

# Pacific Journal of Mathematics

**SOME REMARKS ON HIGH ORDER DERIVATIONS**

YASUNORI ISHIBASHI

## SOME REMARKS ON HIGH ORDER DERIVATIONS

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Let  $k$ ,  $A$  and  $B$  be commutative rings such that  $A$  and  $B$  are  $k$ -algebras. In this paper it is shown that  $\Omega_k^{(q)}(A \otimes_k B)$ , the module of high order differentials of  $A \otimes_k B$  can be expressed by making use of  $\Omega_k^{(q)}(A)$  and  $\Omega_k^{(q)}(B)$ . On the other hand let  $K/k$  be a finite purely inseparable field extension. Sandra Z. Keith has given a criterion for a  $k$ -linear mapping of  $K$  into itself to be a high order derivation of  $K/k$ . The representation of  $\Omega_k^{(q)}(A \otimes_k B)$  is used to show that Keith's result is valid for larger class of algebras.

Let  $k$ ,  $A$  and  $B$  be commutative rings with identities such that  $A$  and  $B$  are  $k$ -algebras.  $A \otimes_k B$  is an  $A$ -algebra (resp. a  $B$ -algebra) via the natural homomorphism  $f_A$  (resp.  $f_B$ ) such that  $f_A(a) = a \otimes 1$  (resp.  $f_B(b) = 1 \otimes b$ ). In [5] Y. Nakai proved that there exists a direct sum decomposition

$$\Omega_k^{(q)}(A \otimes_k B) = \Omega_k^{(q)}(A) \otimes_k B \oplus A \otimes_k \Omega_k^{(q)}(B) \oplus U_{A \otimes_k B}^{(q)}.$$

The submodule  $U_{A \otimes_k B}^{(q)}$  has the universal mapping property with respect to  $q$ th order derivations of  $A \otimes_k B$  which vanish on  $f_A(A)$  and  $f_B(B)$ . In this paper we shall investigate the structure of  $U_{A \otimes_k B}^{(q)}$ . In fact we can express  $U_{A \otimes_k B}^{(q)}$  by making use of  $\Omega_k^{(q)}(A)$  and  $\Omega_k^{(q)}(B)$  when  $k$  is a field.

On the other hand Sandra Z. Keith proved

**THEOREM ([4]).** *Let  $K/k$  be a finite purely inseparable field extension and let  $\varphi$  be a  $k$ -linear mapping of  $K$  into itself. Then we have  $\varphi \in D_0^{(q)}(K/k)$  if and only if  $\delta\varphi \in D_0^{(1)}(K/k) \smile D_0^{(q-1)}(K/k) + D_0^{(2)}(K/k) \smile D_0^{(q-2)}(K/k) + \dots + D_0^{(q-1)}(K/k) \smile D_0^{(1)}(K/k)$ , where  $\delta$  is the Hochschild coboundary operator (cf. [2]) and  $\smile$  denotes the cup-product.*

This gives an alternative inductive definition of  $q$ th order derivations which is meaningful for not-necessarily commutative rings but which possibly differs from Nakai's for commutative rings in general. In this paper we shall use our representation of  $U_{A \otimes_k B}^{(q)}$  to show that Keith's result is generalized to larger class of algebras.

Any ring in this paper is assumed to be commutative and contain 1. Let  $k$  and  $A$  be commutative rings. We say that  $A$  is a  $k$ -algebra if there exists a ring homomorphism  $f$  such that  $f(1) = 1$ . The readers are expected to refer the paper [5] for notations and terminologies.

The author wishes to express his thanks to Professor Y. Nakai for his suggestions and encouragement.

1. Representation of  $U_{A \otimes_k B}^{(q)}$ . Let  $k$ ,  $A$  and  $B$  be rings such that  $A$  and  $B$  are  $k$ -algebras.

LEMMA 1. Let  $D$  be an  $m$ th order derivation of  $A/k$  into an  $A$ -module  $M$  and let  $\Delta$  be an  $n$ th order derivation of  $B/k$  into a  $B$ -module  $N$ . Then  $D \otimes \Delta$  is an  $(m + n)$ th order derivation of  $A \otimes_k B$  into  $M \otimes_k N$ .

*Proof.* We consider the idealizations  $A \oplus M$  and  $B \oplus N$  of  $M$  and  $N$  respectively. Then  $D$  (resp.  $\Delta$ ) is regarded as an  $m$ th (resp.  $n$ th) order derivation of  $A$  (resp.  $B$ ) into  $A \oplus M$  (resp.  $B \oplus N$ ). The mapping  $D \otimes \Delta$  of  $A \otimes_k B$  into  $(A \oplus M) \otimes_k (B \oplus N)$  is decomposed as follows:

$$A \otimes_k B \xrightarrow{D \otimes 1_B} (A \oplus M) \otimes_k B \xrightarrow{1_{A \oplus M} \otimes \Delta} (A \oplus M) \otimes_k (B \oplus N).$$

By Corollary 6.1 in [5],  $D \otimes \Delta$  is an  $(m + n)$ th order derivation. The following lemmas are immediate.

LEMMA 2. In  $A \otimes_k A$  we have

$$\begin{aligned} & (1 \otimes a_1 - a_1 \otimes 1) \cdots (1 \otimes a_q - a_q \otimes 1) \\ &= (1 \otimes a_1 \cdots a_q - a_1 \cdots a_q \otimes 1) \\ &+ \sum_{s=1}^{q-1} (-1)^s \sum_{i_1 < \cdots < i_s} a_{i_1} \cdots a_{i_s} (1 \otimes a_1 \cdots \hat{a}_{i_1} \cdots \hat{a}_{i_s} \cdots a_q \\ &- a_1 \cdots \hat{a}_{i_1} \cdots \hat{a}_{i_s} \cdots a_q \otimes 1). \end{aligned}$$

LEMMA 3. Let  $D$  be a  $q$ th order derivation of  $A \otimes_k B$  into an  $A \otimes_k B$ -module  $M$  vanishing on  $f_A(A)$  and  $f_B(B)$ , where  $f_A$  (resp.  $f_B$ ) is the homomorphism of  $A$  (resp.  $B$ ) into  $A \otimes_k B$  such that  $f_A(a) = a \otimes 1$  (resp.  $f_B(b) = 1 \otimes b$ ). Then we have

$$\begin{aligned} & D(a_1 \cdots a_i \otimes b_1 \cdots b_{q+1-i}) \\ &= \sum_{s=1}^{i-1} (-1)^{s-1} \sum_{\alpha_1 < \cdots < \alpha_s} (a_{\alpha_1} \cdots a_{\alpha_s} \otimes 1) \\ &\quad \times D(a_1 \cdots \hat{a}_{\alpha_1} \cdots \hat{a}_{\alpha_s} \cdots a_i \otimes b_1 \cdots b_{q+1-i}) \\ &+ \sum_{t=1}^{q-i} (-1)^{t-1} \sum_{\beta_1 < \cdots < \beta_t} (1 \otimes b_{\beta_1} \cdots b_{\beta_t}) \\ &\quad \times D(a_1 \cdots a_i \otimes b_1 \cdots \hat{b}_{\beta_1} \cdots \hat{b}_{\beta_t} \cdots b_{q+1-i}) \\ &+ \sum_{\substack{s \leq i-1, t \leq q-i \\ s, t=1}} (-1)^{s+t-1} \sum_{\substack{\alpha_1 < \cdots < \alpha_s \\ \beta_1 < \cdots < \beta_t}} (a_{\alpha_1} \cdots a_{\alpha_s} \otimes b_{\beta_1} \cdots b_{\beta_t}) \\ &\quad \times D(a_1 \cdots \hat{a}_{\alpha_1} \cdots \hat{a}_{\alpha_s} \cdots a_i \otimes b_1 \cdots \hat{b}_{\beta_1} \cdots \hat{b}_{\beta_t} \cdots b_{q+1-i}). \end{aligned}$$

We denote by  $\delta_{A/k}^{(q)}$  the canonical  $q$ th order derivation of  $A$  into  $\Omega_k^{(q)}(A)$ . Unless any confusion arises,  $\delta_{A/k}^{(q)}$  is denoted by  $\delta_A^{(q)}$  or  $\delta^{(q)}$  simply. If  $i \leq j$ , we have the canonical epimorphism  $\varphi_{ij}$  of  $\Omega_k^{(j)}(A)$  onto  $\Omega_k^{(i)}(A)$  given by  $\varphi_{ij}(\delta^{(j)}a) = \delta^{(i)}a$ . Let  $\psi_{ij}$  be the homomorphism of  $\Omega_k^{(j)}(B)$  onto  $\Omega_k^{(i)}(B)$  defined as above. We define the homomorphism  $\Phi_q$  of  $\bigoplus_{i=1}^{q-1} \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-i)}(B)$  into  $\bigoplus_{i=1}^{q-2} \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-1-i)}(B)$  as follows: for  $x \otimes y \in \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(j)}(B)$ ,

$$\Phi_q(x \otimes y) = \begin{cases} \varphi_{q-2, q-1}(x) \otimes y & \text{if } i = q - 1, j = 1 \\ \varphi_{i-1, i}(x) \otimes y - x \otimes \psi_{j-1, j}(y) & \text{if } i, j > 1 \\ -x \otimes \psi_{q-2, q-1}(y) & \text{if } i = 1, j = q - 1. \end{cases}$$

Obviously  $\Phi_q$  is surjective.

**THEOREM 1.** *There exists a natural isomorphism*

- (1)  $U_{A \otimes B/k}^{(2)} \cong \text{Ker } \Phi_2 = \Omega_k^{(1)}(A) \otimes_k \Omega_k^{(1)}(B)$ ,
- (2) for  $q \geq 3$ ,  $U_{A \otimes B/k}^{(q)} \cong \text{Ker } \Phi_q$  if  $k$  is a field.

*Proof.* We consider the mapping  $\delta$  of  $A \otimes_k B$  into  $\bigoplus_{i=1}^{q-1} \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-i)}(B)$  defined by

$$\delta(a \otimes b) = \sum_{i=1}^{q-1} \delta_A^{(i)}a \otimes \delta_B^{(q-i)}b.$$

By Lemma 1 we see that  $\delta$  is a  $q$ th order derivation. Since the image of  $\delta$  is contained in  $\text{Ker } \Phi_q$ ,  $\delta$  induces a  $q$ th order derivation of  $A \otimes_k B$  into  $\text{Ker } \Phi_q$ . The induced one is also denoted by  $\delta$ . Clearly  $\delta$  vanishes on  $f_A(A)$  and  $f_B(B)$ . We have only to prove that the pair  $\{\text{Ker } \Phi_q, \delta\}$  satisfies the universal mapping property with respect to  $q$ th order derivations of  $A \otimes_k B$  which vanish on  $f_A(A)$  and  $f_B(B)$  ([5]). Let  $I_A$  (resp.  $I_B$ ) be the kernel of the contraction mapping:  $A \otimes_k A \rightarrow A$  (resp.  $B \otimes_k B \rightarrow B$ ). We regard  $I_A \otimes_k I_B$  as an  $A \otimes_k B$ -module via

$$(a \otimes b)\{(x \otimes y) \otimes (u \otimes v)\} = (ax \otimes y) \otimes (bu \otimes v).$$

Under our assumption it will be shown that we have a natural isomorphism of  $A \otimes_k B$ -modules

$$\text{Ker } \Phi_q \cong I_A \otimes_k I_B \Big/ \sum_{i=1}^q I_A^i \otimes I_B^{q+1-i},$$

where  $I_A^i \otimes I_B^j$  denotes the image of the canonical homomorphism of  $I_A^i \otimes_k I_B^j$  into  $I_A \otimes_k I_B$ . For  $q = 2$  our assertion is obvious. For  $q \geq 3$  we assume that  $k$  is a field. We define the  $A \otimes_k B$ -linear mapping  $\Psi$  of  $I_A \otimes_k I_B$  into  $\bigoplus_{i=1}^{q-1} \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-i)}(B)$  by

$$\Psi((1 \otimes a - a \otimes 1) \otimes (1 \otimes b - b \otimes 1)) = \sum_{i=1}^{q-1} \delta_A^{(i)} a \otimes \delta_B^{(q-i)} b.$$

Obviously we have  $\text{Im } \Psi \subset \text{Ker } \Phi_q$ . We shall show that  $\Psi$  is an epimorphism of  $I_A \otimes_k I_B$  onto  $\text{Ker } \Phi_q$  with kernel  $\sum_{i=1}^q I_A^i \otimes I_B^{q+1-i}$ . Let  $f \in I_A \otimes_k I_B$  and let  $\pi_i(f)$  denote the canonical image of  $f$  in  $\Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-i)}(B)$ . We assume that  $\sum_{i=1}^{q-1} \pi_i(f_i) \in \text{Ker } \Phi_q$  for  $f_i \in I_A \otimes_k I_B$  ( $1 \leq i \leq q-1$ ). From the definition of  $\Phi_q$  we see that  $f_i - f_{i+1} \in I_A^{i+1} \otimes I_B + I_A \otimes I_B^{q-i}$  ( $1 \leq i \leq q-2$ ). Hence we have  $f_i + \alpha_i = f_{i+1} + \beta_{i+1}$  for some  $\alpha_i \in I_A^{i+1} \otimes I_B$  and  $\beta_{i+1} \in I_A \otimes I_B^{q-i}$  ( $1 \leq i \leq q-2$ ), and so it follows that  $f_1 + \alpha_1 + \dots + \alpha_{q-2} = f_2 + \beta_2 + \alpha_2 + \dots + \alpha_{q-2} = \dots = f_{q-1} + \beta_2 + \dots + \beta_{q-1}$ . Let  $f$  be this equal element of  $I_A \otimes_k I_B$ . Then we have  $\pi_i(f) = \pi_i(f_i)$  and therefore  $\Psi$  is surjective. Next we prove  $\text{Ker } \Psi = \sum_{i=1}^q I_A^i \otimes I_B^{q+1-i}$ . Let us consider an element  $g$  of  $I_A \otimes_k I_B$ . If  $g$  is in  $\text{Ker } \Psi$ , we have  $g \in I_A^{i+1} \otimes I_B + I_A \otimes I_B^{q+1-i}$  ( $1 \leq i \leq q-1$ ) and so  $g = \varepsilon_i + \zeta_i$  for suitable  $\varepsilon_i \in I_A^{i+1} \otimes I_B$  and  $\zeta_i \in I_A \otimes I_B^{q+1-i}$ . On the other hand we get  $\varepsilon_i - \varepsilon_{i+1} = \zeta_{i+1} - \zeta_i \in (I_A^{i+1} \otimes I_B) \cap (I_A \otimes I_B^{q-i}) = I_A^{i+1} \otimes I_B^{q-i}$  since  $k$  is a field. This implies easily  $g \in \sum_{i=1}^q I_A^i \otimes I_B^{q+1-i}$ . We wish to show that the pair  $\{\text{Ker } \Phi_q, \delta\}$  has the universal mapping property. Let  $D$  be a  $q$ th order derivation of  $A \otimes_k B$  into an  $A \otimes_k B$ -module  $M$  vanishing on  $f_A(A)$  and  $f_B(B)$ . Then it suffices to prove that there is an  $A \otimes_k B$ -homomorphism  $\theta$  of  $I_A \otimes_k I_B / \sum_{i=1}^q I_A^i \otimes I_B^{q+1-i}$  into  $M$  satisfying

$$\theta(\pi\{(1 \otimes a - a \otimes 1) \otimes (1 \otimes b - b \otimes 1)\}) = D(a \otimes b),$$

where  $\pi$  is the canonical homomorphism of  $I_A \otimes_k I_B$  onto  $I_A \otimes_k I_B / \sum_{i=1}^q I_A^i \otimes I_B^{q+1-i}$ . We consider the mapping  $\lambda$  of  $(A \otimes_k A) \otimes_k (B \otimes_k B)$  into  $M$  defined by

$$\lambda((x \otimes y) \otimes (u \otimes v)) = (x \otimes u)D(y \otimes v).$$

Since  $D$  vanishes on  $f_A(A)$  and  $f_B(B)$ ,  $\lambda$  induces the mapping of  $I_A \otimes_k I_B$  into  $M$  sending  $(1 \otimes a - a \otimes 1) \otimes (1 \otimes b - b \otimes 1)$  to  $D(a \otimes b)$ . Now it follows from Lemmas 2 and 3 that  $\lambda$  vanishes on  $\sum_{i=1}^q I_A^i \otimes I_B^{q+1-i}$ , and so  $\lambda$  induces the desired mapping  $\theta$ . This completes our proof.

REMARK. If  $\Omega_k^{(i)}(A) = I_A/I_A^{i+1}$  (resp.  $\Omega_k^{(i)}(B) = I_B/I_B^{i+1}$ ) is  $k$ -flat for every  $i$ , we have  $(I_A^{i+1} \otimes I_B) \cap (I_A \otimes I_B^{q-i}) = I_A^{i+1} \otimes I_B^{q-i}$  by [1] (§1, n°6, Proposition 7). In this case our proof shows that we have  $U_{A \otimes B/k}^{(q)} \cong \text{Ker } \Phi_q$  for  $q \geq 3$ .

2. A generalization of the result due to Keith. Let  $k$  and  $A$  be rings such that  $A$  is a  $k$ -algebra. Let  $M$  and  $N$  be  $A$ -modules. We consider the homomorphism  $\omega$  of  $\text{Hom}_A(M, A) \otimes_k \text{Hom}_A(N, A)$  into  $\text{Hom}_{A \otimes_k A}(M \otimes_k N, A)$  given by

$$[\omega(f \otimes g)](m \otimes n) = f(m)g(n)$$

for  $f \in \text{Hom}_A(M, A)$ ,  $g \in \text{Hom}_A(N, A)$ ,  $m \in M$  and  $n \in N$ . Now  $A$  is regarded as an  $A \otimes_k A$ -module via the contraction mapping:  $A \otimes_k A \rightarrow A$ .

LEMMA 4. *If  $M$  is a finite projective  $A$ -module, then  $\omega$  is an epimorphism.*

*Proof.* When  $M$  is a finite free  $A$ -module, our assertion is obvious. If  $M$  is finite  $A$ -projective,  $M$  is a direct summand of a finite free  $A$ -module and hence we see easily that  $\omega$  is an epimorphism.

Let  $\varphi$  and  $\psi$  be  $k$ -linear mappings of  $A$  into itself. The Hochschild coboundary  $\delta\varphi$  of  $\varphi$  is given by  $(\delta\varphi)(a, b) = \varphi(ab) - a\varphi(b) - b\varphi(a)$  for  $a, b \in A$  (cf. [2]). On the other hand the cup-product  $\varphi \smile \psi$  of  $\varphi$  and  $\psi$  is the  $k$ -bilinear mapping of  $A \oplus A$  into  $A$  such that  $(\varphi \smile \psi)(a, b) = \varphi(a)\psi(b)$  for  $a, b \in A$ . Let  $P$  and  $Q$  be  $A$ -submodules of  $\text{Hom}_k(A, A)$ , the set of  $k$ -linear mappings of  $A$  into itself. Then the cup-product  $P \smile Q$  is the set of  $k$ -bilinear mappings of  $A \oplus A$  into  $A$  which are finite sums of mappings of form  $\varphi \smile \psi$  for  $\varphi \in P$  and  $\psi \in Q$ .

THEOREM 2. *Let  $A$  be an algebra over a field  $k$  such that  $\Omega_k^{(i)}(A)$  is a finite projective  $A$ -module for every  $i \geq 1$ . Let  $\varphi$  be a  $k$ -linear mapping of  $A$  into itself. Then we have  $\varphi \in D_0^{(q)}(A/k)$  if and only if  $\delta\varphi \in D_0^{(1)}(A/k) \smile D_0^{(q-1)}(A/k) + D_0^{(2)}(A/k) \smile D_0^{(q-2)}(A/k) + \dots + D_0^{(q-1)}(A/k) \smile D_0^{(1)}(A/k)$ .*

*Proof.* By Theorem 1 we have an exact sequence

$$0 \longrightarrow U_{A \otimes_k A/k}^{(q)} \longrightarrow \bigoplus_{i=1}^{q-1} \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-i)}(A) \xrightarrow{\Phi_q} \bigoplus_{i=1}^{q-2} \Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-1-i)}(A) \longrightarrow 0.$$

Our assumption implies that  $\Omega_k^{(i)}(A) \otimes_k \Omega_k^{(j)}(A)$  is a projective  $A \otimes_k A$ -module, and so the above sequence splits. Hence we have an epimorphism of  $\bigoplus_{i=1}^{q-1} \text{Hom}_{A \otimes_k A}(\Omega_k^{(i)}(A) \otimes_k \Omega_k^{(q-i)}(A), A)$  onto  $\text{Hom}_{A \otimes_k A}(U_{A \otimes_k A/k}^{(q)}, A)$ , where  $A$  is considered as an  $A \otimes_k A$ -module via the contraction mapping:  $A \otimes_k A \rightarrow A$ . Since  $\Omega_k^{(i)}(A)$  is finite  $A$ -projective, Lemma 4 is applicable to see that  $\text{Hom}_A(\Omega_k^{(i)}(A), A) \otimes_k \text{Hom}_A(\Omega_k^{(j)}(A), A)$  is mapped onto  $\text{Hom}_{A \otimes_k A}(\Omega_k^{(i)}(A) \otimes_k \Omega_k^{(j)}(A), A)$ . Thus we get an epimorphism:  $\bigoplus_{i=1}^{q-1} \text{Hom}_A(\Omega_k^{(i)}(A), A) \otimes_k \text{Hom}_A(\Omega_k^{(q-i)}(A), A) \rightarrow \text{Hom}_{A \otimes_k A}(U_{A \otimes_k A/k}^{(q)}, A)$ . Let us consider an element  $\varphi$  of  $D_0^{(q)}(A/k)$ . The contraction mapping of  $A \otimes_k A$  into  $A$  followed by  $\varphi$  is a  $q$ th order

derivation of  $A \otimes_k A/k$  into  $A$ . From the direct sum decomposition of  $\Omega_k^{(q)}(A \otimes_k A)$  it follows that  $\delta\varphi$  gives an element of  $\text{Hom}_{A \otimes_k A}(U_{A \otimes_k A/k}^{(q)}, A)$ . Now only if part is immediate. On the other hand if part is obvious by Proposition 3 of [5].

REMARK. The assumption in Theorem 2 is satisfied in the following two cases, and so in these cases Theorem 2 holds.

- (1)  $A/k$  is a finitely generated field extension.
- (2)  $A$  is a smooth algebra over a field  $k$  ([3] 16.10.1, 16.10.2).

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