

# Pacific Journal of Mathematics

**LOCALLY HOMEOMORPHIC  $\lambda$  CONNECTED PLANE  
CONTINUA**

CHARLES LEMUEL HAGOPIAN

## LOCALLY HOMEOMORPHIC $\lambda$ CONNECTED PLANE CONTINUA

CHARLES L. HAGOPIAN

A continuum  $X$  is  $\lambda$  connected if each two of its points can be joined by a hereditarily decomposable subcontinuum of  $X$ . Suppose that  $X$  and  $Y$  are plane continua and that there is a local homeomorphism that sends  $X$  onto  $Y$ . It follows from Theorem 5 in [2] that  $Y$  is  $\lambda$  connected if  $X$  is  $\lambda$  connected. Here we prove that, conversely, if  $Y$  is  $\lambda$  connected, then  $X$  is  $\lambda$  connected.

A continuous function  $f$  of a topological space  $X$  to a topological space  $Y$  is a *local homeomorphism* if for each point  $x$  of  $X$  there exists an open set  $U$  in  $X$  containing  $x$  such that  $f(U)$  is open in  $Y$  and  $f$  restricted to  $U$  is a homeomorphism of  $U$  onto  $f(U)$ .

A nondegenerate compact connected metric space is a *continuum*.

Let  $X$  be a plane continuum. A continuum  $L$  in  $X$  is said to be a *link* in  $X$  if  $L$  is either the boundary of a complementary domain of  $X$  or the limit of a convergent sequence of complementary domains of  $X$ .

It is known that a plane continuum is  $\lambda$  connected if and only if each of its links is hereditarily decomposable [3, Th. 2].

**THEOREM.** *Suppose that  $X$  and  $Y$  are plane continua and that  $f$  is a local homeomorphism that sends  $X$  onto  $Y$ . Then if one of the two continua  $X$  or  $Y$  is  $\lambda$  connected, so is the other.*

*Proof.* In [2] it is proved that every planar continuous image of a  $\lambda$  connected continuum is  $\lambda$  connected. Hence to establish this theorem it will be sufficient to show that  $Y$  being  $\lambda$  connected implies that  $X$  is  $\lambda$  connected.

Assume that  $Y$  is  $\lambda$  connected and  $X$  is not. By Theorem 2 of [3], there exists an indecomposable continuum  $I$  that is contained in a link in  $X$ . Since  $f$  is a local homeomorphism,  $f(I)$  is a continuum in  $Y$ .

We first show that every subcontinuum of  $Y$  that contains a nonempty open subset of  $f(I)$  contains  $f(I)$ . To accomplish this we suppose that there exists a continuum  $H$  in  $Y$  that contains a nonempty open subset  $G$  of  $f(I)$  and does not contain  $f(I)$ . Define  $p$  to be a point of  $G$ . Let  $q$  be a point of  $f(I) - H$ . There exist points  $x$  and  $y$  of  $I$  and disjoint open sets  $U$  and  $V$  of  $X$  containing  $x$  and  $y$  respectively such that (1)  $f(x) = p$  and  $f(y) = q$ , (2)  $f(V) \cap$

$H = \emptyset$ , and (3)  $f(U \cap I)$  is a subset of  $G$ .

Since  $I$  is contained in a link in  $X$ , every continuum in  $X$  that contains a nonempty open subset of  $I$  contains  $I$  [2, Th. 1]. Hence infinitely many components of  $X - V$  meet  $U \cap I$ . Since  $f^{-1}(H)$  and  $V$  are disjoint in  $X$  and  $f(U \cap I)$  is contained in  $G$ , it follows that  $f^{-1}(H)$  has infinitely many components in  $X$ . According to Whyburn's theorem [6, Th. 7.5, p. 148], each component of  $f^{-1}(H)$  must be mapped onto  $H$  by  $f$ . But since  $X$  is compact and  $f$  is a local homeomorphism, for each point  $z$  of  $Y$ , the set  $f^{-1}(z)$  is finite. Hence we have a contradiction. It follows that every subcontinuum of  $Y$  that contains a nonempty open subset of  $f(I)$  contains  $f(I)$ .

Note that  $f(I)$  is indecomposable [4, Th. 9] and therefore a proper subcontinuum of  $Y$ . By Theorem 2 of [1], there exists a composant  $C$  of  $f(I)$  such that each subcontinuum of  $Y$  that meets both  $C$  and  $Y - f(I)$  contains  $f(I)$ . This contradicts the assumption that  $Y$  is  $\lambda$  connected. Hence  $X$  is  $\lambda$  connected.

*Comment.* We get a false statement when we substitute the word "arcwise" for " $\lambda$ " in the preceding theorem. The so called "Warsaw circle" [5, Ex. 4, p. 230] is an arcwise connected plane continuum that is the image of a nonarcwise connected plane continuum under a local homeomorphism.

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Harm Bart, <i>Spectral properties of locally holomorphic vector-valued functions</i> . . . . .	321
J. Adrian (John) Bondy and Robert Louis Hemminger, <i>Reconstructing infinite graphs</i> . . . . .	331
Bryan Edmund Cain and Richard J. Tondra, <i>Biholomorphic approximation of planar domains</i> . . . . .	341
Richard Carey and Joel David Pincus, <i>Eigenvalues of seminormal operators, examples</i> . . . . .	347
Tyrone Duncan, <i>Absolute continuity for abstract Wiener spaces</i> . . . . .	359
Joe Wayne Fisher and Louis Halle Rowen, <i>An embedding of semiprime P.I.-rings</i> . . . . .	369
Andrew S. Geue, <i>Precompact and collectively semi-precompact sets of semi-precompact continuous linear operators</i> . . . . .	377
Charles Lemuel Hagopian, <i>Locally homeomorphic <math>\lambda</math> connected plane continua</i> . . . . .	403
Darald Joe Hartfiel, <i>A study of convex sets of stochastic matrices induced by probability vectors</i> . . . . .	405
Yasunori Ishibashi, <i>Some remarks on high order derivations</i> . . . . .	419
Donald Gordon James, <i>Orthogonal groups of dyadic unimodular quadratic forms. II</i> . . . . .	425
Geoffrey Thomas Jones, <i>Projective pseudo-complemented semilattices</i> . . . . .	443
Darrell Conley Kent, Kelly Denis McKennon, G. Richardson and M. Schroder, <i>Continuous convergence in <math>C(X)</math></i> . . . . .	457
J. J. Koliha, <i>Some convergence theorems in Banach algebras</i> . . . . .	467
Tsang Hai Kuo, <i>Projections in the spaces of bounded linear operations</i> . . . . .	475
George Berry Leeman, Jr., <i>A local estimate for typically real functions</i> . . . . .	481
Andrew Guy Markoe, <i>A characterization of normal analytic spaces by the homological codimension of the structure sheaf</i> . . . . .	485
Kunio Murasugi, <i>On the divisibility of knot groups</i> . . . . .	491
John Phillips, <i>Perturbations of type I von Neumann algebras</i> . . . . .	505
Billy E. Rhoades, <i>Commutants of some quasi-Hausdorff matrices</i> . . . . .	513
David W. Roeder, <i>Category theory applied to Pontryagin duality</i> . . . . .	519
Maxwell Alexander Rosenlicht, <i>The nonminimality of the differential closure</i> . . . . .	529
Peter Michael Rosenthal, <i>On an inversion theorem for the general Mehler-Fock transform pair</i> . . . . .	539
Alan Saleski, <i>Stopping times for Bernoulli automorphisms</i> . . . . .	547
John Herman Scheuneman, <i>Fundamental groups of compact complete locally affine complex surfaces. II</i> . . . . .	553
Vashishtha Narayan Singh, <i>Reproducing kernels and operators with a cyclic vector. I</i> . . . . .	567
Peggy Strait, <i>On the maximum and minimum of partial sums of random variables</i> . . . . .	585
J. L. Brenner, <i>Maximal ideals in the near ring of polynomials modulo 2</i> . . . . .	595
Ernst Gabor Straus, <i>Remark on the preceding paper: "Ideals in near rings of polynomials over a field"</i> . . . . .	601
Masamichi Takesaki, <i>Faithful states on a <math>C^*</math>-algebra</i> . . . . .	605
R. Michael Tanner, <i>Some content maximizing properties of the regular simplex</i> . . . . .	611
Andrew Bao-hwa Wang, <i>An analogue of the Paley-Wiener theorem for certain function spaces on <math>SL(2, \mathbb{C})</math></i> . . . . .	617
James Juei-Chin Yeh, <i>Inversion of conditional expectations</i> . . . . .	631