotent, or if every 2nd maximal subgroup is 2'-closed.

Theorem B (ii) together with Burnside's $p^{\alpha}q^{\beta}$ Theorem yields:

COROLLARY A. If |G| has exactly 4 prime divisors and π is a set of odd primes, then G is π' -closed if and only if G is π -homogeneous.

The proof of part (ii) of Theorem B uses the following lemma, which follows from a theorem of Baer ([11], Theorem 3.8.2).

LEMMA 2.6. If a group G is 2'-homogeneous then G is 2-closed.

We shall say that G is a D_{π} -group if all the maximal π -subgroups of G are conjugate S_{π} -subgroups of G.

We conjecture that if π is a set of primes, then D_{π} and π -homogeneity imply π '-closure. (The alternating group A_5 , for example, is 5'-homogeneous, but it is not a D_5 -group ([12], p. 143) and it is not 5'-closed.) The following theorem proves this conjecture under additional conditions. THEOREM C. If G is a D_{π} -group and π -homogeneous, then G is π' -closed if one of the following conditions holds:

- (i) $3 \notin \pi(G)$.
- (ii) The proper subgroups of G are π' -closed.

Theorems A, B, and C imply the following corollary about groups all of whose proper subgroups are π' -closed.

COROLLARY B. Let π be a set of primes. Let G be a finite group such that every proper subgroup of G is π' -closed, and assume that any one of the following conditions holds:

(i) $2 \in \pi$ and the π -subgroups of G are 2-closed.

(ii) $2 \notin \pi$ and $3 \notin \pi(G)$.

(iii) $2 \notin \pi$ and the π' -subgroups of G are solvable.

(iv) $2 \notin \pi$ and G has dihedral or abelian S_2 -subgroups.

- (v) $2 \notin \pi$ and every chain of subgroups has length at most 7.
- (vi) G is a D_{π} -group.

Then G is one of the following:

(a) G is π' -closed, or

(b) $\pi = \{p\}, p \ a \ prime, every proper subgroup of G is nilpotent, <math>|G| = p^a q^b, q \ a \ prime, the S_q$ -subgroup of G are cyclic and G is p-closed. (Compare this corollary with ([14], Chap. (iv), Satz 5.4.)

EXAMPLE. Let $\pi = \{2, 3\}$. Every proper subgroup of the alternating group A_5 is π' -closed. But A_5 is neither π' -closed nor solvable.

These results are part of the author's doctoral research at Tel-Aviv University. The author is extremely grateful to his thesis advisor, Professor M. Herzog, for his guidance and encouragement.

The author is also grateful to Dr. Avinoam Mann and Professor J. Muskat for their constructive remarks.

2. *Proofs.* We incorporate a portion of the proofs of Theorems A and B into independent lemmas.

LEMMA 2.1. Let G be either PSL $(2, r^i)$ or $S_z(q)$. Let π be a subset of $\pi(G)$ consisting of odd primes and assume $|\pi| \ge 2$. Then G is not π -homogeneous. Moreover, if P is an S_p -subgroup of PSL $(2, r^i)$ where $p \in \pi$ and $p \neq r$, or P is an S_p -subgroup of $S_z(q)$ where $p \in \pi$ then $2/|N_G(P)/C_G(P)|$.

Proof. If P is an S_p -subgroup of PSL (2, r^t), where $p \in \pi$ and $p \neq r$, then it is well known that $2/|N_G(P)/C_G(P)|$. Therefore, PSL (2, r^t) is not π -homogeneous.

ZVI ARAD

It follows by Theorem 4, Proposition 16, and Theorem 9 of [17] that in $S_z(q)$, $2/|N_G(H)/C_G(H)|$ for every nonidentity subgroup H of $S_z(q)$ of odd order.

The following four basic results concerning π -homogeneous groups were proved in [1].

LEMMA 2.2 ([1], Lemma 2.3). Subgroups, direct products, and epimorphic images of π -homogeneous groups are π -homogeneous.

LEMMA 2.3 ([1], Lemma 2.4). If K is a normal subgroup of the π' -homogeneous group G, and if K and G/K are π -closed, then G is π -closed.

LEMMA 2.4 ([1], Theorem 2.5). The group G is π -closed if, and only if, G is π -separable and π' -homogeneous.

LEMMA 2.5 ([1], Lemma 2.1). π -closed groups are π' -homogeneous.

We now obtain at once

LEMMA 2.6. If a group G is 2'-homogeneous then G is 2-closed.

Proof. Let G be a minimal counterexample. Lemmas 2.2 and 2.3 imply that G is a nonabelian simple group. Let K be the conjugate class of an involution u of G; obviously |K| > 1. Then by Theorem 3.8.2 of [11] there exists $v \in K$, $v \neq u$, such that uv is not a 2-element. If $|uv| = 2^k m$, m > 1 odd, set $t = (uv)^{2^k}$; then |t| = m > 1 is odd. Now $t^u = t^{-1}$; therefore, $N_G(\langle t \rangle)/C_G(\langle t \rangle)$ is not a 2'-group. Hence G is not 2'-homogeneous, a contradiction.

Proof of Theorem A. If G is π' -closed, then without any assumption on π G is π -homogeneous by Lemma 2.5. Therefore, we will prove here that, under the assumptions of Theorem A, if G is π -homogeneous then G is π' -closed. Let $\pi_1 = \pi \cap \pi(G)$. If $2 \notin \pi(G)$ then Lemma 2.4 and [8] imply that G is π' -closed. If $\pi_1 = \{2\}$ this is Frobenius' theorem. Let G be a minimal counterexample. Then G has the following properties:

- (a) G is π_1 -homogeneous, $2 \in \pi_1$ and $|\pi_1| \ge 2$.
- (b) The π_1 -subgroups of G are 2-closed.
- (c) G is not π'_1 -closed.

For the remainder of the proof we shall denote π_1 by π . Lemma 2.2 implies that subgroups and epimorphic images of G are π -homogeneous. Clearly π -subgroups of subgroups of G are 2-closed. Therefore we also have:

(d) Proper subgroups of G are π' -closed (hence solvable, by [8]). We want to prove

(e) G is simple.

Suppose not, and let N be a minimal normal subgroup of G. Since by (d) N is solvable, N is a p-group. If $p \in \pi$ and K/N is a π -subgroup of G/N, then K is a π -subgroup of G. Therefore, the π -subgroups of G/N are 2-closed. G/N is π' -closed, by induction. By Lemma 2.3, G is π' -closed, a contradiction. Assume now that $p \notin \pi$. If K/N is a π -subgroup of G/N, then by the Schur-Zassenhaus theorem $K = K_{\pi}N$ where K_{π} is an S_{π} -subgroup of K. Therefore, K/N has a normal S_2 -subgroup. By induction G/N, and hence G, are π' -closed, a contradiction. Hence G is simple.

Moreover, by (d) G is a minimal simple group. By [21] G is one of the following:

(1) $PSL_2(2^p)$ where p is any prime.

(2) $PSL_2(3^p)$ where p > 2 is any prime.

(3) $\mathrm{PSL}_2(p)$ where p is any prime with p>3, and $p\equiv 2$ or 3 (mod 5).

(4) $S_z(2^p)$ where p is any odd prime.

(5) PSL₃(3).

If G is a group of type (1) or (4), then for $q \in \pi$, q odd $(|\pi| \ge 2)$, there exist Q, a q-subgroup of G, and a 2-element u of G, such that $u \in N_G(Q)$ but $u \notin C_G(Q)$, by Lemma 2.1. Now $T = \langle u \rangle Q$ is a non 2closed π -group, a contradiction.

If G is $PSL_2(r^t)$ of type (2) or (3) and π contains a prime $u \neq r$, 2, then again Lemma 2.1 yields a contradiction. Hence $\pi = \{2, r\}$. Let R be an S_r -subgroup of G. It is well known that $C_G(R) = R$ and that $|N_G(R)| = 1/2(r^t - 1)|R|$. Since G is π -homogeneous we obtain that $1/2(r^t - 1) = 2^{\alpha}$ and therefore $N_G(R)$ is a π -subgroup of G. By assumption $N_G(R)$ is 2-closed, a contradiction.

If G is $PSL_3(3)$, then $\pi(G) = \{2, 3, 13\}$. If $\pi = \{2, 13\}$ then ([14], Satz 7.3, p. 187) implies that $3/|N_G(P)/C_G(P)|$, where P is an S_{13} subgroup of G. Hence G is not π -homogeneous, a contradiction. If G is isomorphic to $PSL_3(3)$ and $\pi = \{2, 3\}$, then a study of the character table of $PSL_3(3)$ implies the existence of a subgroup K of order 54 in $PSL_3(3)$ which is not 2-closed, in contradiction to (b). The proof of Theorem A is now complete.

Before beginning the proof of Theorem B we need several definitions.

A chain of subgroups of G is a set of subgroups of G linearly ordered by inclusion:

$$G = G_0 \supset G_1 \supset \cdots \supset G_k \supset \cdots \supset 1$$
.

The length of a chain is the number of its distinct terms, minus 1.

ZVI ARAD

A subgroup G_k of G is kth maximal if it is the kth term in some chain of proper subgroups, each of which is maximal in its predecessor and k is the smallest such integer.

Proof of Theorem B. Let G be a minimal counterexample.

Proof of (i). Lemmas 2.2 and 2.3 imply that G is simple. By Thompson's classification of simple 3'-groups G isomorphic to $S_z(q)$. Therefore, Lemma 2.1 implies that G is not π -homogeneous, a contradiction.

Proof of (ii). G has the following properties:

(a) G is π -homogeneous, $2 \notin \pi$ and $|\pi \cap \pi(G)| \ge 2$.

(b) The π' -subgroups of G are solvable.

(c) G is not π' -closed.

Lemma 2.2 implies that subgroups and epimorphic images of G are π -homogeneous. Clearly subgroups of G have solvable π' -subgroups. Therefore we also have:

(d) Proper subgroups of G are π -closed (hence solvable, by [8]). We want to prove:

(e) G is simple.

Suppose not, and let N be a minimal normal subgroup of G. Since by (d) N is solvable, N is a p-group. If $p \in \pi'$ and K/N is a π' -subgroup of G/N, then K is a π' -subgroup, so that K is solvable, by hypothesis. Thus K/N is solvable. If $p \in \pi$ and K/N is a π' -subgroup of G/N, then by the Schur-Zassenhaus theorem $K = NK_{\pi'}$ where $K_{\pi'}$ is an $S_{\pi'}$ -subgroup of K. By assumption K/N is solvable. Therefore, G/N has solvable π' -subgroups. By induction G/N, and hence G (by Lemma 2.3), are π' -closed, a contradiction. Hence G is simple. Moreover, by (d) G is a minimal simple group. By [21] G is of one of the 5 types mentioned in the proof of Theorem A.

Lemma 2.1 implies that G is not of type (1), (2), (3) or (4). Frobenius' theorem and Lemma 2.6 imply that G is not $PSL_3(3)$, since $|PSL_3(3)|$ has only 3 prime divisors, a contradiction.

Now, if $N = N_G(H)$ is π' -closed, for any π -subgroup $H \neq 1$ of G, then $N/C_G(H)$ is a π -group. Hence by the preceding paragraph G is π' -closed.

We now obtain at once

Proof of Corollary A. If |G| has only 4 prime divisors; then Frobenius' theorem, Lemma 2.6, and Theorem B (ii), together with Burnside's $p^{\alpha}q^{\beta}$ theorem, yield that G is π' -closed.

We return to the proof of Theorem B.

Proof of (iii). Let G have a dihedral S_2 -subgroup. If there exists $1 \neq N \triangleleft G$, then the S_2 -subgroups of N are of one of the following types: dihedral, cyclic or trivial. In the first case N is π' -closed by induction, in the second case N is 2'-closed and in the third N is solvable by [8]. Lemma 2.4 then implies that in every case N is π' -closed. Similarly G/N is also π' -closed. Therefore, Lemma 2.3 implies that G is π' -closed, a contradiction. Hence G is simple. By Theorem 16.3 of [11] G is isomorphic to either PSL (2, q), q odd, q > 3, or to A_7 . Lemma 2.1 implies that G is isomorphic to A_7 . But $|A_7|$ has only 4 prime divisors, therefore, Corollary A implies that G is π' -closed, a contradiction.

Let G have abelian S_2 -subgroups. Clearly G is simple. Walter [18, 19] proved that one of the following holds:

- (1) G is isomorphic to $L_2(q)$, q > 3, $q \equiv 3, 5 \pmod{8}$ or $q = 2^n$;
- (2) G is isomorphic to J(11); or
- (3) G is of Ree type.

Lemma 2.1 eliminates the first possibility. Now J(11) is of order $2^{3} \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 19$. If P is an S_{p} -subgroup of J(11) for p = 3, 5, 7, 11, 19, then 2/|N(P)/C(P)| by [15]. Hence J(11) is not π -homogeneous, so that G must be of Ree type. Then G is of order $q^3(q-1)(q+1)$ (q^2-q+1) where $q=3^{2k+1}, k\geq 1$. If $3\in\pi$ and P is an S_3 -subgroup of G, then N(P) = PW, where W is cyclic of order q - 1. Now if J is the involution of W, then $J \notin C(P)$. Hence if $3 \in \pi$ then G is not π -homogeneous. We know also [20] that G possesses Abelian Hall subgroups M^+ and M^- of orders q + 1 + 3m and q + 1 - 3m, where $m = 3^k$ and $q^2 - q + 1 = (q + 1 + 3m)(q + 1 - 3m)$. If t is a prime such that either $t/|M^+|$ or $t/|M^-|$ and T is an S_t -subgroup of M^{\pm} , then $N(T) \supseteq N(M^{\pm}) = M^{\pm}W^{\pm}$, where W^{\pm} are cyclic of order 6. But $C(T) = M^{\pm}$. Hence if $t \in \pi$ then G is not π -homogeneous. Now by the definition of G [20] there exist cyclic subgroups R^{\pm} of order $1/2(q \pm 1)$. The normalizer $N_{g}(R_{0})$ of any subgroup $R_{0} \neq 1$ of R^{\pm} is contained in $\langle J \rangle imes L_2(q)$, where J is an involution of G. If R_0 is of odd order then $R_0 \subseteq L_2(q)$ and $2/|N_G(R_0)/C_G(R_0)|$. Therefore, if π contains of primes dividing either q + 1 or q - 1, then G is not π -homogeneous. Since $|G| = q^{3}(q-1)(q+1)(q^{2}-q+1)$ where $q = 3^{2k+1}, k \ge 1$ 1, (iii) follows.

Proof of (iv). Lemmas 2.2 and 2.3 imply that G is simple. Gagen's theorem [9] and Harada's theorem [13] imply that G is isomorphic to one of the following groups: PSU_3 (3), PSU_3 (5), A_7 , M_{11} , J(11), or PSL (2, q), for certain values of q. The last possibility is eliminated by Lemma 2.1. In the proof of (iii) we found that J(11)is not π -homogeneous. Since the remaining groups have orders with at most 4 prime divisors, they are π' -closed, by Corollary A and Lemma 2.6.

Proof of Theorem C. Let G be a minimal counterexample. In both cases Lemmas 2.2, 2.3, and ([14], Chap. (iv), Hilf. 7.2, p. 444) imply that G is simple. Therefore, if (i) $3 \notin \pi(G)$ then, assuming Thompson's classification of simple 3'-groups, G is isomorphic to $S_z(q)$. If in addition $2 \notin \pi$ then Theorem B implies that G is π' -closed, a contradiction. If $2 \in \pi$ then Theorem 9 of [17] implies that G is not a D_{π} -group, again a contradiction. In case (ii) Theorem 3.1 of [7] implies that G is π' -closed. This contradiction completes the proof of Theorem C.

It is well known that if every proper subgroup of G is p'-closed but G is not p'-closed, then every proper subgroup of G is nilpotent, $|G| = p^{\alpha}q^{\beta}$, q a prime, and the S_q -subgroups of G are cyclic (see [14], Chap. (iv), Satz 5.4, p. 434).

Theorems A, B, and C imply the same conclusion under additional conditions for groups every proper subgroup of which is π' -closed.

Proof of Corollary B. Let G be a minimal counterexample. If G is not π' -closed, then Theorems A, B, and C imply that there exist S, a π -subgroup of G, and x, a π' -element of G, such that $x \in N_G(S)$ but $x \notin C_G(S)$. Therefore, Theorem 6.2.2 of [11] implies that there exists a prime p in π and P, an S_p -subgroup of S, such that $x \in N_G(P)$ but $x \notin C_G(P)$. Set $T = P \langle x \rangle$. If $T \subset G$, then by hypothesis $T = P \times$ $\langle x \rangle$ and $x \in C_G(P)$, a contradiction. If $T = G = P \langle x \rangle$, then every proper subgroup of G is by hypothesis p'-closed, but G itself is not p'-closed. Hence ([14], Chap. (iv), Satz 5.4, p. 434) implies (b).

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Received December 12, 1972 and in revised form August 23, 1973.

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PACIFIC JOURNAL OF MATHEMATICS

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Printed in Japan by Intarnational Academic Printing Co., Ltd., Tokyo, Japan

R. A. BEAUMONT University of Washington Seattle, Washington 98105

Pacific Journal of Mathematics Vol. 51, No. 1 November, 1974

Zvi Arad, π -homogeneity and π' -closure of finite groups	1
Ivan Baggs, A connected Hausdorff space which is not contained in a maximal connected space	11
Eric Bedford, The Dirichlet problem for some overdetermined systems on the unit ball in C^n	19
R. H. Bing, Woodrow Wilson Bledsoe and R. Daniel Mauldin, Sets generated by rectangles	27
Carlo Cecchini and Alessandro Figà-Talamanca, <i>Projections of uniqueness for</i> $L^{p}(G)$	37
Gokulananda Das and Ram N. Mohapatra, <i>The non absolute Nörlund summability of</i> Fourier series	49
Frank Rimi DeMever. On separable polynomials over a commutative ring	57
Richard Detmer. Sets which are tame in arcs in F^3	67
William Frh Dietrich Ideals in convolution algebras on Abelian groups	75
Bryce I Elkins A Galois theory for linear topological rings	89
William Alan Feldman, A characterization of the topology of compact convergence on $C(X)$	100
Hillel Halkin Gershenson, A problem in compact Lie groups and framed	109
Cobolaism	121
Mamin L. Croonbarg. Stuigtly logal solutions of Displanting soughting	131
Infarvin J. Greenberg, Strictly local solutions of Diophantine equations	143
Domingo Antonio Horroro, Inner functione un den uniform topology	155
Johningo Antonio Henero, Inner junctions under uniform topology	107
Marvin Stanford Keeper, Oscillatory solutions and multi point boundary value	1//
functions for certain nth-order linear ordinary differential equations	187
John Cronan Kieffer, A simple proof of the Moy-Perez generalization of the Shannon-McMillan theorem	203
Joong Ho Kim, <i>Power invariant rings</i>	207
Gangaram S. Ladde and V. Lakshmikantham, On flow-invariant sets	215
Roger T. Lewis, Oscillation and nonoscillation criteria for some self-adjoint even order linear differential operators	221
Jürg Thomas Marti, On the existence of support points of solid convex sets	235
John Rowlay Martin, Determining knot types from diagrams of knots	241
James Jerome Metzger, Local ideals in a topological algebra of entire functions	251
K. C. O'Moore, Intrinsic artensions of prime rings	251
K. C. O Meara, Intrinsic extensions of prime rings	237
Statiley Foreda, A note on the continuity of best polynomial approximations	271
of mans	273
Fric Peter Smith The Garabedian function of an arbitrary compact set	275
Arne Stray, Pointwise bounded approximation by functions satisfying a side	207
	301
Jonn St. Clair Werth, Jr., <i>Maximal pure subgroups of torsion complete abelian</i> <i>p-groups</i>	307
Robert S. Wilson, On the structure of finite rings. II	317
Kari Ylinen, The multiplier algebra of a convolution measure algebra	327