Pacific Journal of Mathematics

LOCALLY HOMEOMORPHIC λ CONNECTED PLANE CONTINUA

CHARLES LEMUEL HAGOPIAN

Vol. 52, No. 2 February 1974

PACIFIC JOURNAL OF MATHEMATICS
Vol. 52, No. 2, 1974

LOCALLY HOMEOMORPHIC λ CONNECTED PLANE CONTINUA

CHARLES L. HAGOPIAN

A continuum X is λ connected if each two of its points can be joined by a hereditarily decomposable subcontinuum of X. Suppose that X and Y are plane continua and that there is a local homeomorphism that sends X onto Y. It follows from Theorem 5 in [2] that Y is λ connected if X is λ connected. Here we prove that, conversely, if Y is λ connected, then X is λ connected.

A continuous function f of a topological space X to a topological space Y is a *local homeomorphism* if for each point x of X there exists an open set U in X containing x such that $f(U)$ is open in Y and f restricted to U is a homeomorphism of U onto $f(U)$.

A nondegenerate compact connected metric space is a *continuum*.

Let X be a plane continuum. A continuum L in X is said to be a *link* in X if L is either the boundary of a complementary domain of X or the limit of a convergent sequence of complementary domains of X.

It is known that a plane continuum is λ connected if and only if each of its links is hereditarily decomposable [3, Th. 2].

THEOREM. *Suppose that X and Y are plane continua and that f is a local homeomorphism that sends X onto Y. Then if one of the two continua X or Y is λ connected, so is the other.*

Proof. In [2] it is proved that every planar continuous image of a λ connected continuum is λ connected. Hence to establish this theorem it will be sufficient to show that Y being λ connected implies that X is λ connected.

Assume that Y is λ connected and X is not. By Theorem 2 of [3], there exists an indecomposable continuum I that is contained in a link in X. Since f is a local homeomorphism, $f(I)$ is a continuum in Y.

We first show that every subcontinuum of Y that contains a nonempty open subset of $f(I)$ contains $f(I)$. To accomplish this we suppose that there exists a continuum H in Y that contains a nonempty open subset G of $f(I)$ and does not contain $f(I)$. Define p to be a point of G. Let q be a point of $f(I) - H$. There exist points x and y of I and disjoint open sets U and V of X containing x and y respectively such that (1) $f(x) = p$ and $f(y) = q$, (2) $f(V) \cap$

$H = \emptyset$, and (3) $f(U \cap I)$ is a subset of G.

Since I is contained in a link in X, every continuum in X that contains a nonempty open subset of I contains I [2, Th. 1]. Hence infinitely many components of $X - V$ meet $U \cap I$. Since $f^{-1}(H)$ and V are disjoint in X and $f(U \cap I)$ is contained in G, it follows that $f^{-1}(H)$ has infinitely many components in X. According to Whyburn's theorem [6, Th. 7.5, p. 148], each component of $f^{-1}(H)$ must be mapped onto H by f. But since X is compact and f is a local homeomorphism, for each point z of Y, the set $f^{-1}(z)$ is finite. Hence we have a contradiction. It follows that every subcontinuum of Y that contains a nonempty open subset of $f(I)$ contains $f(I)$.

Note that $f(I)$ is indecomposable [4, Th. 9] and therefore a proper subcontinuum of Y. By Theorem 2 of [1], there exists a composant C of $f(I)$ such that each subcontinuum of Y that meets both C and $Y - f(I)$ contains $f(I)$. This contradicts the assumption that Y is λ connected. Hence X is λ connected.

Comment. We get a false statement when we substitute the word "arcwise" for "λ" in the preceding theorem. The so called "Warsaw circle" [5, Ex. 4, p. 230] is an arcwise connected plane continuum that is the image of a nonarcwise connected plane continuum under a local homeomorphism.

References

1. C. L. Hagopian, *Planar images of decomposable continua*, Pacific J. Math., **42** (1972), 329-331.
2. ———, *λ connected plane continua*, Trans. Amer. Math. Soc., **191** (1974), 277-287.
3. ———, *Planar λ connected continua*, Proc. Amer. Math. Soc., **39** (1973), 190-194.
4. F. B. Jones, *Concerning non-aposyndetic continua*, Amer. J. Math., **70** (1948), 403-413.
5. R. L. Moore, *Foundations of Point-set Theory*, Amer. Math. Soc. Colloq. Publ., Vol. 13, Providence, Rhode Island, 1962, revised edition.
6. G. T. Whyburn, *Analytic Topology*, Amer. Math. Soc. Colloq. Publ., Vol. 28, Providence, Rhode Island, 1942.

Received November 13, 1972.

CALIFORNIA STATE UNIVERSITY, SACRAMENTO

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be in typed form or offset-reproduced, (not dittoed), double spaced with large margins. Underline Greek letters in red, German in green, and script in blue. The first paragraph or two must be capable of being used separately as a synopsis of the entire paper. Items of the bibliography should not be cited there unless absolutely necessary, in which case they must be identified by author and Journal, rather than by item number. Manuscripts, in duplicate if possible, may be sent to any one of the four editors. Please classify according to the scheme of Math. Rev. Index to Vol. **39**. All other communications to the editors should be addressed to the managing editor, or Elaine Barth, University of California, Los Angeles, California, 90024.

100 reprints are provided free for each article, only if page charges have been substantially paid Additional copies may be obtained at cost in multiples of 50.

The *Pacific of Journal Mathematics* is issued monthly as of January 1966. Regular subscription rate: $72.00 a year (6 Vols., 12 issues). Special rate: $36.00 a year to individual members of supporting institutions.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley, California, 94708.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), 270, 3-chome Totsuka-cho, Shinjuku-ku, Tokyo 160, Japan.

Pacific Journal of Mathematics

Vol. 52, No. 2 February, 1974