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THE NON-MINIMALITY OF INDUCED CENTRAL REPRESENTATIONS

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Let G be a finite p -group and \mathfrak{G} a minimal faithful permutation representation of G possessing the minimal number of generators of the centre of G transitive constituents. One surmises that the induced representation, \mathfrak{G}' , of the centre of G , is minimal. The conjecture is validated subject to either of the hypotheses $|G| \leq p^5$ except $G = Q_8 \times Z_4$ or $Z(G) \cong n$ copies of the cyclic group of order p^m and is trivial when G is abelian. However, a group of order p^6 shows the conjecture to be false for p odd, also. The converse problem of extending minimal representations of $Z(G)$ to minimal representations of G is also, in general, not possible.

NOTATION. G a finite group, $Z(G)$ is the centre of G , $d(Z(G))$ is the minimal number of generators of $Z(G)$. When G is a p -group $\Omega_i(G) = \langle g \in G \mid g^p = e \rangle$. Zp^m is the cyclic group of order p^m . $\mu(G)$ is the least natural number n such that G can be embedded in the symmetric group of degree n .

Let $\mathfrak{G} = \{G_1, \dots, G_n\}$ be a collection of subgroups of a finite group G and X_i be the set of distinct cosets of G_i in G . The transitive action of G on X_i defines a permutation representation of G on the set $X = \bigcup_{i=1}^n X_i$ with kernel core $(\bigcap_{i=1}^n G_i)$. A faithful representation is called minimal in case $|X| = \sum_{i=1}^n |G : G_i|$ is minimal over all faithful \mathfrak{G} . Suppose now that G is a p -group and $d = d(Z(G))$. Then by [1] Theorem 3 $n = d$ for $p \neq 2$ whilst when $p = 2$ $21/2d \leq n \leq d$, the upper bound being attained. It is assumed throughout that $n = d$ thereby imposing a restriction on \mathfrak{G} only when $p = 2$.

The problem is approached by first classifying minimal representations \mathfrak{G} , say, of finite abelian p -groups (with a restriction on \mathfrak{G} if $p = 2$) and then observing two elementary properties regarding the structure of $G_i \cap Z(G)$.

1. Minimal representations of abelian groups.

THEOREM 1. *Let G be a finite abelian p -group with $n \geq 2$. Suppose $\mathfrak{G} = \{G_1, \dots, G_n\}$ is a minimal faithful permutation representation of G and $K_i = \bigcap_{\substack{j=1 \\ j \neq i}}^n G_j$, then*

$$G = \bigtimes_{i=1}^n K_i \quad \text{and} \quad G_i = \prod_{\substack{j=1 \\ j \neq i}}^n K_j .$$

NOTE. Any \mathfrak{G} of this form is a minimal representation of G , so this theorem characterizes minimal representations of abelian p -groups, $p \neq 2$.

Proof. If $G = Z_1 \times \cdots \times Z_n$ with Z_i cyclic then we know that the G_i can be reordered so that $G_i \cap Z_i = E$ (see [2], Lemma 2). Hence $|G:G_i| \geq |Z_i|$. Suppose for some k , $|G:G_k| > |Z_k|$, then

$$\mu(G) = \sum_{i=1}^n |G:G_i| > \sum_{i=1}^n |Z_i| = \mu(G)$$

so that $|G:G_i| = |Z_i|$, for all $1 \leq i \leq n$. Now

$$\begin{aligned} |G:K_i| &= \left| G: \bigcap_{\substack{j=1 \\ j \neq i}}^n G_j \right| \leq \prod_{\substack{j=1 \\ j \neq i}}^n |G:G_j|, \text{ Pointcaré's theorem} \\ &= \prod_{\substack{j=1 \\ j \neq i}}^n |Z_j| = |G:Z_i|. \end{aligned}$$

It follows that $|K_i| \geq |Z_i|$ and $|\times_{i=1}^n K_i| \geq \prod_{i=1}^n |Z_i| = |G|$ so that $G = \times_{i=1}^n K_i$ and $|K_i| = |Z_i|$ (see [3], Lemma 0). Also, $G_i \cong \prod_{\substack{j=1 \\ j \neq i}}^n K_j$ but $|G: \prod_{\substack{j=1 \\ j \neq i}}^n K_j| = |K_i| = |Z_i| = |G:G_i|$ and the lemma is now clear.

From the proof of [1], Proposition 2 we conclude that whenever G and H have coprime orders any stabilizer in a minimal representation of $G \times H$ has the form $G_1 \times H$ or $G \times H_1$, $G_1 \leq G$, $H_1 \leq H$. By decomposing an abelian group A into the direct product of its Sylow p -subgroups we easily generalize Theorem 1 to classify minimal representations of abelian groups (of odd order).

2. Induced central representation. Throughout this section whenever $\mathfrak{G} = \{G_1, \dots, G_n\}$, $n = d(Z(G))$.

LEMMA 2. No generator of $G_i \cap Z(G)$ is a p -power of any element in $Z(G)$ provided \mathfrak{G} is minimal.

Proof. Let $H_i = (\bigcap_{\substack{j=1 \\ j \neq i}}^n G_j) \cap Z(G)$. Since $G_i \cong H_1 \times \cdots \times H_{i-1} \times H_{i+1} \times \cdots \times H_n$, see [3] lemma, it follows that $d(G_i \cap Z(G)) = n - 1$. Suppose $G_i \cap Z(G) = \langle x_k | k \in I \rangle$ and $x_j = y^p$, for some j . Then $|I| \geq n - 1$. Define $Y = \langle x_k, y | k \in I \setminus \{j\} \rangle \cong G_i \cap Z(G)$. Clearly, $\Omega_1(Y) = \Omega_1(G_i \cap Z(G))$ and $Y G_i \cap Z(G) = Y$. Thus, the representation $\{G_1, \dots, G_{i-1}, Y G_i, G_{i+1}, \dots, G_n\}$ is faithful. The minimality of \mathfrak{G} yields $Y G_i = G_i$ so that $Y = G_i \cap Z(G)$. It follows that $x_j \in \langle x_k | k \in I \setminus \{j\} \rangle$, contradicting that x_i is a generator of G .

The next lemma is easy to verify.

LEMMA 3. Let $A = \mathbf{X}_{i=1}^n \langle a_i \rangle$ be an abelian p -group with $d(A) = n$. If $B \leq A$ with $d(B) = n - 1$ such that no generator of B is a p -power of any element of A then

(i) $B = \langle a_j \mid j \in N \setminus \{s\}, \text{ some } s \rangle$, where $N = \{k \mid 1 \leq k \leq n\}$

or

(ii) $B = \langle a_r a_s^{ir}, a_k \mid r \in J, k \in K, J \cup K = N \setminus \{s\}, \text{ some } s, J \cap K = \emptyset \rangle$.

COROLLARY. If $Z(G) = Z_1 \times \dots \times Z_n$ with $Z_i = \langle z_i \rangle$ cyclic then

$G_i \cap Z(G) = \langle z_j \mid j \in N \setminus \{s\} \rangle$

or

$G_i \cap Z(G) = \langle z_r z_s^{ir}, z_k \mid r \in J, k \in K, J \cup K = N \setminus \{s\}, J \cap K = \emptyset \rangle$.

Proof. By Lemma 2 $G_i \cap Z(G)$ and $Z(G)$ satisfy the conditions of Lemma 3.

Write $\mathfrak{G}' = \{G_1 \cap Z(G), \dots, G_n \cap Z(G)\}$ then:

LEMMA 4. \mathfrak{G}' is minimal whenever $Z(G) \cong n$ copies of Z_p^m .

Proof. $n = 1$ is trivial. For $n \neq 1$, by the corollary to Lemma 3 we deduce $|Z: G_i \cap Z(G)| = p^m, 1 \leq i \leq n$, yielding $\deg \mathfrak{G}' = np^m$ and \mathfrak{G}' is minimal.

THEOREM 5. If $|G| \leq p^5$ then \mathfrak{G}' is minimal, except for the case $p = 2, G = Q_8 \times Z_4$, the direct product of the quaternionic group of order 8 and the cyclic group of order 4.

Proof. We already have the result if G is abelian or $Z(G)$ is isomorphic to n copies of Z_p^m . This leaves the case: $|G| = p^5, Z(G) = \langle z_1 \rangle \times \langle z_2 \rangle \cong Z_{p^2} \times Z_p$. If $G = H \times K$ and is non-abelian then $K \cong Z_p$ or $K \cong Z_{p^2}$. Let $\mathfrak{G} = \{G_1, G_2\}$ be a minimal faithful representation of G . By [3], $\mu(G) = \mu(H) + \mu(K)$. When $K \cong Z_p, |G: G_1| = p$, say, and $G_1 \cap Z(H) \neq E$. By the corollary to Lemma 3, $G_1 \cong Z(H)$, so that \mathfrak{G}' is minimal. If $K \cong Z_{p^2}$, then except for the case $p = 2$ and $H \cong Q_8, \mu(H) = p^2$. Therefore, $\mu(G) = p^2 + p^2$ and $|G_1| = |G_2| = p^3$. As above, \mathfrak{G}' not minimal implies $G_1 \cap Z(H) = E = G_2 \cap Z(H)$. It follows that $G = G_1 Z(H) = G_2 Z(H)$ and G_1, G_2 are normal subgroups of G . Hence, $G_1 \cap G_2$ is a nontrivial normal subgroup of G , contradicting the faithfulness of \mathfrak{G} . When $G = Q_8 \times Z_4$, suppose $Q_8 = \langle x, y \mid x^2 = y^2, x^y = x^{-1} \rangle, Z_4 = \langle z \mid z^4 = e \rangle$. Then $\mathfrak{G} = \{Q_8, \langle xz \rangle\}$ is minimal but $\mathfrak{G}' = \{\langle x^2 \rangle, \langle x^2 z^2 \rangle\}$ is not. Under the hypothesis $G \neq Q_8 \times Z_4$, (a) any counterexample is not a nontrivial direct

product. We also have, (b) g^p is central for all $g \in G$, since $G/Z \cong Z_p \times Z_p$. By Lemma 2, since $|G_1 \cap Z(G)| = p = |G_2 \cap Z(G)|$, we may assume without loss of generality that $G_1 \cap Z(G) = \langle z_2 \rangle$, $G_2 \cap Z(G) = \langle z_1^r z_2 \rangle$ where $(r, p) = 1$ because $G_i \cong \langle z_i^p \rangle$ implies $G_i \cong \langle z_i \rangle$. Also, if $|G_i| = p^3$ then $G_i \cap Z_{p^2} = E$ yields $G = G_i Z_{p^2}$: Let $g \in G_i$, $h \in G$ then $h = g_1 z$, $g_1 \in G_i$, $z \in Z_{p^2}$ hence $g^h = g^{g_1 z} = g^{g_1} z \in G_i$ so G_i is normal in G and $G = G_i \times Z_{p^2}$, contradicting (a). We deduce, (c) $|G_i| \leq p^2$, $i = 1, 2$ and $\mu(G) \geq 2p^3$.

Let M be a maximal subgroup of G containing $Z(G)$, then M is abelian and has one of the forms:

- (i) $M = \langle a \rangle \times \langle b \rangle \times \langle c \rangle \cong Z_{p^2} \times Z_p \times Z_p$,
- (ii) $M = \langle a \rangle \times \langle b \rangle \cong Z_{p^3} \times Z_p$,
- (iii) $M = \langle a \rangle \times \langle b \rangle \cong Z_{p^2} \times Z_{p^2}$.

Case (i). We can choose a, b, c so that $Z(G) = \langle a \rangle \times \langle b \rangle$ and then $[\langle a, c \rangle \cap \langle b, c \rangle] \cap Z(G) = \langle c \rangle \cap Z(G) = E$ giving $\mu(G) \leq p^2 + p^3 < 2p^3$, contradicting (c). Case (ii). $Z(G) = \langle a^p \rangle \times \langle b \rangle$. Suppose $G/M = \langle cM \rangle$. $c^p = e$ implies case (i) holds. $c^p \neq e$ then $c^p = a^{pr} b^s$ by (b). If $p \mid r$, let $c_1 = ca^{-r} \notin M$ then $c_1^p = b^s$ and $\{\langle a \rangle, \langle c_1, b \rangle\}$ is faithful of degree less than $2p^3$. Hence for all $c \in G \setminus M$ $\langle c \rangle \cap \langle a \rangle \neq E$. Let $\mathfrak{G} = \{G_1, G_2\}$ be minimal then by Lemma 2, $G_i \cap \langle a \rangle = E$ and it follows that $|G_i| = p$, contradicting the minimality of \mathfrak{G} . Case (iii). Without loss of generality we may assume $Z(G) = \langle a \rangle \times \langle b^p \rangle$. Suppose $G/M = \langle cM \rangle$. $c^p = e$ implies case (i) holds. If $c^{p^2} \neq e$ then $\langle c \rangle \cap \langle a \rangle = E$ or $\langle c \rangle \cap \langle b \rangle = E$ so that $|c| = p^3$ and $\{\langle c \rangle, \langle a \rangle\}$ or $\{\langle c \rangle, \langle b \rangle\}$ is faithful of degree less than $2p^3$. This leaves the case $c^{p^2} = e$. c^p is central, $c^p = a^{pr} b^{ps}$, say, but $(ca^{-r})^p = b^{ps}$ and $ca^{-r} \notin M$. As above, $b^{ps} = e$ reduces to case (i). We may now assume that

$$G = \langle a, b, c \mid a^{p^2} = b^{p^2} = c^{p^2} = e = [a, b] = [a, c], b^p = c^p, [b, c] = a^{2u} b^{pv} \rangle.$$

If $a^{2u} = e$ then G is a nontrivial direct product. If $b^{pv} \neq e$ we can choose a so that $[b, c] = (a^p b^p)^v$ then $[ab, ac] = [b, c] = (ab)^{pv}$ but $G = \langle a, ab, ac \rangle$ and we proceed as above. By suitable choice of a it remains to eliminate the case $[b, c] = a^p$. Since $(b^{-1}c)^p = [b, c]^{-1/2} p(p+1)$, when $p \neq 2$ $(b^{-1}c)^p = e$ and when $p = 2$ $(ab^{-1}c)^2 = e$. In either case G/M can be generated by an element of order p . This completes the argument.

While attacking groups of order p^6 by identical methods to Theorem 5, one obtains the following counterexample.

THEOREM 6. *Let $G = \langle a, b, c \mid a^{p^3} = b^{p^2} = c^p = 1 = [a, b] = [a, c], [c, b] = a^{p^2} \rangle$ then*

- (i) $|G| = p^8$ and $Z(G) = \langle a \rangle \times \langle b^p \rangle \cong Z_{p^3} \times Z_p$,
- (ii) G is not a nontrivial direct product,
- (iii) $\mu(G) = p^2 + p^4$,

(iv) $\mathfrak{G} = \{\langle ab, c \rangle, \langle b \rangle\}$ is a minimal representation of G , but $\mathfrak{G}' = \{\langle ab, c \rangle \cap Z(G), \langle b \rangle \cap Z(G)\}$ is not minimal.

Proof. (i) For $1 \leq i \leq p^2$ define $\alpha_i, \beta_i, \gamma_i$ by

$$\alpha_i: (r, i, s) \mapsto (r, i, s + 1)$$

$$\beta_i: (r, i, s) \mapsto (r + s, i, s + 2)$$

$$\gamma_i: (r, i, s) \mapsto (r + 1, i, s)$$

$1 \leq r, s \leq p, \text{ mod } p$ in the first and third components [i.e., $\alpha_1 = ((1, 1, 1)(1, 1, 2) \cdots (1, 1, p))(2, 1, 1)(2, 1, 2) \cdots (2, 1, p) \cdots ((p, 1, 1) \cdots (p, 1, p))$]. $\alpha_i, \beta_i, \gamma_i$ each have order p and $[\alpha_i, \beta_i] = \gamma_i$. Define λ, μ, ν as follows

$$\lambda: (r, i, s) \mapsto \begin{cases} (r, i + 1, s), & 1 \leq i \leq p^2 \\ (r + 1, 1, s), & i = p^2 \end{cases}$$

$$\mu = (12 \cdots p^2) \prod_{i=1}^{p^2} \beta_i$$

$$\nu = \prod_{i=1}^{p^2} \alpha_i .$$

λ, μ, ν satisfy $\lambda^{p^3} = \mu^{p^2} = \nu^p = 1 = [\lambda, \mu] = [\lambda, \nu], [\nu, \mu] = \lambda^{p^2}$. Clearly any element of G has the form $a^i b^j c^k, 0 \leq i < p^3, 0 \leq j < p^2, 0 \leq k < p$ and the representation shows that these are distinct and (i) follows.

(ii) Suppose $G = H \times K$, then $Z(G) = Z(H) \times Z(K)$. We may assume $Z(H) \cong Z_p$ and $Z(K) = \langle ab^{p^s} \rangle \cong Z_{p^s}$. $K \cap \langle b \rangle = E$ implies $|K| \leq p^4$. If $|K| = p^4$ K and H are abelian and consequently G is abelian. It follows that $|K| = |H| = p^3$. Therefore, there exist $h \in H$ and $r, 0 \leq r < p^3$ such that $c = (ab^{p^s})^r h$ then $[h, b] = [(ab^{p^s})^r h, b]$ (since $(ab^{p^s})^r$ is central) $= [c, b] = a^{p^2}$. But H is normal in G and so $a^{p^2} = [h, b] \in H \cap K$, a contradiction.

(iii) Let $\mathfrak{G} = \{G_1, G_2\}$ be a minimal faithful representation of G . This always exists by [1], Theorem 3. If $|G:G_i| = p$ then G_i is normal in G and G is a nontrivial direct product. Therefore, $|G:G_i| \geq p^2, i = 1, 2$. For some $i, G_i \cap \langle a \rangle = E$, since \mathfrak{G} is faithful suppose, say, $G_1 \cap \langle a \rangle = E$. If $|G_1| = p^3, G = G_1 \times \langle a \rangle$ since a is central. Hence $\mu(G) \geq p^2 + p^4$ but (i) exhibits a faithful representation of degree $p^2 + p^4$. The final part of the theorem is now easy.

The converse problem: Given $\mathfrak{G}' = \{Z_1, \dots, Z_n\}, n = d(Z(G))$ a minimal representation of $Z(G)$, does there exist a minimal representation $\mathfrak{G} = \{G_1, \dots, G_n\}$ of G such that $G_i \cap Z(G) = Z_i$? The answer to this question is quickly found to be negative.

LEMMA 7. Let $G = H \times K$ where $H = \langle a, b \mid a^p = b^p = [a, b] \rangle$ and $K = \langle c \mid c^p = e \rangle$ then $\mathfrak{G}' = \{\langle a^p c \rangle, \langle c \rangle\}$ is a minimal representation of $Z(G)$ which cannot be extended to a minimal representation of G .

Proof. When $p \neq 2$ H is the non-abelian group of order p^3 containing an element of order p^2 and when $p = 2$ H is the quaternionic group of order 8. $Z(H) = \langle a^p \rangle$ and \mathfrak{G}' is obviously minimal. Now

$$\begin{aligned} (a^i b^j)^p &= b^{jp} (b^{-jp} a^i b^{jp}) (b^{-j(p-1)} a^i b^{j(p-1)}) \dots (b^{-j} a^i b^j), \quad j \neq 0 \\ &= a^{(i+j)p + ij p(1+\dots+p)}, \text{ since } a^b = a^{p+1}, (a^i)^{b^m} = a^{i(m p+1)}. \end{aligned}$$

Case I. $p \neq 2$ then $p \mid (1 + \dots + p) = 1/2 p(p+1)$ and

$$(*) \quad (a^i b^j c^k)^p = a^{(i+j)p} \text{ for all } i, j, k.$$

Every element of G has the form $a^i b^j c^k$, $0 \leq i < p^2$, $0 \leq j, k < p$. If $G_1 \cong \langle a^p c \rangle$ then $a^i b^j c^k \in G_1$ implies that $i + j = 0 \pmod{p}$ i.e., $j = rp - i$ consequently for each choice of i there is only one choice for j . It follows that $|G_1| \leq p^2$ and $|G:G_1| \geq p^2$ since $G_1 \cap \langle c \rangle = E$. By (*), $(ab^{p-1})^p = a^{p^2} = e$, $\langle ab^{p-1} \rangle \cap Z(H) = E$ and trivially $\mu(H) = p^2$. By [3], $\mu(G) = \mu(H) + \mu(K) = p^2 + p$. $G_2 \cong \langle c \rangle$ so $Z(H) \cap G_2 = E$ and $\{H, G_2\}$ is faithful. Therefore, $|G:H| + |G:G_2| \geq \mu(G) = p^2 + p$ and $|G:G_2| \geq p^2$. Hence $\deg \{G_1, G_2\} = |G:G_1| + |G:G_2| \geq 2p^2 > \mu(G)$ proving $\{G_1, G_2\}$ is not minimal.

Case II. $p = 2$, $\mu(H) = 8$ and $\mu(G) = \mu(H) + \mu(K) = 10$, by [3]. (*) becomes

$$(a^i b^j c^k)^2 = a^{(i+j)2 + ij^2} = \begin{cases} e, & i, j \text{ both even} \\ a^2, & \text{otherwise.} \end{cases}$$

One easily checks that $G_1 = \langle a^2 c \rangle$, $G_2 = \langle c \rangle$ and $\deg \{G_1, G_2\} = 16 > \mu(G)$ which proves the lemma.

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