

Pacific Journal of Mathematics

AN EMBEDDING OF SEMIPRIME P.I.-RINGS

JOE WAYNE FISHER AND LOUIS HALLE ROWEN

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Let us say an extension R' of a ring R is a *quotient ring* of R if every regular element of R is invertible in R' . In this note we construct a class of quotient rings of semiprime *P.I.*-rings and use this construction to find rapid proofs of several facts about semiprime *P.I.*-rings.

1. Preliminaries. Throughout this paper R will denote a *semiprime P.I.-ring* with unity and center C , i.e., R has no nonzero nilpotent ideals and the standard polynomial

$$S_{2n}(X_1, \dots, X_{2n}) = \sum_{\pi} (\text{sgn } \pi) X_{\pi(1)} \cdots X_{\pi(2n)},$$

the sum taken over all permutations π of $(1, \dots, 2n)$, is an identity of R for suitable n (the minimal such n is the *degree* of R). Formanek [5] has constructed a polynomial $g_n(X_1, \dots, X_{n+1})$ which is central for all semiprime *P.I.*-rings of degree n , and Rowen [11] has used these central polynomials to prove

THEOREM A. *Any nonzero ideal of R intersects C nontrivially.*

Let $S = \{c \in C: cr \neq 0 \text{ for all nonzero } r \text{ in } R\}$. Define an equivalence relation on $R \times S$ by saying $(r_1, s_1) \sim (r_2, s_2)$ if $r_1 s_2 = r_2 s_1$, and let rs^{-1} denote the equivalence class of (r, s) . Then $R_s = \{rs^{-1}: (r, s) \in R \times S\}$ is a ring when endowed with the (well-defined) operations $r_1 s_1^{-1} + r_2 s_2^{-1} = (r_1 s_2 + r_2 s_1)(s_1 s_2)^{-1}$, called the *ring of central quotients* of R . The following theorem is a direct consequence of Theorem A (cf., Rowen [11, §2]):

THEOREM B. *If R is a prime *P.I.*-ring of degree n , then R_s is simple Artinian of dimension n^2 over its center C_s , C_s is the quotient field of C , and R_s satisfies the identities of R .*

Theorem B often enables us to study R by examining R_s . If R is a semiprime *P.I.*-ring of degree n and satisfies the ascending chain condition on annihilators of two-sided ideals, then R_s is the classical semisimple Artinian ring of left and right quotients of R (cf., [12]). Unfortunately, this situation fails for semiprime *P.I.*-rings in general, so one is led to study other extensions of R . The purpose of this paper is to introduce a straightforward type of extension of R and to deduce from it properties of semiprime *P.I.*-rings and their classical quotient rings (if these exist). This paper

subsumes Fisher [4]. First we shall derive some easy known properties of R .

For a subset A of R , let $\text{Ann}_R(A)$ denote $\{r \in R \mid Ar = 0\}$. Also we say an ideal A of R is *essential* if for every nonzero ideal B of R , $A \cap B \neq 0$. Since R is semiprime, $A \cap B = 0$ if and only if $AB = 0$. The following lemma is known by Martindale [9].

LEMMA 1. (i) *If E is an essential ideal of C , then ER is an essential ideal of R .*

(ii) *If J is a left ideal of R with $\text{Ann}_R(J) = 0$, then $J \cap C$ is essential in C , so J contains an essential ideal of R .*

Proof. (i) Suppose that $A \cap E = 0$ for some ideal A of R . Then $(A \cap C) \cap E = A \cap (C \cap E) = A \cap E = 0$, implying $A \cap C = 0$. Hence $A = 0$ by Theorem A and thus ER is essential.

(ii) Viewed as a ring (without 1), J is clearly a *P.I.*-ring and can easily be shown to be semiprime. We claim that $J \cap C = \text{cent } J$. Indeed $J \cap C \subseteq \text{cent } J$ and if $a \in \text{cent } J$, then for all r in R and for all x in J , $(ra - ar)x = rax - a(rx) = rax - r(xa) = rax - rax = 0$. Hence $(ra - ar) \in \text{Ann}_R(J) = 0$ and so $a \in C$.

Now let B be an ideal of C such that $(J \cap C) \cap B = 0$. Then $(J \cap C \cap BR)^2 \subseteq (J \cap C)BR = B(J \cap C)R \subseteq (B \cap (J \cap C))R = 0$ and so $(J \cap C \cap BR)^2 = 0$. Since $J \cap C$ has no nonzero nilpotent elements, we have $J \cap C \cap BR = 0$, i.e., $(J \cap C) \cap (J \cap BR) = 0$. But by Theorem A applied to the semiprime ring J (with center $J \cap C$), $J \cap BR = 0$. This implies $RJB = BRJ \subseteq J \cap BR = 0$, so $B \subseteq \text{Ann}_R(RJ) = \text{Ann } J = 0$. Hence $J \cap C$ is essential in C . The rest of the lemma follows from (i).

2. Definition and elementary properties of $T(R)$. For the remainder of this paper, we assume that the semiprime *P.I.*-ring R has degree n . This implies that every prime factor ring of R has degree equal to or less than n . The *degree* of a prime ideal P of R is defined as the degree of R/P .

Let \mathcal{P} be a collection (indexed by Λ) of prime ideals P_λ of R such that $\bigcap \{P_\lambda; \lambda \in \Lambda\} = 0$. For each λ in Λ , set $R_\lambda = R/P_\lambda$, let Q_λ equal the simple Artinian ring of central quotients of R_λ , and let Q be the complete direct product $\prod \{Q_\lambda; \lambda \in \Lambda\}$. There is a natural embedding $R \rightarrow \prod R_\lambda \rightarrow Q$ and we shall often view R as a subring of Q under this embedding. Hence R satisfies the identities of Q . On the other hand, any identity f of R is an identity of each R_λ , and is an identity of each Q_λ by Theorem B; hence f is an identity of $Q = \prod Q_\lambda$. Consequently, R and Q satisfy the same identities.

Clearly Q is *von Neumann regular*, i.e., for any $x \in Q$, there is some y in Q such that $xyx = x$.

As remarked above, each Q_λ has degree $\leq n$. Let $A_j = \{\lambda \in A: Q_\lambda \text{ has degree } j\}$ and let $\bar{Q}_j = \prod\{Q_\lambda: \lambda \in A_j\}$. Then \bar{Q}_j is a semiprimitive ring of degree j with the property that every nonzero homomorphic image of \bar{Q}_j has degree j . This is equivalent to saying, by the Artin [2]-Procesi [10] theorem, that \bar{Q}_j is an Azumaya algebra of rank j . Hence Q is a finite direct sum of the Azumaya algebras \bar{Q}_j of finite rank j .

LEMMA 2. *Any nonzero homomorphic image $\psi(Q)$ of Q is von Neumann regular. Moreover, $\psi(Q)$ is the finite direct sum of the Azumaya algebras $\psi(\bar{Q}_j)$ of finite rank j , and each identity of R is an identity of $\psi(Q)$.*

Proof. Every homomorphic image of a von Neumann ring is von Neumann regular. Also, every homomorphic image of $\psi(\bar{Q}_j)$ is a homomorphic image of \bar{Q}_j , thereby having rank j ; hence $\psi(\bar{Q}_j)$ is Azumaya of rank j , and clearly $\psi(Q)$ is the direct sum of $\psi(\bar{Q}_j)$ for $j = 1, \dots, n$. The last assertion is immediate.

For any x in Q , let x_λ denote the component of x in Q_λ and let $W_x = \{\lambda \in A: x_\lambda \neq 0\}$. Set $V = \{x \in Q: \bigcap \{P_\lambda: \lambda \in W_x\} \text{ is an essential ideal of } R\}$. Now V is an ideal of Q because, taking x, y in V and q in Q , $W_{x \pm y} \subseteq W_x \cup W_y$; $W_{qx} \subseteq W_x$; $W_{xq} \subseteq W_x$. Let us define $T(R, \mathcal{P}) = Q/V$. From Lemma 2 we have that $T(R, \mathcal{P})$ is a finite direct sum of Azumaya algebras of finite rank and is von Neumann regular.

THEOREM 1. (i) *There is a canonical imbedding $R \rightarrow T(R, \mathcal{P})$ given by $R \rightarrow Q \rightarrow Q/V$.*

(ii) *Half regular elements of R are both left and right invertible in $T(R, \mathcal{P})$.*

(iii) *$T(R, \mathcal{P})$ satisfies precisely the same identities as R .*

Proof. (i) We need show only that $R \cap V = 0$. If $r \in R \cap V$, then $\bigcap \{P_\lambda: \lambda \in W_r\}$ is essential in R and so $\bigcap \{P_\lambda: r \in P_\lambda\} = 0$. Hence $r = 0$.

(ii) Let r in R have right annihilator zero. Then $\text{Ann}_R(Rr) = 0$ and Rr contains an essential ideal E of C by Lemma 1(ii). Let $W'_r = \{\lambda: P_\lambda \not\supseteq E\}$. Clearly $W'_r \subseteq W_r$. Moreover, for any λ in W'_r there is an x_λ in Q_λ such that $0 \neq x_\lambda r_\lambda \in \text{cent } Q_\lambda$. Since $\text{cent } Q_\lambda$ is a field, there is d_λ in $\text{cent } Q_\lambda$ such that $d_\lambda x_\lambda r_\lambda = 1_\lambda$. Furthermore, $r_\lambda d_\lambda x_\lambda = 1_\lambda$ because Q_λ is simple Artinian. Define y in Q as follows: $y_\lambda = 0$ for $\lambda \notin W'_r$ and $y_\lambda = d_\lambda x_\lambda$ for $\lambda \in W'_r$. Then $(yr - 1)_\lambda = 0$ and $(ry - 1)_\lambda = 0$ for all λ in W'_r . Thus $\bigcap \{P_\lambda: \lambda \in W_{yr-1}\} \supseteq \bigcap \{P_\lambda: \lambda \notin W'_r\} \supseteq$

E. It follows from Lemma 1(i) that $yr - 1 \in V$; likewise $ry - 1 \in V$. Hence, for \bar{y} the image of y in $T(R, \mathcal{P})$, we have $\bar{y}r = 1$ and $r\bar{y} = 1$ in $T(R, \mathcal{P})$.

(iii) $T(R, \mathcal{P})$ satisfies each identity of R by Lemma 2; conversely, by (i), each identity of $T(R, \mathcal{P})$ is an identity of R .

The following theorem of Herstein-Small [8] is a consequence of Theorem 1.

COROLLARY 1. *Half regular elements of R are regular.*

Proof. If r in R is, say, right regular, then for some $y \in T(R, \mathcal{P})$ we have $ry = 1$. Hence r is left regular.

COROLLARY 2. *If R has a classical left ring of quotients R' , then R' satisfies the same polynomial identities as R .*

Proof. In view of Theorem 1(ii) the canonical embedding of R into $T(R, \mathcal{P})$ extends to an embedding of R' into $T(R, \mathcal{P})$. Hence R' satisfies the identities of $T(R, \mathcal{P})$ which are precisely the identities of R .

Note that this construction of $T(R, \mathcal{P})$ is related to constructions of Amitsur [1] and Goldie [7]. Also, those versed in logic may wish to regard $T(R, \mathcal{P})$ as the "reduced product" (cf., [6]) of the simple Artinian rings $\{Q_\lambda: \lambda \in A\}$ by the filter $\{A - W_x: x \in V\}$.

3. Definition and structure of $T(R)$. Now we consider an interesting special case of $T(R, \mathcal{P})$. Index the set of all the prime ideals of R by a set \bar{A} with $\bar{A}_i = \{\lambda \in \bar{A}: P_\lambda \text{ has degree } i\}$ for $i = 1, \dots, n$. Set $\bar{N}_i = \bigcap \{P_\lambda: \lambda \in \bar{A}_i\}$ (if $\bar{A}_i = \emptyset$ then $\bar{N}_i = R$), $A_i = \{\lambda \in \bar{A}_i: P_\lambda \not\subseteq \bigcap_{j=i+1}^n \bar{N}_j\}$, $\mathcal{P}_i = \{P_\lambda: \lambda \in A_i\}$, $\mathcal{P} = \mathcal{P}_1 \cup \dots \cup \mathcal{P}_n$, $A = A_1 \cup \dots \cup A_n$. Clearly $\bigcap \{P: P \in \mathcal{P}\} = \bar{N}_1 \cap \dots \cap \bar{N}_n = 0$. We define $T(R)$ to be $T(R, \mathcal{P})$. Note that $A_n = \bar{A}_n$ and that $A = A_n$ if and only if $\bar{N}_n = 0$.

Let $N_i = \bigcap \{P: P \in \mathcal{P}_i\}$ and let $R_i = R/N_i$. Note that $N_n = \bar{N}_n$. Clearly R is a subdirect product of the R_i and this subdirect decomposition is unique with respect to the properties that each of the nonzero subdirect factors has a degree different from each of the other subdirect factors and that for any subdirect factor of degree j , the intersection of its prime ideals of degree j is zero. Our aim is to show how the structure of $T(R)$ is linked to this decomposition. As in Rowen [12], let a polynomial be called *regular* if it is linear in some indeterminant, and let the *central kernel* of a ring be the additive subgroup generated by the values taken (in the center) by regular central polynomials of the ring. The central kernel is an ideal of the center C . If the central kernel is essential in C , we

say that R has *essential central kernel*. Let I be the central of R , let $B = N_1 \cap \cdots \cap N_{n-1}$, and let $R'_n = R/B$. It is shown in Rowen [12] that for $\lambda \in \bar{A}$, $I \not\subseteq P_\lambda$ if and only if $\lambda \in A_n$.

LEMMA 3. (i) $(RI + N_n)/N_n$ is an essential ideal of R_n .
 (ii) $(N_n + B)/B$ is an essential ideal of R'_n .
 (iii) A semiprime ring R of degree j has essential central kernel if and only if the intersection of its prime ideals of degree j is zero.

Proof. (i) Suppose that $[(A + N_n)/N_n] \cap [(RI + N_n)/N_n] = 0$ for some ideal A of R . Then $ARI \subseteq N_n \subseteq P_\lambda$ for each $\lambda \in A_n$. Since $I \not\subseteq P_\lambda$ for $\lambda \in A_n$, we have $A \subseteq \bigcap \{P_\lambda : \lambda \in A_n\} = N_n$. So

$$(A + N_n)/N_n = 0.$$

(ii) Suppose that $[(A + B)/B] \cap [(N_n + B)/B] = 0$ for some ideal A of R . Then $AN_n \subseteq B = N_1 \cap \cdots \cap N_{n-1} \subseteq P_\lambda$ for each $\lambda \in A - A_n$. By definition $P_\lambda \not\subseteq N_n$ for $\lambda \in A - A_n$, so $A \subseteq \bigcap \{P_\lambda : \lambda \in A - A_n\} = B$. So $(A + B)/B = 0$.

(iii) Let \bar{N}_j be the intersection of the prime ideals of degree j . Since every prime ideal of degree $< j$ contains I , we have $I \cap \bar{N}_j = 0$. Since I is essential in C , we have $\bar{N}_j \cap C = 0$, hence $N_j = 0$ by Theorem A. The reverse implication is immediate from (i) and Lemma 1.

Lemma 3(iii) gives us a neater characterization of R_1, \dots, R_n . Namely, the nonzero R_i are uniquely determined if we are to express R as a subdirect product of minimal length of rings with essential central kernel.

LEMMA 4. (i) Suppose that J is an ideal of R and $N_n \subseteq J$. Then J is essential in R if and only if J/N_n is essential in R_n .

(ii) Suppose $B \subseteq J$. Then J is essential in R if and only if J/B is essential in R'_n .

Proof. (i) (\Rightarrow) Suppose that $J/N_n \cap [(A + N_n)/N_n] = 0$ for some ideal A of R . Then $JA \subseteq N_n$ and so $B \cap JA = 0$. Now since $I \subseteq P_\lambda$ for each $\lambda \in A - A_n$, we have $RI \subseteq \bigcap \{P_\lambda : \lambda \in A - A_n\} \subseteq B$ and $RI \cap JA = 0$, or $IJA = 0$. Hence $(J \cap AI)^2 \subseteq (JAI)^2 = 0$ and $J \cap AI = 0$ since R is semiprime. By hypothesis, we then see $AI = 0$, so $A \subseteq N_n$ by Lemma 3(i). Consequently $(A + N_n)/N_n = 0$.

Conversely suppose that $J \cap A = 0$ for some ideal A of R . Then $JA = 0 \subseteq N_n$, so $A \subseteq N_n$ by hypothesis. Thus $A^2 \subseteq N_n A \subseteq JA = 0$ and so $A = 0$.

(ii) (\Rightarrow) Suppose that $J/B \cap [(A + B)/B] = 0$. Then $JA \subseteq B$, or

$JAN_n \subseteq B \cap N_n = 0$ which implies $AN_n = 0$. Hence $A \subseteq B$ by Lemma 3(ii) and so $(A + B)/B = 0$. The proof of the converse is analogous to that in (i).

THEOREM 2. $T(R) \cong T(R_1) \oplus \cdots \oplus T(R_n)$.

Proof. We use induction on $n = \text{degree of } R$. The assertion is true for $n = 2$. Since R'_n has degree $\leq n - 1$, we have by our induction hypothesis that $T(R'_n) \cong T(R_1) \oplus \cdots \oplus T(R_{n-1})$. Let $\bar{Q}_n = \Pi\{Q_\lambda: \lambda \in A_n\}$, $\bar{Q}'_n = \Pi\{Q_\lambda: \lambda \in A - A_n\}$, $V_n = V \cap \bar{Q}_n$, and $V'_n = V_n \cap \bar{Q}'_n$. Clearly $V = V_n \oplus V'_n$ and $T(R) = Q/V \cong \bar{Q}_n \oplus \bar{Q}'_n/V \cong \bar{Q}'_n/V_n \oplus \bar{Q}'_n/V'_n$. But Lemma 4(i) shows $\bar{Q}_n/V_n \cong T(R_n)$ and Lemma 4(ii) shows $\bar{Q}'_n/V'_n \cong T(R'_n)$. Thus $T(R) \cong T(R_n) \oplus T(R'_n) \cong T(R_1) \oplus \cdots \oplus T(R_{n-1}) \oplus T(R_n)$.

Theorem 2 enables us to reduce the study of $T(R)$ to rings with essential central kernel.

THEOREM 3. *Let R be a semiprime P.I.-ring of degree n with essential central kernel. Then $T(R)$ is an Azumaya algebra of rank n^2 and $T(C) \cong \text{center } (T(R))$.*

Proof. By Lemma 3(iii), $N_n = 0$. Hence $T(R)$ is a homomorphic image of $\Pi\{Q_\lambda: \lambda \in A_n\}$. Therefore, $T(R)$ is Azumaya of rank n^2 . Write $C_\lambda = \text{center } Q_\lambda$ for $\lambda \in A$. Since $\Pi\{Q_\lambda: \lambda \in A_n\}$ is an Azumaya algebra of rank n^2 , we have the following fact which we will need later, $\text{cent } [(\Pi_{\lambda \in A_n} Q_\lambda)/(V \cap \Pi_{\lambda \in A_n} Q_\lambda)] = (\Pi_{\lambda \in A_n} C_\lambda + V \cap \Pi_{\lambda \in A_n} Q_\lambda)/(V \cap \Pi_{\lambda \in A_n} Q_\lambda)$.

We claim that the homomorphism $\varphi: (\Pi_{\lambda \in A} Q_\lambda)/V \rightarrow (\Pi_{\lambda \in A_n} Q_\lambda)/(V \cap \Pi_{\lambda \in A_n} Q_\lambda)$, induced by the projection, $\Pi_{\lambda \in A} Q_\lambda \rightarrow \Pi_{\lambda \in A_n} Q_\lambda$, is an isomorphism. Indeed, suppose that $0 \neq x + V$ for x in $\Pi_{\lambda \in A} Q_\lambda$. Then $\bigcap \{P_\lambda: \lambda \in W_x\}$ is not essential. Since each prime of degree $< n$ contains I and $I \subseteq \bigcap \{P_\lambda: \lambda \in W_x \cap (A - A_n)\}$ is essential, we conclude that $\bigcap \{P_\lambda: \lambda \in W_x \cap A_n\}$ is not essential and $0 \neq x + (V \cap \Pi_{\lambda \in A_n} Q_\lambda)$. Consequently φ is an isomorphism.

Now by Rowen [12, Theorem 3] there exists a 1:1 correspondence of $\{P_\lambda: \lambda \in A_n\}$ and the set of prime ideals of C , not containing I , given by $P_\lambda \mapsto P_\lambda \cap C$. We claim that $T(C) \cong (\Pi_{\lambda \in A_n} C_\lambda)/(V \cap \Pi_{\lambda \in A_n} C_\lambda)$. The proof of this is similar to the one in the preceding paragraph because every prime in C which is not in $\{P_\lambda \cap C: \lambda \in A_n\}$ contains I which is essential in C .

Finally we have all the requisite pieces to obtain

$$\begin{aligned}
T(C) &\cong (\Pi_{\lambda \in \Lambda_n} C_\lambda) / (V \cap \Pi_{\lambda \in \Lambda_n} C_\lambda) \\
&\cong (\Pi_{\lambda \in \Lambda_n} C_\lambda + V \cap \Pi_{\lambda \in \Lambda_n} Q_\lambda) / (V \cap \Pi_{\lambda \in \Lambda_n} Q_\lambda) \\
&\cong \text{cent} [(\Pi_{\lambda \in \Lambda_n} Q_\lambda) / (V \cap \Pi_{\lambda \in \Lambda_n} Q_\lambda)] \\
&\cong \text{cent} ((\Pi_{\lambda \in \Lambda} Q_\lambda) / V) = \text{cent} (T(R)) .
\end{aligned}$$

REMARK 1. Given \mathcal{P} as in §2, let $\varphi: Q \rightarrow T(R, \mathcal{P})$ be the canonical homomorphism. Then there is a partial order on {ideals A of Q : $\text{Ker } \varphi \subseteq A$ and $R \cap A = 0$ }. So there exists a maximal such ideal \bar{A} . Then $Q/\bar{A} \cong T(R, \mathcal{P})/(\bar{A}/(\text{Ker } \varphi))$ is an extension of R which has all the aforementioned properties of $T(R, \mathcal{P})$, and, moreover, any ideal of Q/\bar{A} intersects R (viewed as a subring) nontrivially.

REMARK 2. Suppose that R has an involution $(*)$. Then, for any prime P of degree j , there is a prime P^* of degree j and an isomorphism $R/P \rightarrow R/P^*$ given by $r + P \rightarrow r^* + P^*$. This isomorphism extends to the algebra of central quotients, and one can check that in the definition of $T(R)$, an involution is induced in Q . Moreover, V is stable under this involution, so $T(R)$ inherits an involution which coincides with $(*)$ on R . Hence the embedding $R \rightarrow T(R)$ is actually an embedding in the category of rings with involution.

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