DETERMINING KNOT TYPES FROM DIAGRAMS OF KNOTS

JOHN ROWLAY MARTIN
DETERMINING KNOT TYPES FROM DIAGRAMS
OF KNOTS

JOHN R. MARTIN

The word of knot and the characteristics of its double points, both of which may be read from the diagram of knot, are used to give necessary and sufficient conditions for two (oriented) knots to belong to the same (oriented) knot type.

1. Introduction. A knot is a circle imbedded as a polygon in 3-dimensional space $\mathbb{R}^3$. Two knots $K$ and $L$ are called equivalent if there is an autohomoeomorphism $h$ of $\mathbb{R}^3$ such that $h(K) = L$. Each equivalence class is called a knot type. For oriented knots a stronger equivalence may be defined: Two oriented knots $K$ and $L$ are called O-equivalent if there is an orientation preserving autohomoeomorphism $h$ of $\mathbb{R}^3$ such that $h$ maps $K$ onto $L$ so their orientations match. In this case each equivalence class is called an oriented knot type.

In [8] D. E. Penney uses the diagram of a knot to define a “word” for the knot and obtain sufficient conditions for two knots to belong to the same knot type. Penney’s results have been generalized by L. B. Treybig in [12] where the concept of the “boundary collection” of a knot is used to give necessary conditions for two knots to belong to the same knot type. The purpose of this paper is to use the word of a knot and the characteristics of its double points, both of which may be read from the diagram of a knot, to give necessary and sufficient conditions for two (oriented) knots to belong to the same (oriented) knot type.

The preliminaries needed for our main results are given in §2. In §3 a relationship between the word of a knot and the characteristics of its double points is derived. This relationship is used to obtain Theorem 3.4 which yields sufficient conditions for two oriented knots to belong to the same oriented knot type. In §4 the following two principal results are obtained: Theorem 4.3 states that two prime knots $K$ and $L$ are equivalent iff there exists a certain finite sequence of words relating a word of $K$ to a word of $L$. Theorem 4.4 states that two oriented knots $K$ and $L$ are O-equivalent iff there exists a certain finite sequence of words and characteristics relating the word and characteristic of $K$ to those of $L$.

We remark that in each of Theorems 4.3 and 4.4 the sufficiency part follows from the results developed in §3 while the necessity is an immediate consequence of the classical work of Alexander and Briggs [1].

In §5 the group of a prime word is defined and this is used in
Theorem 5.1 to give necessary and sufficient conditions for a group to be a knot group.

2. Preliminaries and basic lemma. Let $K$ be an oriented knot in regular position with respect to the parallel projection $p$ from $\mathbb{R}^3$ onto the $xy$-plane $\mathbb{R}^2$. Suppose $K$ has $n$ double points $d_1, \ldots, d_n$ and the corresponding overcrossing (undercrossing) points are denoted by $o_1, \ldots, o_n(u_1, \ldots, u_n)$. With each $o_i(u_i)$ we associate the syllable $d_i(u_i^{-1})$. The oriented knot $K$ determines a cyclically ordered sequence of $2n$ crossing points. By arbitrarily designating one crossing point as the first and replacing each point in the resulting sequence by its associated syllable we obtain a word for $K$ (Fig. 1). Words $x_1^{a_1} \cdots x_{2m}^{a_{2m}}$ and $y_1^{b_1} \cdots y_{2m}^{b_{2m}}$ are defined to be equivalent if $m = n$ and there is a cyclic permutation $\alpha$ of $1, \ldots, 2m$ such that $x_i = x_j$ iff $y_{\alpha(i)} = y_{\alpha(j)}$ and $a_i = b_{\alpha(i)}, a_j = b_{\alpha(j)}$. A nonempty word of a knot is called prime if it contains no proper segment $S$ such that $s \in S$ iff $s^{-1} \in S$. (For instance, $ab^{-1}ca^{-1}bc^{-1}$ is prime but $xab^{-1}ca^{-1}bc^{-1}x^{-1}$ is not.)

We remark that the results of [6] or [10] can be used to give necessary and sufficient conditions for a given word to be the word associated with the diagram of an oriented knot.

To each double point $d$ of a diagram of an oriented knot we

![Figure 1](image1.png)

$ab^{-1}ca^{-1}bc^{-1}$

**Figure 1**

![Figure 2](image2.png)

$\varepsilon(d) = 1$

$\varepsilon(d) = -1$

**Figure 2**
associate an integer $\varepsilon(d) = \pm 1$, called the characteristic of $d$ (see p. 18 of [9]), according as the directed underpass does or does not cross under the directed overpass from left to right so as to preserve a left-handed screw. We note that $\varepsilon$ is independent of the orientation assigned to the knot (Fig. 2).

We shall call the directed arc between two successive crossing points of an oriented knot $K$ a boundary arc. Thus to each pair of consecutive syllables of a word for $K$ there corresponds a unique boundary arc on $K$. We now state a lemma which follows from the proof of Theorem 2 of [8].

**Basic Lemma 2.1.** Let $K$ and $L$ be two oriented knots possessing equivalent prime words. Then there is an isotopic deformation of $R'$, fixed on a neighborhood of double points of $K$, which transforms $K$ into a knot $K'$ such that

(a) The oriented projections $p(K')$ and $p(L)$ are plane equivalent under an autohomeomorphism of $R^2$ which maps projected boundary arcs of $K'$ onto the corresponding projected boundary arcs of $L$.

(b) $K$ and $L$ belong to the same knot type.

3. The prime factorization of a word. If $W$ is a nonempty word which is not prime, then it can be written in the form $ABC$ where $AC$, $B$ are nonempty words such that $b \in B$ if and only if $b^{-1} \in B$. Clearly $AC$ and $B$ themselves are words for some knot. Moreover, $W$ can be written in the above form where $B$ is a prime word. If $AC$ is not prime the process may be repeated and so on. In this manner every nonempty word $W$ may be factored into a finite number of prime words, called the *prime factors* of $W$. Furthermore, it is easy to see that this factorization is unique except possibly for the order of the prime factors.

**Definition 3.1.** Let $P(Q)$ denote the set of prime factors of the nonempty word $U(V)$. Then $U$ and $V$ have equivalent prime factorizations iff there is a bijection $h$ between $P$ and $Q$ such that $W \in P$ implies $h(W)$ is equivalent to $W$.

**Lemma 3.2.** Let $K(K')$ be a knot with double points $d_1, \ldots, d_n$ and prime word $W(W')$. Suppose $W$ is equivalent to $W'$ and $d_i$ corresponds to $d'_i (i = 1, \ldots, n)$ under this equivalence. Then $\varepsilon(d_i) = \varepsilon(d'_i)$ implies $\varepsilon(d_i) = \varepsilon(d'_i)$ for $i = 2, \ldots, n$.

**Proof.** By Lemma 2.1 we may assume that there is an autohomeomorphism $h$ of $R^2$ mapping the oriented projection $p(K)$ onto the oriented projection $p(K')$ so that the projected boundary arcs of
K are mapped onto the corresponding projected boundary arcs of K'. It is well known (p. 158 of [3]) that we may color the unbounded region of \( p(K)(p(K')) \) white and then alternately color the bounded regions of \( p(K)(p(K')) \) black and white so that every projected boundary arc of \( K(K') \) lies on the boundary of precisely one black region and one white region. We may suppose that the double points of K have been labelled so that there is a projected boundary arc \( d_1 d_2 \) joining \( d_1 \) to \( d_2 \). The arc \( d_1 d_2 \) lies on the boundary of precisely one black region, say \( D \). Since \( K \) has a prime word it follows from Theorem 9 of [10] that \( \text{Bd} \ D \) is a simple closed curve, and by Theorem 7 of [10] the union of a finite number of projected boundary arcs form an arc \( d_2 b d_1 \) such that \( \text{Bd} \ D = d_1 d_2 \cup d_2 b d_1 \).

Since \( h \) preserves projected overpasses (underpasses) and their orientations, \( \varepsilon(d_1) = \varepsilon(d_1') \) implies that the projected overpass at \( d_1 \) can be rotated through black regions by a clockwise rotation onto the projected underpass at \( d_1 \) iff the projected overpass at \( d_1 \) can be rotated through black regions by a clockwise rotation onto the projected underpass at \( d_1' \). It then follows the preceding statement is true if \( d_1 (d_1') \) is replaced by \( d_2 (d_2') \), and therefore \( \varepsilon(d_1) = \varepsilon(d_1') \). By successively applying this procedure \( n - 1 \) times we obtain \( \varepsilon(d_i) = \varepsilon(d_i') \) for \( i = 2, \ldots, n \).

Since the mirror image of a knot \( K \) can be rotated to yield a knot \( L \) such that \( K \) and \( L \) have equivalent words and the characteristics of corresponding double points have opposite signs, we obtain

**Corollary 3.3.** Let \( K(K') \) be a knot with double points \( d_1, \ldots, d_n(d_1', \ldots, d_n') \) and prime word \( W(W') \). Suppose \( W \) is equivalent to \( W' \) and \( d_i \) corresponds to \( d_i'(i = 1, \ldots, n) \) under this equivalence. Then either \( \varepsilon(d_i) = \varepsilon(d_i')(i = 1, \ldots, n) \) or \( \varepsilon(d_i) = -\varepsilon(d_i')(i = 1, \ldots, n) \).

**Theorem 3.4.** Let \( K(L) \) be an oriented knot with word \( U(V) \) whose prime factors are \( U_1, \ldots, U_m(V_1, \ldots, V_m) \). Suppose \( U \) and \( V \) have equivalent prime factorizations with \( U_i \) being equivalent to \( V_i \) \( (i = 1, \ldots, m) \). Then, if the characteristic of a double point of \( U \) is equal to the characteristic of the corresponding double point of \( V \) \( (i = 1, \ldots, m) \), the oriented knots \( K \) and \( L \) belong to the same oriented knot type.

**Proof.** Now \( K(L) \) can be represented as a composition of oriented knots \( K_1, \ldots, K_m(L_1, \ldots, L_m) \) whose words are \( U_1, \ldots, U_m(V_1, \ldots, V_m) \). Since the set of oriented knot types form a commutative semigroup (indeed, with unique factorization) [p. 140, 3], it suffices to prove the theorem in the case when \( U = U_1 \) and \( V = V_1 \).

By Lemma 2.1 we may assume that there is an autohomeomorphism \( h \) of \( R^2 \) mapping the oriented projection \( p(K) \) onto the oriented pro-
jection $p(L)$ so that the projected boundary arcs of $K$ are mapped onto the corresponding boundary arcs of $L$. Let $R_i(R_s)$ denote the unbounded region of $K(L)$. Without loss of generality we may assume that $\text{Bd } R_i (i = 1, 2)$ lies on a circle $C_i$ except in a neighborhood of double points. By Lemma 3.2 the characteristics of corresponding double points are equal. It follows that if $d_i d_k$ is a projected boundary arc of $K$, then $d_i d_k$ induces a clockwise orientation on $C_i$ iff $h(d_i d_k)$ induces a clockwise orientation on $C_s$. Hence we may suppose that $h$ is the identity on $R^3 \setminus \text{Int } C$ where $C$ denotes a circle in $R^3$ whose interior contains $p(K)$ and $p(L)$. Let $f(x, y, z) = (h, (x, y), z)$. Then $f$ is an orientation preserving automorphism of $R^3$. Since $f(K)$ and $L$ obviously belong to the same oriented knot type the result follows.

If in Theorem 3.4 it is only assumed that $U$ and $V$ have equivalent prime factorizations, then it follows from (b) of Lemma 2.1 that $K(L)$ can be represented as a composition of knots $K_i, \ldots, K_m(L_i, \ldots, L_m)$ where $K_i$ and $L_i$ belong to the same knot type ($i = 1, \ldots, m$). Consequently we have

**Theorem 3.5.** Let $K$ and $L$ be two knots which may be given orientations so that their words have equivalent prime factorizations. If either $K$ or $L$ is a prime knot, then they belong to the same knot type.

We note that it is possible for inequivalent nonprime knots to have words possessing equivalent prime factorizations. For instance, it is well known that the square and granny knots belong to different knot types ([2]). However, diagrams can be chosen for these two knots so as to yield identical words.

4. Diagram transformations. We remark that due to the work of Graeub ([5]) and Moise ([7]) the classical notion of equivalence for (oriented) knots (see [1]) agrees with the definitions given in §1. In [1] Alexander and Briggs have shown that two oriented knots belong to the same oriented knot type iff the diagram of one can be transformed into the diagram of the other by a finite sequence of the following types of deformations or their inverses:

1) A boundary arc acquires a loop which creates a new double
(II) One boundary arc passes under another with the creation of two new double points (Fig. 4).

(III) If there is a region bounded by three double points and three projected boundary arcs where the endpoints of one of the boundary arcs are undercrossing points, then any one of the boundary arcs may be deformed past the crossing points determined by the other two boundary arcs (Fig. 5).

The above diagram transformations determine the following possible transformations between the associated words.

(1) \( A \rightarrow Ax^{x^{-e}}(e = \pm 1) \)

(II) \( AB \rightarrow Ax^{x^{-e}}Bx^{-y^{-e}}(e = \pm 1) \)

(III) \( Ax_{1}y_{1}x_{2}z_{1}C_{1}y_{1}^{-1}z_{1}^{-1} \rightarrow Ay_{1}z_{2}B_{2}x_{2}^{-1}C_{2}y_{2}^{-1} \)
\( Ay_{1}x_{2}C_{2}x_{2}^{-1}y_{2}^{-1} \rightarrow Ax_{2}y_{2}B_{2}x_{2}^{-1}C_{2}y_{2}^{-1} \)
\( Ax_{1}y_{2}x_{2}C_{2}x_{2}^{-1}y_{2}^{-1} \rightarrow Ay_{2}z_{2}B_{2}x_{2}^{-1}C_{2}y_{2}^{-1} \)
\( Ay_{1}x_{2}C_{2}x_{2}^{-1}y_{2}^{-1} \rightarrow Ax_{2}y_{2}B_{2}x_{2}^{-1}C_{2}y_{2}^{-1} \)

The words in (III) may occur with the order of the pairs \( x_{i}y_{i}(x_{i}y_{i}) \), \( x_{i}^{-1}z_{i}, y_{i}^{-1}z_{i}^{-1}(x_{i}^{-1}y_{i}^{-1}) \) being permuted provided that the analogous rearrangement is made for the pairs \( y_{i}x_{i}(x_{i}y_{i}), z_{i}x_{i}^{-1}, z_{i}^{-1}y_{i}^{-1}(y_{i}^{-1}x_{i}^{-1}) \).

We now list three additional types of transformations of words.

(0) Any transformation between two equivalent words.

(IV) A transformation which results from a reversal of the orientation of the knot,

i.e., \( x_{1}^{e_{1}} \cdots x_{2n}^{e_{n}} \rightarrow x_{2n}^{e_{2n}} \cdots x_{1}^{e_{1}} \).
(V) A transformation which results from a reflection through the $xy$-plane,

$$x_{2n}^{e_1} \cdots x_{2n}^{e_n} \longrightarrow x_{2n}^{-e_1} \cdots x_{2n}^{-e_n}.$$ 

**Definition 4.1.** A sequence $W_1, W_2, \ldots, W_n$ of words of knots is called a fundamental sequence of words between $U$ and $V$ iff $W_i = U, W_n = V$ and $W_i$ is obtained from $W_{i-1}(i = 2, \ldots, n)$ by a transformation of one of the types (0)-(V).

**Definition 4.2.** A finite sequence of words and characteristics of double points of knots is said to be fundamental if each word and its successor are related by a transformation of one of the types (0)-(III) and under this transformation corresponding double points have the same characteristic.

**Theorem 4.3.** Let $K, L$ be two prime knots with words $U, V$ respectively. Then $K$ and $L$ belong to the same knot type iff there is a fundamental sequence of words between $U$ and $V$.

**Proof.** (Necessity) Suppose $K$ and $L$ are prime knots which belong to the same knot type. We regard $K, L$ as oriented knots with the orientations being determined by $U, V$ respectively. Let $\sigma$ denote reversal of knot orientation and let $\rho(x, y, z) = (x, y, -z)$. Then the oriented knot $K$ must belong to the same oriented knot type as one of the oriented knots $L, \rho L, \sigma L, \sigma \rho L$. Hence the necessity of the theorem follows from [1].

(Sufficiency) Let $W_1, W_2, \ldots, W_n$ be a fundamental sequence of words between $U$ and $V$. Let $K_1, \ldots, K_{n-1}$ be a sequence of knots with words $W_2, \ldots, W_{n-1}$ respectively. If $W_2$ is obtained from $W_1$ by a type (0) transformation it is easy to show that $W_i$ and $W_{i+1}$ have equivalent prime factorizations. Hence $K$ is equivalent to $K_2$ by Theorem 3.5. If $W_2$ is not obtained from $W_1$ by a type (0) transformation, then it is evident that there exist equivalent knots $L_i, L_2$ with words $W_i, W_2$ respectively. By Theorem 3.5 $K$ is equivalent to $L_1$ and $L_2$ is equivalent to $K_2$. Thus $K$ is equivalent to $K_2$. Using an inductive argument we obtain the desired result that $K$ and $L$ belong to the same knot type.

By using an argument similar to that used above and evoking Theorem 3.4 instead of Theorem 3.5 we obtain

**Theorem 4.4.** Two oriented knots $K$ and $L$ belong to the same oriented knot type iff there is a fundamental sequence relating a
word and the characteristics of the double points of $K$ to a word and 
the characteristics of the double points of $L$.

REMARKS 4.5. L. B. Treybig has conjectured that if $U$ and $V$ are 
words of length $\leq m$ which may be connected by a fundamental 
sequence of words, then there is an integer $N(m)$ and a fundamental 
sequence of words between $U$ and $V$ such that each word in the 
sequence has length $\leq N(m)$. In [13] Treybig obtains some partial 
results in this direction. The proof of such a conjecture together 
with Theorem 4.4 would provide an algorithm for determining oriented 
knot types.

5. The group of a prime word. By Corollary 3.3 there are 
precisely two sets of characteristics associated with a prime word, 
one set being the negatives of the other. Given a prime word, a set 
of characteristics associated with its alphabet may be obtained by 
simply constructing any diagram associated with the word.

Let $W = b_1^1 b_2^2 \cdots b_n^{2n}$ be a prime word. A segment (consider $W$ 
to be cyclically ordered) of the form $b_i \cdots b_{i+k}$ where $e_{i-1} = e_{i+k+1} = -1$ 
is called an over segment, and a segment of the form $b_j^{-1} \cdots b_{j+m}^{-1}$ 
where $e_{j-1} = e_{j+m+1} = 1$ is called and under segment. The group of 
$W$ is presented as follows: The alphabet of $W$ is the set of generators. For each over segment $b_i \cdots b_{i+k}$ we associate the relation 
$b_i = \cdots = b_{i+k}$. For each under segment $b_j^{-1} \cdots b_{j+m}^{-1}$ we associate the 
relation $b_{j-1} b_j^{(b_j)} \cdots b_{j+m-1} b_j^{(b_j+m)} b_{j+m+1}^{-1} b_{j+m}^{(b_j+m)} \cdots b_j^{(b_j)} = 1$. The above generators and relations present a group which we shall call the group 
of $W$.

THEOREM 5.1. A group is a knot group iff it has a presentation 
which presents the group of a prime word.

Proof. To prove the theorem it suffices to show that every 
oriented knot type has a representative diagram which possesses a 
prime word, and that the group of a prime word $W$ is the group of 
any knot whose diagram yields $W$.

Let $K$ be an oriented knot with word $W$, and let $W_1, \ldots, W_m$ 
denote the prime factors of $W$. Then $K$ can be represented as a 
composition of oriented knots $K_1, \ldots, K_m$ possessing prime words $W_1, 
\ldots, W_m$ respectively. If $K$ is a prime knot it follows that only one 
of $K_1, \ldots, K_m$ belongs to a nontrivial knot type. Hence it can be 
assumed that every prime knot possesses a prime word. Thus given 
an oriented knot $K$, we may assume that $K$ is a composition of 
oriented prime knots $K_1, \ldots, K_m$ with prime words $W_1, \ldots, W_m$ respectively, and that the word of $K$ is $W_1 W_2 \cdots W_m$. An isotopic deformation
of $R^n$ can be chosen so as to deform an arc in $K_i$ so that it passes under $K_n, \ldots, K_1$ in such a way that $K$ is transformed into a new knot $K'$ which possesses a prime word.

Now suppose $K$ is an oriented knot with prime word $W$. We may assume that the characteristics associated with $W$ are the characteristics of $K$. (For otherwise, the characteristics would be associated with the diagram of a knot $\alpha \rho K$ where $\alpha$ is a rotation and $\rho(x, y, z) = (x, y, -z)$. Since $K$ and $\alpha \rho K$ are equivalent, it would suffice to show that the presentation for the group of $W$ presents the group of $\alpha \rho K$.) By using the Tietze operations we can eliminate all the generators in the presentation except for the first letter in each over segment. It is then easy to check that the resulting presentation is precisely the over presentation of the group of $K$ given by Fox and Torres (p. 212, [4]).

**REFERENCES**


Received October 10, 1972. This paper represents a portion of the author's Ph. D. dissertation, written at Tulane University under the direction of Professor L. B. Treybig, and was supported partly by the National Research Council of Canada (grant A8205).

TULANE UNIVERSITY

AND

THE UNIVERSITY OF SASKATCHEWAN (SASKATOON CAMPUS)
<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zvi Arad, π-homogeneity and π′-closure of finite groups</td>
<td>1</td>
</tr>
<tr>
<td>Ivan Baggs, A connected Hausdorff space which is not contained in a maximal connected space</td>
<td>11</td>
</tr>
<tr>
<td>Eric Bedford, The Dirichlet problem for some overdetermined systems on the unit ball in ( C^n )</td>
<td>19</td>
</tr>
<tr>
<td>R. H. Bing, Woodrow Wilson Bledsoe and R. Daniel Mauldin, Sets generated by rectangles</td>
<td>27</td>
</tr>
<tr>
<td>Carlo Cecchini and Alessandro Figà-Talamanca, Projections of uniqueness for ( L^p(G) )</td>
<td>37</td>
</tr>
<tr>
<td>Gokulananda Das and Ram N. Mohapatra, The non absolute Nörlund summability of Fourier series</td>
<td>49</td>
</tr>
<tr>
<td>Frank Rimi DeMeyer, On separable polynomials over a commutative ring</td>
<td>57</td>
</tr>
<tr>
<td>Richard Detmer, Sets which are tame in arcs in ( E^3 )</td>
<td>67</td>
</tr>
<tr>
<td>William Erb Dietrich, Ideals in convolution algebras on Abelian groups</td>
<td>75</td>
</tr>
<tr>
<td>Bryce L. Elkins, A Galois theory for linear topological rings</td>
<td>89</td>
</tr>
<tr>
<td>William Alan Feldman, A characterization of the topology of compact convergence on ( C(X) )</td>
<td>109</td>
</tr>
<tr>
<td>Hillel Halkin Gershenson, A problem in compact Lie groups and framed cobordism</td>
<td>121</td>
</tr>
<tr>
<td>Samuel R. Gordon, Associators in simple algebras</td>
<td>131</td>
</tr>
<tr>
<td>Marvin J. Greenberg, Strictly local solutions of Diophantine equations</td>
<td>143</td>
</tr>
<tr>
<td>Jon Craig Helton, Product integrals and inverses in normed rings</td>
<td>155</td>
</tr>
<tr>
<td>Domingo Antonio Herrero, Inner functions under uniform topology</td>
<td>167</td>
</tr>
<tr>
<td>Jerry Alan Johnson, Lipschitz spaces</td>
<td>177</td>
</tr>
<tr>
<td>Marvin Stanford Keener, Oscillatory solutions and multi-point boundary value functions for certain nth-order linear ordinary differential equations</td>
<td>187</td>
</tr>
<tr>
<td>John Cronan Kieffer, A simple proof of the Moy-Perez generalization of the Shannon-McMillan theorem</td>
<td>203</td>
</tr>
<tr>
<td>Joong Ho Kim, Power invariant rings</td>
<td>207</td>
</tr>
<tr>
<td>Gangaram S. Ladde and V. Lakshmikantham, On flow-invariant sets</td>
<td>215</td>
</tr>
<tr>
<td>Roger T. Lewis, Oscillation and nonoscillation criteria for some self-adjoint even order linear differential operators</td>
<td>221</td>
</tr>
<tr>
<td>Jürg Thomas Marti, On the existence of support points of solid convex sets</td>
<td>235</td>
</tr>
<tr>
<td>John Rowlay Martin, Determining knot types from diagrams of knots</td>
<td>241</td>
</tr>
<tr>
<td>James Jerome Metzger, Local ideals in a topological algebra of entire functions characterized by a non-radial rate of growth</td>
<td>251</td>
</tr>
<tr>
<td>K. C. O’Meara, Intrinsic extensions of prime rings</td>
<td>257</td>
</tr>
<tr>
<td>Stanley Poreda, A note on the continuity of best polynomial approximations</td>
<td>271</td>
</tr>
<tr>
<td>Robert John Sacker, Asymptotic approach to periodic orbits and local prolongations of maps</td>
<td>273</td>
</tr>
<tr>
<td>Eric Peter Smith, The Garabedian function of an arbitrary compact set</td>
<td>289</td>
</tr>
<tr>
<td>Arne Stray, Pointwise bounded approximation by functions satisfying a side condition</td>
<td>301</td>
</tr>
<tr>
<td>John St. Clair Werth, Jr., Maximal pure subgroups of torsion complete abelian ( p )-groups</td>
<td>307</td>
</tr>
<tr>
<td>Robert S. Wilson, On the structure of finite rings. II</td>
<td>317</td>
</tr>
<tr>
<td>Kari Ylinen, The multiplier algebra of a convolution measure algebra</td>
<td>327</td>
</tr>
</tbody>
</table>