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π -HOMOGENEITY AND π' -CLOSURE OF FINITE GROUPS

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The purpose of this paper is to present a proof, under additional conditions, of the following conjecture: Let π be a set of primes, and let all π -subgroups of G be 2-closed. (If $2 \notin \pi$, this condition is satisfied.) If G is π -homogeneous, then G is π' -closed.

All groups considered here are *finite*. If π is a set of prime numbers, we say that the element x of a group G is a π -*element* if $|x|$ is divisible only by primes in π . In particular, one may speak of a p -*element*, p a prime. Similarly, a group G is called a π -*group* if $|G|$ is divisible only by primes in π . In addition, $\pi(G)$ will denote the set of primes dividing $|G|$. The set of primes not in π will be denoted by π' . A group G is termed π -*closed*, if the subset of G consisting of π -elements is a subgroup of G . We say that a group G is π -*homogeneous* if $N_G(H)/C_G(H)$ is a π -group for every nonidentity π -subgroup H of G .

It is well known that π' -closed groups are π -homogeneous. The converse, in general, does not hold. For instance, A_5 is not 5-closed, but it is 5'-homogeneous.

For $\pi = \{p\}$, p a prime, the conjecture reduces to Frobenius' theorem ([11], Theorem 7.4.5).

The conjecture is closely connected to other well known problems in group theory. The proof of the conjecture would imply the solution of Baer's problem [3] (see also [5], p. 117), the answer to which is not known.

Baer's Problem. Let $\pi \subseteq \pi(G)$. Suppose that G is π and π' -homogeneous. Is G a direct product of a π -group and a π' -group?

In order to show the connection with Frobenius' problem, we need some additional notation. For any prime p , we denote by $|G|_p$ the highest power of the prime p that divides $|G|$. Define G to be *weakly π -closed* if for every subgroup U of G the number of π -elements of U is exactly $\prod_{p \in \pi} |U|_p$.

Baer proved that if G is weakly π -closed then G is π' -homogeneous ([2], Lemma 2). Therefore, in the case that $2 \in \pi$, the proof of the above conjecture would imply also a solution of Frobenius' problem ([2], p. 325).

Frobenius' Problem. Let G be a weakly π -closed group. Is G π -closed?

Our first result is that the conjecture holds if $2 \in \pi$.

THEOREM A. Let π be a set of primes which includes 2. Assume that all π -subgroups of G are 2-closed. Then G is π' -closed if and only if G is π -homogeneous. (Compare with [2], Satze A, A*.)

In the next omnibus theorem, $2 \notin \pi$. The proofs of Theorems B and C, as well as the proof of Corollary B, rely on the recent classification of simple 3'-groups by J. Thompson.

THEOREM B. Let π be a set of odd primes. Then G is π' -closed if G is π -homogeneous and any one of the following conditions holds:

(i) $3 \notin \pi(G)$.

(ii) The π' -subgroups of G are solvable (hence if $N_G(H)$ is π' -closed for every nonidentity π -subgroup of G and the π' -subgroups of G are solvable, then G is π' -closed).

(iii) G has dihedral or abelian S_2 -subgroups.

(iv) Every chain of subgroups has length at most 7.

A similar result holds if every 3rd maximal subgroup is nilpotent, or if every 2nd maximal subgroup is 2'-closed.

Theorem B (ii) together with Burnside's $p^a q^b$ Theorem yields:

COROLLARY A. If $|G|$ has exactly 4 prime divisors and π is a set of odd primes, then G is π' -closed if and only if G is π -homogeneous.

The proof of part (ii) of Theorem B uses the following lemma, which follows from a theorem of Baer ([11], Theorem 3.8.2).

LEMMA 2.6. If a group G is 2'-homogeneous then G is 2-closed.

We shall say that G is a D_π -group if all the maximal π -subgroups of G are conjugate S_π -subgroups of G .

We conjecture that if π is a set of primes, then D_π and π -homogeneity imply π' -closure. (The alternating group A_5 , for example, is 5'-homogeneous, but it is not a D_5 -group ([12], p. 143) and it is not 5'-closed.) The following theorem proves this conjecture under additional conditions.

THEOREM C. *If G is a D_π -group and π -homogeneous, then G is π' -closed if one of the following conditions holds:*

- (i) $3 \notin \pi(G)$.
- (ii) *The proper subgroups of G are π' -closed.*

Theorems A, B, and C imply the following corollary about groups all of whose proper subgroups are π' -closed.

COROLLARY B. *Let π be a set of primes. Let G be a finite group such that every proper subgroup of G is π' -closed, and assume that any one of the following conditions holds:*

- (i) $2 \in \pi$ and the π -subgroups of G are 2-closed.
- (ii) $2 \notin \pi$ and $3 \notin \pi(G)$.
- (iii) $2 \notin \pi$ and the π' -subgroups of G are solvable.
- (iv) $2 \notin \pi$ and G has dihedral or abelian S_2 -subgroups.
- (v) $2 \notin \pi$ and every chain of subgroups has length at most 7.
- (vi) G is a D_π -group.

Then G is one of the following:

- (a) G is π' -closed, or
 - (b) $\pi = \{p\}$, p a prime, every proper subgroup of G is nilpotent, $|G| = p^a q^b$, q a prime, the S_q -subgroup of G are cyclic and G is p -closed.
- (Compare this corollary with ([14], Chap. (iv), Satz 5.4.)

EXAMPLE. Let $\pi = \{2, 3\}$. Every proper subgroup of the alternating group A_5 is π' -closed. But A_5 is neither π' -closed nor solvable.

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2. *Proofs.* We incorporate a portion of the proofs of Theorems A and B into independent lemmas.

LEMMA 2.1. *Let G be either $\text{PSL}(2, r^t)$ or $S_2(q)$. Let π be a subset of $\pi(G)$ consisting of odd primes and assume $|\pi| \geq 2$. Then G is not π -homogeneous. Moreover, if P is an S_p -subgroup of $\text{PSL}(2, r^t)$ where $p \in \pi$ and $p \neq r$, or P is an S_p -subgroup of $S_2(q)$ where $p \in \pi$ then $2 \mid |N_G(P)/C_G(P)|$.*

Proof. If P is an S_p -subgroup of $\text{PSL}(2, r^t)$, where $p \in \pi$ and $p \neq r$, then it is well known that $2 \mid |N_G(P)/C_G(P)|$. Therefore, $\text{PSL}(2, r^t)$ is not π -homogeneous.

It follows by Theorem 4, Proposition 16, and Theorem 9 of [17] that in $S_z(q)$, $2/|N_G(H)/C_G(H)|$ for every nonidentity subgroup H of $S_z(q)$ of odd order.

The following four basic results concerning π -homogeneous groups were proved in [1].

LEMMA 2.2 ([1], Lemma 2.3). *Subgroups, direct products, and epimorphic images of π -homogeneous groups are π -homogeneous.*

LEMMA 2.3 ([1], Lemma 2.4). *If K is a normal subgroup of the π' -homogeneous group G , and if K and G/K are π -closed, then G is π -closed.*

LEMMA 2.4 ([1], Theorem 2.5). *The group G is π -closed if, and only if, G is π -separable and π' -homogeneous.*

LEMMA 2.5 ([1], Lemma 2.1). *π -closed groups are π' -homogeneous.*

We now obtain at once

LEMMA 2.6. *If a group G is $2'$ -homogeneous then G is 2-closed.*

Proof. Let G be a minimal counterexample. Lemmas 2.2 and 2.3 imply that G is a nonabelian simple group. Let K be the conjugate class of an involution u of G ; obviously $|K| > 1$. Then by Theorem 3.8.2 of [11] there exists $v \in K$, $v \neq u$, such that uv is not a 2-element. If $|uv| = 2^k m$, $m > 1$ odd, set $t = (uv)^{2^k}$; then $|t| = m > 1$ is odd. Now $t^u = t^{-1}$; therefore, $N_G(\langle t \rangle)/C_G(\langle t \rangle)$ is not a $2'$ -group. Hence G is not $2'$ -homogeneous, a contradiction.

Proof of Theorem A. If G is π' -closed, then without any assumption on π G is π -homogeneous by Lemma 2.5. Therefore, we will prove here that, under the assumptions of Theorem A, if G is π -homogeneous then G is π' -closed. Let $\pi_1 = \pi \cap \pi(G)$. If $2 \notin \pi(G)$ then Lemma 2.4 and [8] imply that G is π' -closed. If $\pi_1 = \{2\}$ this is Frobenius' theorem. Let G be a minimal counterexample. Then G has the following properties:

- (a) G is π_1 -homogeneous, $2 \in \pi_1$ and $|\pi_1| \geq 2$.
- (b) The π_1 -subgroups of G are 2-closed.
- (c) G is not π'_1 -closed.

For the remainder of the proof we shall denote π_1 by π . Lemma 2.2 implies that subgroups and epimorphic images of G are π -homogeneous. Clearly π -subgroups of subgroups of G are 2-closed. Therefore we also have:

(d) Proper subgroups of G are π' -closed (hence solvable, by [8]).

We want to prove

(e) G is simple.

Suppose not, and let N be a minimal normal subgroup of G . Since by (d) N is solvable, N is a p -group. If $p \in \pi$ and K/N is a π -subgroup of G/N , then K is a π -subgroup of G . Therefore, the π -subgroups of G/N are 2-closed. G/N is π' -closed, by induction. By Lemma 2.3, G is π' -closed, a contradiction. Assume now that $p \notin \pi$. If K/N is a π -subgroup of G/N , then by the Schur-Zassenhaus theorem $K = K_\pi N$ where K_π is an S_π -subgroup of K . Therefore, K/N has a normal S_2 -subgroup. By induction G/N , and hence G , are π' -closed, a contradiction. Hence G is simple.

Moreover, by (d) G is a minimal simple group. By [21] G is one of the following:

(1) $\text{PSL}_2(2^p)$ where p is any prime.

(2) $\text{PSL}_2(3^p)$ where $p > 2$ is any prime.

(3) $\text{PSL}_2(p)$ where p is any prime with $p > 3$, and $p \equiv 2$ or $3 \pmod{5}$.

(4) $S_z(2^p)$ where p is any odd prime.

(5) $\text{PSL}_3(3)$.

If G is a group of type (1) or (4), then for $q \in \pi$, q odd ($|\pi| \geq 2$), there exist Q , a q -subgroup of G , and a 2-element u of G , such that $u \in N_G(Q)$ but $u \notin C_G(Q)$, by Lemma 2.1. Now $T = \langle u \rangle Q$ is a non 2-closed π -group, a contradiction.

If G is $\text{PSL}_2(r^t)$ of type (2) or (3) and π contains a prime $u \neq r, 2$, then again Lemma 2.1 yields a contradiction. Hence $\pi = \{2, r\}$. Let R be an S_r -subgroup of G . It is well known that $C_G(R) = R$ and that $|N_G(R)| = 1/2(r^t - 1)|R|$. Since G is π -homogeneous we obtain that $1/2(r^t - 1) = 2^\alpha$ and therefore $N_G(R)$ is a π -subgroup of G . By assumption $N_G(R)$ is 2-closed, a contradiction.

If G is $\text{PSL}_3(3)$, then $\pi(G) = \{2, 3, 13\}$. If $\pi = \{2, 13\}$ then ([14], Satz 7.3, p. 187) implies that $3 \mid |N_G(P)/C_G(P)|$, where P is an S_{13} -subgroup of G . Hence G is not π -homogeneous, a contradiction. If G is isomorphic to $\text{PSL}_3(3)$ and $\pi = \{2, 3\}$, then a study of the character table of $\text{PSL}_3(3)$ implies the existence of a subgroup K of order 54 in $\text{PSL}_3(3)$ which is not 2-closed, in contradiction to (b). The proof of Theorem A is now complete.

Before beginning the proof of Theorem B we need several definitions.

A chain of subgroups of G is a set of subgroups of G linearly ordered by inclusion:

$$G = G_0 \supset G_1 \supset \cdots \supset G_k \supset \cdots \supset 1.$$

The length of a chain is the number of its distinct terms, minus 1.

A subgroup G_k of G is k th maximal if it is the k th term in some chain of proper subgroups, each of which is maximal in its predecessor and k is the smallest such integer.

Proof of Theorem B. Let G be a minimal counterexample.

Proof of (i). Lemmas 2.2 and 2.3 imply that G is simple. By Thompson's classification of simple 3'-groups G isomorphic to $S_z(q)$. Therefore, Lemma 2.1 implies that G is not π -homogeneous, a contradiction.

Proof of (ii). G has the following properties:

- (a) G is π -homogeneous, $2 \notin \pi$ and $|\pi \cap \pi(G)| \geq 2$.
- (b) The π' -subgroups of G are solvable.
- (c) G is not π' -closed.

Lemma 2.2 implies that subgroups and epimorphic images of G are π -homogeneous. Clearly subgroups of G have solvable π' -subgroups. Therefore we also have:

- (d) Proper subgroups of G are π -closed (hence solvable, by [8]).

We want to prove:

- (e) G is simple.

Suppose not, and let N be a minimal normal subgroup of G . Since by (d) N is solvable, N is a p -group. If $p \in \pi'$ and K/N is a π' -subgroup of G/N , then K is a π' -subgroup, so that K is solvable, by hypothesis. Thus K/N is solvable. If $p \in \pi$ and K/N is a π' -subgroup of G/N , then by the Schur-Zassenhaus theorem $K = NK_{\pi'}$, where $K_{\pi'}$ is an $S_{\pi'}$ -subgroup of K . By assumption K/N is solvable. Therefore, G/N has solvable π' -subgroups. By induction G/N , and hence G (by Lemma 2.3), are π' -closed, a contradiction. Hence G is simple. Moreover, by (d) G is a minimal simple group. By [21] G is of one of the 5 types mentioned in the proof of Theorem A.

Lemma 2.1 implies that G is not of type (1), (2), (3) or (4). Frobenius' theorem and Lemma 2.6 imply that G is not $\text{PSL}_3(3)$, since $|\text{PSL}_3(3)|$ has only 3 prime divisors, a contradiction.

Now, if $N = N_G(H)$ is π' -closed, for any π -subgroup $H \neq 1$ of G , then $N/C_G(H)$ is a π -group. Hence by the preceding paragraph G is π' -closed.

We now obtain at once

Proof of Corollary A. If $|G|$ has only 4 prime divisors; then Frobenius' theorem, Lemma 2.6, and Theorem B (ii), together with Burnside's $p^a q^b$ theorem, yield that G is π' -closed.

We return to the proof of Theorem B.

Proof of (iii). Let G have a dihedral S_2 -subgroup. If there exists $1 \neq N \triangleleft G$, then the S_2 -subgroups of N are of one of the following types: dihedral, cyclic or trivial. In the first case N is π' -closed by induction, in the second case N is $2'$ -closed and in the third N is solvable by [8]. Lemma 2.4 then implies that in every case N is π' -closed. Similarly G/N is also π' -closed. Therefore, Lemma 2.3 implies that G is π' -closed, a contradiction. Hence G is simple. By Theorem 16.3 of [11] G is isomorphic to either $\text{PSL}(2, q)$, q odd, $q > 3$, or to A_7 . Lemma 2.1 implies that G is isomorphic to A_7 . But $|A_7|$ has only 4 prime divisors, therefore, Corollary A implies that G is π' -closed, a contradiction.

Let G have abelian S_2 -subgroups. Clearly G is simple. Walter [18, 19] proved that one of the following holds:

- (1) G is isomorphic to $L_2(q)$, $q > 3$, $q \equiv 3, 5 \pmod{8}$ or $q = 2^n$;
- (2) G is isomorphic to $J(11)$; or
- (3) G is of Ree type.

Lemma 2.1 eliminates the first possibility. Now $J(11)$ is of order $2^3 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 19$. If P is an S_p -subgroup of $J(11)$ for $p = 3, 5, 7, 11, 19$, then $2 \mid |N(P)/C(P)|$ by [15]. Hence $J(11)$ is not π -homogeneous, so that G must be of Ree type. Then G is of order $q^3(q-1)(q+1)(q^2-q+1)$ where $q = 3^{2k+1}$, $k \geq 1$. If $3 \in \pi$ and P is an S_3 -subgroup of G , then $N(P) = PW$, where W is cyclic of order $q-1$. Now if J is the involution of W , then $J \notin C(P)$. Hence if $3 \in \pi$ then G is not π -homogeneous. We know also [20] that G possesses Abelian Hall subgroups M^+ and M^- of orders $q+1+3m$ and $q+1-3m$, where $m = 3^k$ and $q^2 - q + 1 = (q+1+3m)(q+1-3m)$. If t is a prime such that either $t \mid |M^+|$ or $t \mid |M^-|$ and T is an S_t -subgroup of M^\pm , then $N(T) \cong N(M^\pm) = M^\pm W^\pm$, where W^\pm are cyclic of order 6. But $C(T) = M^\pm$. Hence if $t \in \pi$ then G is not π -homogeneous. Now by the definition of G [20] there exist cyclic subgroups R^\pm of order $1/2(q \pm 1)$. The normalizer $N_G(R_0)$ of any subgroup $R_0 \neq 1$ of R^\pm is contained in $\langle J \rangle \times L_2(q)$, where J is an involution of G . If R_0 is of odd order then $R_0 \cong L_2(q)$ and $2 \mid |N_G(R_0)/C_G(R_0)|$. Therefore, if π contains of primes dividing either $q+1$ or $q-1$, then G is not π -homogeneous. Since $|G| = q^3(q-1)(q+1)(q^2-q+1)$ where $q = 3^{2k+1}$, $k \geq 1$, (iii) follows.

Proof of (iv). Lemmas 2.2 and 2.3 imply that G is simple. Gagen's theorem [9] and Harada's theorem [13] imply that G is isomorphic to one of the following groups: $\text{PSU}_3(3)$, $\text{PSU}_3(5)$, A_7 , M_{11} , $J(11)$, or $\text{PSL}(2, q)$, for certain values of q . The last possibility is eliminated by Lemma 2.1. In the proof of (iii) we found that $J(11)$ is not π -homogeneous. Since the remaining groups have orders with at most 4 prime divisors, they are π' -closed, by Corollary A and

Lemma 2.6.

Proof of Theorem C. Let G be a minimal counterexample. In both cases Lemmas 2.2, 2.3, and ([14], Chap. (iv), Hilf. 7.2, p. 444) imply that G is simple. Therefore, if (i) $3 \in \pi(G)$ then, assuming Thompson's classification of simple 3'-groups, G is isomorphic to $S_z(q)$. If in addition $2 \in \pi$ then Theorem B implies that G is π' -closed, a contradiction. If $2 \in \pi$ then Theorem 9 of [17] implies that G is not a D_π -group, again a contradiction. In case (ii) Theorem 3.1 of [7] implies that G is π' -closed. This contradiction completes the proof of Theorem C.

It is well known that if every proper subgroup of G is p' -closed but G is not p' -closed, then every proper subgroup of G is nilpotent, $|G| = p^\alpha q^2$, q a prime, and the S_q -subgroups of G are cyclic (see [14], Chap. (iv), Satz 5.4, p. 434).

Theorems A, B, and C imply the same conclusion under additional conditions for groups every proper subgroup of which is π' -closed.

Proof of Corollary B. Let G be a minimal counterexample. If G is not π' -closed, then Theorems A, B, and C imply that there exist S , a π -subgroup of G , and x , a π' -element of G , such that $x \in N_G(S)$ but $x \notin C_G(S)$. Therefore, Theorem 6.2.2 of [11] implies that there exists a prime p in π and P , an S_p -subgroup of S , such that $x \in N_G(P)$ but $x \notin C_G(P)$. Set $T = P\langle x \rangle$. If $T \subset G$, then by hypothesis $T = P \times \langle x \rangle$ and $x \in C_G(P)$, a contradiction. If $T = G = P\langle x \rangle$, then every proper subgroup of G is by hypothesis p' -closed, but G itself is not p' -closed. Hence ([14], Chap. (iv), Satz 5.4, p. 434) implies (b).

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