

Pacific Journal of Mathematics

SOME REMARKS ON HIGH ORDER DERIVATIONS

YASUNORI ISHIBASHI

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Let k , A and B be commutative rings such that A and B are k -algebras. In this paper it is shown that $\Omega_k^{(q)}(A \otimes_k B)$, the module of high order differentials of $A \otimes_k B$ can be expressed by making use of $\Omega_k^{(q)}(A)$ and $\Omega_k^{(q)}(B)$. On the other hand let K/k be a finite purely inseparable field extension. Sandra Z. Keith has given a criterion for a k -linear mapping of K into itself to be a high order derivation of K/k . The representation of $\Omega_k^{(q)}(A \otimes_k B)$ is used to show that Keith's result is valid for larger class of algebras.

Let k , A and B be commutative rings with identities such that A and B are k -algebras. $A \otimes_k B$ is an A -algebra (resp. a B -algebra) via the natural homomorphism f_A (resp. f_B) such that $f_A(a) = a \otimes 1$ (resp. $f_B(b) = 1 \otimes b$). In [5] Y. Nakai proved that there exists a direct sum decomposition

$$\Omega_k^{(q)}(A \otimes_k B) = \Omega_k^{(q)}(A) \otimes_k B \oplus A \otimes_k \Omega_k^{(q)}(B) \oplus U_{A \otimes_k B|k}^{(q)}.$$

The submodule $U_{A \otimes_k B|k}^{(q)}$ has the universal mapping property with respect to q th order derivations of $A \otimes_k B$ which vanish on $f_A(A)$ and $f_B(B)$. In this paper we shall investigate the structure of $U_{A \otimes_k B|k}^{(q)}$. In fact we can express $U_{A \otimes_k B|k}^{(q)}$ by making use of $\Omega_k^{(i)}(A)$ and $\Omega_k^{(j)}(B)$ when k is a field.

On the other hand Sandra Z. Keith proved

THEOREM ([4]). *Let K/k be a finite purely inseparable field extension and let φ be a k -linear mapping of K into itself. Then we have $\varphi \in D_0^{(q)}(K/k)$ if and only if $\delta\varphi \in D_0^{(1)}(K/k) \smile D_0^{(q-1)}(K/k) + D_0^{(2)}(K/k) \smile D_0^{(q-2)}(K/k) + \dots + D_0^{(q-1)}(K/k) \smile D_0^{(1)}(K/k)$, where δ is the Hochschild coboundary operator (cf. [2]) and \smile denotes the cup-product.*

This gives an alternative inductive definition of q th order derivations which is meaningful for not-necessarily commutative rings but which possibly differs from Nakai's for commutative rings in general. In this paper we shall use our representation of $U_{A \otimes_k B|k}^{(q)}$ to show that Keith's result is generalized to larger class of algebras.

Any ring in this paper is assumed to be commutative and contain 1. Let k and A be commutative rings. We say that A is a k -algebra if there exists a ring homomorphism f such that $f(1) = 1$. The readers are expected to refer the paper [5] for notations and terminologies.

The author wishes to express his thanks to Professor Y. Nakai for his suggestions and encouragement.

1. Representation of $U_{A \otimes B/k}^{(q)}$. Let k , A and B be rings such that A and B are k -algebras.

LEMMA 1. Let D be an m th order derivation of A/k into an A -module M and let Δ be an n th order derivation of B/k into a B -module N . Then $D \otimes \Delta$ is an $(m + n)$ th order derivation of $A \otimes_k B$ into $M \otimes_k N$.

Proof. We consider the idealizations $A \oplus M$ and $B \oplus N$ of M and N respectively. Then D (resp. Δ) is regarded as an m th (resp. n th) order derivation of A (resp. B) into $A \oplus M$ (resp. $B \oplus N$). The mapping $D \otimes \Delta$ of $A \otimes_k B$ into $(A \oplus M) \otimes_k (B \oplus N)$ is decomposed as follows:

$$A \otimes_k B \xrightarrow{D \otimes 1_B} (A \oplus M) \otimes_k B \xrightarrow{1_{A \oplus M} \otimes \Delta} (A \oplus M) \otimes_k (B \oplus N).$$

By Corollary 6.1 in [5], $D \otimes \Delta$ is an $(m + n)$ th order derivation. The following lemmas are immediate.

LEMMA 2. In $A \otimes_k A$ we have

$$\begin{aligned} & (1 \otimes a_1 - a_1 \otimes 1) \cdots (1 \otimes a_q - a_q \otimes 1) \\ &= (1 \otimes a_1 \cdots a_q - a_1 \cdots a_q \otimes 1) \\ &+ \sum_{s=1}^{q-1} (-1)^s \sum_{i_1 < \cdots < i_s} a_{i_1} \cdots a_{i_s} (1 \otimes a_1 \cdots \hat{a}_{i_1} \cdots \hat{a}_{i_s} \cdots a_q \\ &- a_1 \cdots \hat{a}_{i_1} \cdots \hat{a}_{i_s} \cdots a_q \otimes 1). \end{aligned}$$

LEMMA 3. Let D be a q th order derivation of $A \otimes_k B$ into an $A \otimes_k B$ -module M vanishing on $f_A(A)$ and $f_B(B)$, where f_A (resp. f_B) is the homomorphism of A (resp. B) into $A \otimes_k B$ such that $f_A(a) = a \otimes 1$ (resp. $f_B(b) = 1 \otimes b$). Then we have

$$\begin{aligned} & D(a_1 \cdots a_i \otimes b_1 \cdots b_{q+1-i}) \\ &= \sum_{s=1}^{i-1} (-1)^{s-1} \sum_{\alpha_1 < \cdots < \alpha_s} (a_{\alpha_1} \cdots a_{\alpha_s} \otimes 1) \\ &\quad \times D(a_1 \cdots \hat{a}_{\alpha_1} \cdots \hat{a}_{\alpha_s} \cdots a_i \otimes b_1 \cdots b_{q+1-i}) \\ &+ \sum_{t=1}^{q-i} (-1)^{t-1} \sum_{\beta_1 < \cdots < \beta_t} (1 \otimes b_{\beta_1} \cdots b_{\beta_t}) \\ &\quad \times D(a_1 \cdots a_i \otimes b_1 \cdots \hat{b}_{\beta_1} \cdots \hat{b}_{\beta_t} \cdots b_{q+1-i}) \\ &+ \sum_{\substack{s \leq i-1, t \leq q-i \\ s, t=1}} (-1)^{s+t-1} \sum_{\substack{\alpha_1 < \cdots < \alpha_s \\ \beta_1 < \cdots < \beta_t}} (a_{\alpha_1} \cdots a_{\alpha_s} \otimes b_{\beta_1} \cdots b_{\beta_t}) \\ &\quad \times D(a_1 \cdots \hat{a}_{\alpha_1} \cdots \hat{a}_{\alpha_s} \cdots a_i \otimes b_1 \cdots \hat{b}_{\beta_1} \cdots \hat{b}_{\beta_t} \cdots b_{q+1-i}). \end{aligned}$$

We denote by $\delta_{A/k}^{(q)}$ the canonical q th order derivation of A into $\Omega_k^{(q)}(A)$. Unless any confusion arises, $\delta_{A/k}^{(q)}$ is denoted by $\delta_A^{(q)}$ or $\delta^{(q)}$ simply. If $i \leq j$, we have the canonical epimorphism φ_{ij} of $\Omega_k^{(j)}(A)$ onto $\Omega_k^{(i)}(A)$ given by $\varphi_{ij}(\delta^{(j)}a) = \delta^{(i)}a$. Let ψ_{ij} be the homomorphism of $\Omega_k^{(j)}(B)$ onto $\Omega_k^{(i)}(B)$ defined as above. We define the homomorphism Φ_q of $\bigoplus_{i=1}^{q-1} \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-i)}(B)$ into $\bigoplus_{i=1}^{q-2} \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-1-i)}(B)$ as follows: for $x \otimes y \in \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(j)}(B)$,

$$\Phi_q(x \otimes y) = \begin{cases} \varphi_{q-2, q-1}(x) \otimes y & \text{if } i = q - 1, j = 1 \\ \varphi_{i-1, i}(x) \otimes y - x \otimes \psi_{j-1, j}(y) & \text{if } i, j > 1 \\ -x \otimes \psi_{q-2, q-1}(y) & \text{if } i = 1, j = q - 1. \end{cases}$$

Obviously Φ_q is surjective.

THEOREM 1. *There exists a natural isomorphism*

- (1) $U_{A \otimes B/k}^{(2)} \cong \text{Ker } \Phi_2 = \Omega_k^{(1)}(A) \otimes_k \Omega_k^{(1)}(B)$,
- (2) for $q \geq 3$, $U_{A \otimes B/k}^{(q)} \cong \text{Ker } \Phi_q$ if k is a field.

Proof. We consider the mapping δ of $A \otimes_k B$ into $\bigoplus_{i=1}^{q-1} \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-i)}(B)$ defined by

$$\delta(a \otimes b) = \sum_{i=1}^{q-1} \delta_A^{(i)}a \otimes \delta_B^{(q-i)}b.$$

By Lemma 1 we see that δ is a q th order derivation. Since the image of δ is contained in $\text{Ker } \Phi_q$, δ induces a q th order derivation of $A \otimes_k B$ into $\text{Ker } \Phi_q$. The induced one is also denoted by δ . Clearly δ vanishes on $f_A(A)$ and $f_B(B)$. We have only to prove that the pair $\{\text{Ker } \Phi_q, \delta\}$ satisfies the universal mapping property with respect to q th order derivations of $A \otimes_k B$ which vanish on $f_A(A)$ and $f_B(B)$ ([5]). Let I_A (resp. I_B) be the kernel of the contraction mapping: $A \otimes_k A \rightarrow A$ (resp. $B \otimes_k B \rightarrow B$). We regard $I_A \otimes_k I_B$ as an $A \otimes_k B$ -module via

$$(a \otimes b)\{(x \otimes y) \otimes (u \otimes v)\} = (ax \otimes y) \otimes (bu \otimes v).$$

Under our assumption it will be shown that we have a natural isomorphism of $A \otimes_k B$ -modules

$$\text{Ker } \Phi_q \cong I_A \otimes_k I_B / \sum_{i=1}^q I_A^i \otimes I_B^{q+1-i},$$

where $I_A^i \otimes I_B^j$ denotes the image of the canonical homomorphism of $I_A^i \otimes_k I_B^j$ into $I_A \otimes_k I_B$. For $q = 2$ our assertion is obvious. For $q \geq 3$ we assume that k is a field. We define the $A \otimes_k B$ -linear mapping Ψ of $I_A \otimes_k I_B$ into $\bigoplus_{i=1}^{q-1} \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-i)}(B)$ by

$$\Psi((1 \otimes a - a \otimes 1) \otimes (1 \otimes b - b \otimes 1)) = \sum_{i=1}^{q-1} \delta_A^{(i)} a \otimes \delta_B^{(q-i)} b.$$

Obviously we have $\text{Im } \Psi \subset \text{Ker } \Phi_q$. We shall show that Ψ is an epimorphism of $I_A \otimes_k I_B$ onto $\text{Ker } \Phi_q$ with kernel $\sum_{i=1}^q I_A^i \otimes I_B^{q+1-i}$. Let $f \in I_A \otimes_k I_B$ and let $\pi_i(f)$ denote the canonical image of f in $\Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-i)}(B)$. We assume that $\sum_{i=1}^{q-1} \pi_i(f_i) \in \text{Ker } \Phi_q$ for $f_i \in I_A \otimes_k I_B$ ($1 \leq i \leq q-1$). From the definition of Φ_q we see that $f_i - f_{i+1} \in I_A^{i+1} \otimes I_B + I_A \otimes I_B^{q-i}$ ($1 \leq i \leq q-2$). Hence we have $f_i + \alpha_i = f_{i+1} + \beta_{i+1}$ for some $\alpha_i \in I_A^{i+1} \otimes I_B$ and $\beta_{i+1} \in I_A \otimes I_B^{q-i}$ ($1 \leq i \leq q-2$), and so it follows that $f_1 + \alpha_1 + \cdots + \alpha_{q-2} = f_2 + \beta_2 + \alpha_2 + \cdots + \alpha_{q-2} = \cdots = f_{q-1} + \beta_2 + \cdots + \beta_{q-1}$. Let f be this equal element of $I_A \otimes_k I_B$. Then we have $\pi_i(f) = \pi_i(f_i)$ and therefore Ψ is surjective. Next we prove $\text{Ker } \Psi = \sum_{i=1}^q I_A^i \otimes I_B^{q+1-i}$. Let us consider an element g of $I_A \otimes_k I_B$. If g is in $\text{Ker } \Psi$, we have $g \in I_A^{i+1} \otimes I_B + I_A \otimes I_B^{q+1-i}$ ($1 \leq i \leq q-1$) and so $g = \varepsilon_i + \zeta_i$ for suitable $\varepsilon_i \in I_A^{i+1} \otimes I_B$ and $\zeta_i \in I_A \otimes I_B^{q+1-i}$. On the other hand we get $\varepsilon_i - \varepsilon_{i+1} = \zeta_{i+1} - \zeta_i \in (I_A^{i+1} \otimes I_B) \cap (I_A \otimes I_B^{q-i}) = I_A^{i+1} \otimes I_B^{q-i}$ since k is a field. This implies easily $g \in \sum_{i=1}^q I_A^i \otimes I_B^{q+1-i}$. We wish to show that the pair $\{\text{Ker } \Phi_q, \delta\}$ has the universal mapping property. Let D be a q th order derivation of $A \otimes_k B$ into an $A \otimes_k B$ -module M vanishing on $f_A(A)$ and $f_B(B)$. Then it suffices to prove that there is an $A \otimes_k B$ -homomorphism θ of $I_A \otimes_k I_B / \sum_{i=1}^q I_A^i \otimes I_B^{q+1-i}$ into M satisfying

$$\theta(\pi\{(1 \otimes a - a \otimes 1) \otimes (1 \otimes b - b \otimes 1)\}) = D(a \otimes b),$$

where π is the canonical homomorphism of $I_A \otimes_k I_B$ onto $I_A \otimes_k I_B / \sum_{i=1}^q I_A^i \otimes I_B^{q+1-i}$. We consider the mapping Λ of $(A \otimes_k A) \otimes_k (B \otimes_k B)$ into M defined by

$$\Lambda((x \otimes y) \otimes (u \otimes v)) = (x \otimes u)D(y \otimes v).$$

Since D vanishes on $f_A(A)$ and $f_B(B)$, Λ induces the mapping of $I_A \otimes_k I_B$ into M sending $(1 \otimes a - a \otimes 1) \otimes (1 \otimes b - b \otimes 1)$ to $D(a \otimes b)$. Now it follows from Lemmas 2 and 3 that Λ vanishes on $\sum_{i=1}^q I_A^i \otimes I_B^{q+1-i}$, and so Λ induces the desired mapping θ . This completes our proof.

REMARK. If $\Omega_k^{(i)}(A) = I_A/I_A^{i+1}$ (resp. $\Omega_k^{(i)}(B) = I_B/I_B^{i+1}$) is k -flat for every i , we have $(I_A^{i+1} \otimes I_B) \cap (I_A \otimes I_B^{q-i}) = I_A^{i+1} \otimes I_B^{q-i}$ by [1] (§1, n°6, Proposition 7). In this case our proof shows that we have $U_{A \otimes_k B/k}^{(q)} \cong \text{Ker } \Phi_q$ for $q \geq 3$.

2. A generalization of the result due to Keith. Let k and A be rings such that A is a k -algebra. Let M and N be A -modules. We consider the homomorphism ω of $\text{Hom}_A(M, A) \otimes_k \text{Hom}_A(N, A)$ into $\text{Hom}_{A \otimes_k A}(M \otimes_k N, A)$ given by

$$[\omega(f \otimes g)](m \otimes n) = f(m)g(n)$$

for $f \in \text{Hom}_A(M, A)$, $g \in \text{Hom}_A(N, A)$, $m \in M$ and $n \in N$. Now A is regarded as an $A \otimes_k A$ -module via the contraction mapping: $A \otimes_k A \rightarrow A$.

LEMMA 4. *If M is a finite projective A -module, then ω is an epimorphism.*

Proof. When M is a finite free A -module, our assertion is obvious. If M is finite A -projective, M is a direct summand of a finite free A -module and hence we see easily that ω is an epimorphism.

Let φ and ψ be k -linear mappings of A into itself. The Hochschild coboundary $\delta\varphi$ of φ is given by $(\delta\varphi)(a, b) = \varphi(ab) - a\varphi(b) - b\varphi(a)$ for $a, b \in A$ (cf. [2]). On the other hand the cup-product $\varphi \smile \psi$ of φ and ψ is the k -bilinear mapping of $A \oplus A$ into A such that $(\varphi \smile \psi)(a, b) = \varphi(a)\psi(b)$ for $a, b \in A$. Let P and Q be A -submodules of $\text{Hom}_k(A, A)$, the set of k -linear mappings of A into itself. Then the cup-product $P \smile Q$ is the set of k -bilinear mappings of $A \oplus A$ into A which are finite sums of mappings of form $\varphi \smile \psi$ for $\varphi \in P$ and $\psi \in Q$.

THEOREM 2. *Let A be an algebra over a field k such that $\Omega_k^{(i)}(A)$ is a finite projective A -module for every $i \geq 1$. Let φ be a k -linear mapping of A into itself. Then we have $\varphi \in D_0^{(q)}(A/k)$ if and only if $\delta\varphi \in D_0^{(1)}(A/k) \smile D_0^{(q-1)}(A/k) + D_0^{(2)}(A/k) \smile D_0^{(q-2)}(A/k) + \dots + D_0^{(q-1)}(A/k) \smile D_0^{(1)}(A/k)$.*

Proof. By Theorem 1 we have an exact sequence

$$0 \longrightarrow U_{A \otimes_k A/k}^{(q)} \longrightarrow \bigoplus_{i=1}^{q-1} \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-i)}(A) \xrightarrow{\Phi_q} \bigoplus_{i=1}^{q-2} \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-1-i)}(A) \longrightarrow 0 .$$

Our assumption implies that $\Omega_k^{(i)}(A) \otimes_k \Omega_k^{(j)}(A)$ is a projective $A \otimes_k A$ -module, and so the above sequence splits. Hence we have an epimorphism of $\bigoplus_{i=1}^{q-1} \text{Hom}_{A \otimes_k A}(\Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-i)}(A), A)$ onto $\text{Hom}_{A \otimes_k A}(U_{A \otimes_k A/k}^{(q)}, A)$, where A is considered as an $A \otimes_k A$ -module via the contraction mapping: $A \otimes_k A \rightarrow A$. Since $\Omega_k^{(i)}(A)$ is finite A -projective, Lemma 4 is applicable to see that $\text{Hom}_A(\Omega_k^{(i)}(A), A) \otimes_k \text{Hom}_A(\Omega_k^{(j)}(A), A)$ is mapped onto $\text{Hom}_{A \otimes_k A}(\Omega_k^{(i)}(A) \otimes_k \Omega_k^{(j)}(A), A)$. Thus we get an epimorphism: $\bigoplus_{i=1}^{q-1} \text{Hom}_A(\Omega_k^{(i)}(A), A) \otimes_k \text{Hom}_A(\Omega_k^{(q-i)}(A), A) \rightarrow \text{Hom}_{A \otimes_k A}(U_{A \otimes_k A/k}^{(q)}, A)$. Let us consider an element φ of $D_0^{(q)}(A/k)$. The contraction mapping of $A \otimes_k A$ into A followed by φ is a q th order

derivation of $A \otimes_k A/k$ into A . From the direct sum decomposition of $\Omega_k^{(q)}(A \otimes_k A)$ it follows that $\delta\varphi$ gives an element of $\text{Hom}_{A \otimes_k A}(U_{A \otimes_k A}^{(q)}, A)$. Now only if part is immediate. On the other hand if part is obvious by Proposition 3 of [5].

REMARK. The assumption in Theorem 2 is satisfied in the following two cases, and so in these cases Theorem 2 holds.

- (1) A/k is a finitely generated field extension.
- (2) A is a smooth algebra over a field k ([3] 16.10.1, 16.10.2).

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Received August 1, 1973.

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The *Pacific Journal of Mathematics* is issued monthly as of January 1966. Regular subscription rate: \$72.00 a year (6 Vols., 12 issues). Special rate: \$36.00 a year to individual members of supporting institutions.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley, California, 94708.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), 270, 3-chome Totsuka-cho, Shinjuku-ku, Tokyo 160, Japan.

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Pacific Journal of Mathematics

Vol. 52, No. 2

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