

# Pacific Journal of Mathematics

## **THE NON-MINIMALITY OF INDUCED CENTRAL REPRESENTATIONS**

DAVID LEE WRIGHT

## THE NON-MINIMALITY OF INDUCED CENTRAL REPRESENTATIONS

D. WRIGHT

Let  $G$  be a finite  $p$ -group and  $\mathfrak{G}$  a minimal faithful permutation representation of  $G$  possessing the minimal number of generators of the centre of  $G$  transitive constituents. One surmises that the induced representation,  $\mathfrak{G}'$ , of the centre of  $G$ , is minimal. The conjecture is validated subject to either of the hypotheses  $|G| \leq p^5$  except  $G = Q_8 \times Z_4$  or  $Z(G) \cong n$  copies of the cyclic group of order  $p^m$  and is trivial when  $G$  is abelian. However, a group of order  $p^8$  shows the conjecture to be false for  $p$  odd, also. The converse problem of extending minimal representations of  $Z(G)$  to minimal representations of  $G$  is also, in general, not possible.

NOTATION.  $G$  a finite group,  $Z(G)$  is the centre of  $G$ ,  $d(Z(G))$  is the minimal number of generators of  $Z(G)$ . When  $G$  is a  $p$ -group  $\Omega_i(G) = \langle g \in G \mid g^{p^i} = e \rangle$ .  $Zp^m$  is the cyclic group of order  $p^m$ .  $\mu(G)$  is the least natural number  $n$  such that  $G$  can be embedded in the symmetric group of degree  $n$ .

Let  $\mathfrak{G} = \{G_1, \dots, G_n\}$  be a collection of subgroups of a finite group  $G$  and  $X_i$  be the set of distinct cosets of  $G_i$  in  $G$ . The transitive action of  $G$  on  $X_i$  defines a permutation representation of  $G$  on the set  $X = \bigcup_{i=1}^n X_i$  with kernel core  $(\bigcap_{i=1}^n G_i)$ . A faithful representation is called minimal in case  $|X| = \sum_{i=1}^n |G : G_i|$  is minimal over all faithful  $\mathfrak{G}$ . Suppose now that  $G$  is a  $p$ -group and  $d = d(Z(G))$ . Then by [1] Theorem 3  $n = d$  for  $p \neq 2$  whilst when  $p = 2$   $1/2d \leq n \leq d$ , the upper bound being attained. It is assumed throughout that  $n = d$  thereby imposing a restriction on  $\mathfrak{G}$  only when  $p = 2$ .

The problem is approached by first classifying minimal representations  $\mathfrak{G}$ , say, of finite abelian  $p$ -groups (with a restriction on  $\mathfrak{G}$  if  $p = 2$ ) and then observing two elementary properties regarding the structure of  $G_i \cap Z(G)$ .

### 1. Minimal representations of abelian groups.

**THEOREM 1.** *Let  $G$  be a finite abelian  $p$ -group with  $n \geq 2$ . Suppose  $\mathfrak{G} = \{G_1, \dots, G_n\}$  is a minimal faithful permutation representation of  $G$  and  $K_i = \bigcap_{\substack{j=1 \\ j \neq i}}^n G_j$ , then*

$$G = \bigtimes_{i=1}^n K_i \quad \text{and} \quad G_i = \prod_{\substack{j=1 \\ j \neq i}}^n K_j.$$

NOTE. Any  $\mathfrak{G}$  of this form is a minimal representation of  $G$ , so this theorem characterizes minimal representations of abelian  $p$ -groups,  $p \neq 2$ .

*Proof.* If  $G = Z_1 \times \cdots \times Z_n$  with  $Z_i$  cyclic then we know that the  $G_i$  can be reordered so that  $G_i \cap Z_i = E$  (see [2], Lemma 2). Hence  $|G:G_i| \geq |Z_i|$ . Suppose for some  $k$ ,  $|G:G_k| > |Z_k|$ , then

$$\mu(G) = \sum_{i=1}^n |G:G_i| > \sum_{i=1}^n |Z_i| = \mu(G)$$

so that  $|G:G_i| = |Z_i|$ , for all  $1 \leq i \leq n$ . Now

$$\begin{aligned} |G:K_i| &= \left| G: \prod_{\substack{j=1 \\ j \neq i}}^n G_j \right| \leq \prod_{\substack{j=1 \\ j \neq i}}^n |G:G_j|, \text{ Pointcaré's theorem} \\ &= \prod_{\substack{j=1 \\ j \neq i}}^n |Z_j| = |G:Z_i|. \end{aligned}$$

It follows that  $|K_i| \geq |Z_i|$  and  $|\times_{i=1}^n K_i| \geq \prod_{i=1}^n |Z_i| = |G|$  so that  $G = \times_{i=1}^n K_i$  and  $|K_i| = |Z_i|$  (see [3], Lemma 0). Also,  $G_i \cong \prod_{\substack{j=1 \\ j \neq i}}^n K_j$  but  $|G: \prod_{\substack{j=1 \\ j \neq i}}^n K_j| = |K_i| = |Z_i| = |G:G_i|$  and the lemma is now clear.

From the proof of [1], Proposition 2 we conclude that whenever  $G$  and  $H$  have coprime orders any stabilizer in a minimal representation of  $G \times H$  has the form  $G_1 \times H$  or  $G \times H_1$ ,  $G_1 \leq G$ ,  $H_1 \leq H$ . By decomposing an abelian group  $A$  into the direct product of its Sylow  $p$ -subgroups we easily generalize Theorem 1 to classify minimal representations of abelian groups (of odd order).

2. Induced central representation. Throughout this section whenever  $\mathfrak{G} = \{G_1, \dots, G_n\}$ ,  $n = d(Z(G))$ .

LEMMA 2. No generator of  $G_i \cap Z(G)$  is a  $p$ -power of any element in  $Z(G)$  provided  $\mathfrak{G}$  is minimal.

*Proof.* Let  $H_i = (\prod_{\substack{j=1 \\ j \neq i}}^n G_j) \cap Z(G)$ . Since  $G_i \cong H_1 \times \cdots \times H_{i-1} \times H_{i+1} \times \cdots \times H_n$ , see [3] lemma, it follows that  $d(G_i \cap Z(G)) = n - 1$ . Suppose  $G_i \cap Z(G) = \langle x_k \mid k \in I \rangle$  and  $x_j = y^p$ , for some  $j$ . Then  $|I| \geq n - 1$ . Define  $Y = \langle x_k, y \mid k \in I \setminus \{j\} \rangle \cong G_i \cap Z(G)$ . Clearly,  $\Omega_1(Y) = \Omega_1(G_i \cap Z(G))$  and  $YG_i \cap Z(G) = Y$ . Thus, the representation  $\{G_1, \dots, G_{i-1}, YG_i, G_{i+1}, \dots, G_n\}$  is faithful. The minimality of  $\mathfrak{G}$  yields  $YG_i = G_i$  so that  $Y = G_i \cap Z(G)$ . It follows that  $x_j \in \langle x_k \mid k \in I \setminus \{j\} \rangle$ , contradicting that  $x_i$  is a generator of  $G$ .

The next lemma is easy to verify.

LEMMA 3. Let  $A = \times_{i=1}^n \langle a_i \rangle$  be an abelian  $p$ -group with  $d(A) = n$ . If  $B \leq A$  with  $d(B) = n - 1$  such that no generator of  $B$  is a  $p$ -power of any element of  $A$  then

(i)  $B = \langle a_j \mid j \in N \setminus \{s\}, \text{ some } s \rangle$ , where  $N = \{k \mid 1 \leq k \leq n\}$

or

(ii)  $B = \langle a_r a_s^r, a_k \mid r \in J, k \in K, J \cup K = N \setminus \{s\}, \text{ some } s, J \cap K = \emptyset \rangle$ .

COROLLARY. If  $Z(G) = Z_1 \times \dots \times Z_n$  with  $Z_i = \langle z_i \rangle$  cyclic then

$$G_i \cap Z(G) = \langle z_j \mid j \in N \setminus \{s\} \rangle$$

or

$$G_i \cap Z(G) = \langle z_r z_s^r, z_k \mid r \in J, k \in K, J \cup K = N \setminus \{s\}, J \cap K = \emptyset \rangle.$$

*Proof.* By Lemma 2  $G_i \cap Z(G)$  and  $Z(G)$  satisfy the conditions of Lemma 3.

Write  $\mathfrak{G}' = \{G_1 \cap Z(G), \dots, G_n \cap Z(G)\}$  then:

LEMMA 4.  $\mathfrak{G}'$  is minimal whenever  $Z(G) \cong n$  copies of  $Z_p^m$ .

*Proof.*  $n = 1$  is trivial. For  $n \neq 1$ , by the corollary to Lemma 3 we deduce  $|Z:G_i \cap Z(G)| = p^m$ ,  $1 \leq i \leq n$ , yielding  $\deg \mathfrak{G}' = np^m$  and  $\mathfrak{G}'$  is minimal.

THEOREM 5. If  $|G| \leq p^5$  then  $\mathfrak{G}'$  is minimal, except for the case  $p = 2$ ,  $G = Q_8 \times Z_4$ , the direct product of the quaternionic group of order 8 and the cyclic group of order 4.

*Proof.* We already have the result if  $G$  is abelian or  $Z(G)$  is isomorphic to  $n$  copies of  $Z_p^m$ . This leaves the case:  $|G| = p^5$ ,  $Z(G) = \langle z_1 \rangle \times \langle z_2 \rangle \cong Z_{p^2} \times Z_p$ . If  $G = H \times K$  and is non-abelian then  $K \cong Z_p$  or  $K \cong Z_{p^2}$ . Let  $\mathfrak{G} = \{G_1, G_2\}$  be a minimal faithful representation of  $G$ . By [3],  $\mu(G) = \mu(H) + \mu(K)$ . When  $K \cong Z_p$ ,  $|G:G_1| = p$ , say, and  $G_1 \cap Z(H) \neq E$ . By the corollary to Lemma 3,  $G_1 \cong Z(H)$ , so that  $\mathfrak{G}'$  is minimal. If  $K \cong Z_{p^2}$ , then except for the case  $p = 2$  and  $H \cong Q_8$ ,  $\mu(H) = p^2$ . Therefore,  $\mu(G) = p^2 + p^2$  and  $|G_1| = |G_2| = p^3$ . As above,  $\mathfrak{G}'$  not minimal implies  $G_1 \cap Z(H) = E = G_2 \cap Z(H)$ . It follows that  $G = G_1 Z(H) = G_2 Z(H)$  and  $G_1, G_2$  are normal subgroups of  $G$ . Hence,  $G_1 \cap G_2$  is a nontrivial normal subgroup of  $G$ , contradicting the faithfulness of  $\mathfrak{G}$ . When  $G = Q_8 \times Z_4$ , suppose  $Q_8 = \langle x, y \mid x^2 = y^2, x^y = x^{-1} \rangle$ ,  $Z_4 = \langle z \mid z^4 = e \rangle$ . Then  $\mathfrak{G} = \{Q_8, \langle xz \rangle\}$  is minimal but  $\mathfrak{G}' = \{\langle x^2 \rangle, \langle x^2 z^2 \rangle\}$  is not. Under the hypothesis  $G \cong Q_8 \times Z_4$ , (a) any counterexample is not a nontrivial direct

product. We also have, (b)  $g^p$  is central for all  $g \in G$ , since  $G/Z \cong Z_p \times Z_p$ . By Lemma 2, since  $|G_1 \cap Z(G)| = p = |G_2 \cap Z(G)|$ , we may assume without loss of generality that  $G_1 \cap Z(G) = \langle z_2 \rangle$ ,  $G_2 \cap Z(G) = \langle z_1^r z_2 \rangle$  where  $(r, p) = 1$  because  $G_i \cong \langle z_i^p \rangle$  implies  $G_i \cong \langle z_i \rangle$ . Also, if  $|G_i| = p^3$  then  $G_i \cap Z_{p^2} = E$  yields  $G = G_i Z_{p^2}$ : Let  $g \in G_i$ ,  $h \in G$  then  $h = g_1 z$ ,  $g_1 \in G_i$ ,  $z \in Z_{p^2}$  hence  $g^h = g^{g_1 z} = g^{g_1} z \in G_i$  so  $G_i$  is normal in  $G$  and  $G = G_i \times Z_{p^2}$ , contradicting (a). We deduce, (c)  $|G_i| \leq p^2$ ,  $i = 1, 2$  and  $\mu(G) \geq 2p^3$ .

Let  $M$  be a maximal subgroup of  $G$  containing  $Z(G)$ , then  $M$  is abelian and has one of the forms:

- (i)  $M = \langle a \rangle \times \langle b \rangle \times \langle c \rangle \cong Z_{p^2} \times Z_p \times Z_p$ ,
- (ii)  $M = \langle a \rangle \times \langle b \rangle \cong Z_{p^3} \times Z_p$ ,
- (iii)  $M = \langle a \rangle \times \langle b \rangle \cong Z_{p^2} \times Z_{p^2}$ .

Case (i). We can choose  $a, b, c$  so that  $Z(G) = \langle a \rangle \times \langle b \rangle$  and then  $[\langle a, c \rangle \cap \langle b, c \rangle] \cap Z(G) = \langle c \rangle \cap Z(G) = E$  giving  $\mu(G) \leq p^2 + p^3 < 2p^3$ , contradicting (c). Case (ii).  $Z(G) = \langle a^p \rangle \times \langle b \rangle$ . Suppose  $G/M = \langle cM \rangle$ .  $c^p = e$  implies case (i) holds.  $c^p \neq e$  then  $c^p = a^{pr} b^s$  by (b). If  $p \mid r$ , let  $c_1 = ca^{-r} \in M$  then  $c_1^p = b^s$  and  $\{\langle a \rangle, \langle c_1, b \rangle\}$  is faithful of degree less than  $2p^3$ . Hence for all  $c \in G \setminus M$   $\langle c \rangle \cap \langle a \rangle \neq E$ . Let  $\mathfrak{G} = \{G_1, G_2\}$  be minimal then by Lemma 2,  $G_i \cap \langle a \rangle = E$  and it follows that  $|G_i| = p$ , contradicting the minimality of  $\mathfrak{G}$ . Case (iii). Without loss of generality we may assume  $Z(G) = \langle a \rangle \times \langle b^p \rangle$ . Suppose  $G/M = \langle cM \rangle$ .  $c^p = e$  implies case (i) holds. If  $c^{p^2} \neq e$  then  $\langle c \rangle \cap \langle a \rangle = E$  or  $\langle c \rangle \cap \langle b \rangle = E$  so that  $|c| = p^3$  and  $\{\langle c \rangle, \langle a \rangle\}$  or  $\{\langle c \rangle, \langle b \rangle\}$  is faithful of degree less than  $2p^3$ . This leaves the case  $c^{p^2} = e$ .  $c^p$  is central,  $c^p = a^{pr} b^{ps}$ , say, but  $(ca^{-r})^p = b^{ps}$  and  $ca^{-r} \notin M$ . As above,  $b^{ps} = e$  reduces to case (i). We may now assume that

$$G = \langle a, b, c \mid a^{p^2} = b^{p^2} = c^{p^2} = e = [a, b] = [a, c], b^p = c^p, [b, c] = a^{pu} b^{pv} \rangle.$$

If  $a^{pu} = e$  then  $G$  is a nontrivial direct product. If  $b^{pv} \neq e$  we can choose  $a$  so that  $[b, c] = (a^p b^p)^v$  then  $[ab, ac] = [b, c] = (ab)^{pv}$  but  $G = \langle a, ab, ac \rangle$  and we proceed as above. By suitable choice of  $a$  it remains to eliminate the case  $[b, c] = a^p$ . Since  $(b^{-1}c)^p = [b, c]^{-1/2 p(p+1)}$ , when  $p \neq 2$   $(b^{-1}c)^p = e$  and when  $p = 2$   $(ab^{-1}c)^2 = e$ . In either case  $G/M$  can be generated by an element of order  $p$ . This completes the argument.

While attacking groups of order  $p^6$  by identical methods to Theorem 5, one obtains the following counterexample.

**THEOREM 6.** *Let  $G = \langle a, b, c \mid a^{p^3} = b^{p^2} = c^p = 1 = [a, b] = [a, c], [c, b] = a^{p^2} \rangle$  then*

- (i)  $|G| = p^6$  and  $Z(G) = \langle a \rangle \times \langle b^p \rangle \cong Z_{p^3} \times Z_p$ ,
- (ii)  $G$  is not a nontrivial direct product,
- (iii)  $\mu(G) = p^2 + p^4$ ,

(iv)  $\mathfrak{G} = \{\langle ab, c \rangle, \langle b \rangle\}$  is a minimal representation of  $G$ , but  $\mathfrak{G}' = \{\langle ab, c \rangle \cap Z(G), \langle b \rangle \cap Z(G)\}$  is not minimal.

*Proof.* (i) For  $1 \leq i \leq p^2$  define  $\alpha_i, \beta_i, \gamma_i$  by

$$\alpha_i: (r, i, s) \mapsto (r, i, s + 1)$$

$$\beta_i: (r, i, s) \mapsto (r + s, i, s + 2)$$

$$\gamma_i: (r, i, s) \mapsto (r + 1, i, s)$$

$1 \leq r, s \leq p$ , mod  $p$  in the first and third components [i.e.,  $\alpha_1 = ((1, 1, 1)(1, 1, 2) \cdots (1, 1, p))(2, 1, 1)(2, 1, 2) \cdots (2, 1, p) \cdots ((p, 1, 1) \cdots (p, 1, p))$ ].  $\alpha_i, \beta_i, \gamma_i$  each have order  $p$  and  $[\alpha_i, \beta_i] = \gamma_i$ . Define  $\lambda, \mu, \nu$  as follows

$$\lambda: (r, i, s) \mapsto \begin{cases} (r, i + 1, s), & 1 \leq i \leq p^2 \\ (r + 1, 1, s), & i = p^2 \end{cases}$$

$$\mu = (12 \cdots p^2) \prod_{i=1}^{p^2} \beta_i$$

$$\nu = \prod_{i=1}^{p^2} \alpha_i .$$

$\lambda, \mu, \nu$  satisfy  $\lambda^{p^3} = \mu^{p^2} = \nu^p = 1 = [\lambda, \mu] = [\lambda, \nu], [\nu, \mu] = \lambda^{p^2}$ . Clearly any element of  $G$  has the form  $a^i b^j c^k, 0 \leq i < p^3, 0 \leq j < p^2, 0 \leq k < p$  and the representation shows that these are distinct and (i) follows.

(ii) Suppose  $G = H \times K$ , then  $Z(G) = Z(H) \times Z(K)$ . We may assume  $Z(H) \cong Z_p$  and  $Z(K) = \langle ab^{p^s} \rangle \cong Z_{p^3}$ .  $K \cap \langle b \rangle = E$  implies  $|K| \leq p^4$ . If  $|K| = p^4$   $K$  and  $H$  are abelian and consequently  $G$  is abelian. It follows that  $|K| = |H| = p^3$ . Therefore, there exist  $h \in H$  and  $r, 0 \leq r < p^3$  such that  $c = (ab^{p^s})^r h$  then  $[h, b] = [(ab^{p^s})^r h, b]$  (since  $(ab^{p^s})^r$  is central)  $= [c, b] = a^{p^2}$ . But  $H$  is normal in  $G$  and so  $a^{p^2} = [h, b] \in H \cap K$ , a contradiction.

(iii) Let  $\mathfrak{G} = \{G_1, G_2\}$  be a minimal faithful representation of  $G$ . This always exists by [1], Theorem 3. If  $|G:G_i| = p$  then  $G_i$  is normal in  $G$  and  $G$  is a nontrivial direct product. Therefore,  $|G:G_i| \geq p^2, i = 1, 2$ . For some  $i, G_i \cap \langle a \rangle = E$ , since  $\mathfrak{G}$  is faithful suppose, say,  $G_1 \cap \langle a \rangle = E$ . If  $|G_1| = p^3, G = G_1 \times \langle a \rangle$  since  $a$  is central. Hence  $\mu(G) \geq p^2 + p^4$  but (i) exhibits a faithful representation of degree  $p^2 + p^4$ . The final part of the theorem is now easy.

The converse problem: Given  $\mathfrak{G}' = \{Z_1, \dots, Z_n\}, n = d(Z(G))$  a minimal representation of  $Z(G)$ , does there exist a minimal representation  $\mathfrak{G} = \{G_1, \dots, G_n\}$  of  $G$  such that  $G_i \cap Z(G) = Z_i$ ? The answer to this question is quickly found to be negative.

LEMMA 7. Let  $G = H \times K$  where  $H = \langle a, b \mid a^p = b^p = [a, b] \rangle$  and  $K = \langle c \mid c^p = e \rangle$  then  $\mathfrak{G}' = \{\langle a^p c \rangle, \langle c \rangle\}$  is a minimal representation of  $Z(G)$  which cannot be extended to a minimal representation of  $G$ .

*Proof.* When  $p \neq 2$   $H$  is the non-abelian group of order  $p^3$  containing an element of order  $p^2$  and when  $p = 2$   $H$  is the quaternionic group of order 8.  $Z(H) = \langle a^p \rangle$  and  $\mathfrak{G}'$  is obviously minimal. Now

$$\begin{aligned} (a^i b^j)^p &= b^{j^p} (b^{-j^p} a^i b^{j^p}) (b^{-j^{p-1}} a^i b^{j^{p-1}}) \dots (b^{-j} a^i b^j), \quad j \neq 0 \\ &= a^{(i+j)p + ij^p(1+\dots+p)}, \text{ since } a^b = a^{p+1}, (a^i)^{b^m} = a^{i(mp+1)}. \end{aligned}$$

Case I.  $p \neq 2$  then  $p \mid (1 + \dots + p) = 1/2 p(p + 1)$  and

$$(*) \quad (a^i b^j c^k)^p = a^{(i+j)p} \text{ for all } i, j, k.$$

Every element of  $G$  has the form  $a^i b^j c^k$ ,  $0 \leq i < p^2$ ,  $0 \leq j, k < p$ . If  $G_1 \cong \langle a^p c \rangle$  then  $a^i b^j c^k \in G_1$  implies that  $i + j = 0 \pmod p$  i.e.,  $j = rp - i$  consequently for each choice of  $i$  there is only one choice for  $j$ . It follows that  $|G_1| \leq p^2$  and  $|G:G_1| \geq p^2$  since  $G_1 \cap \langle c \rangle = E$ . By (\*),  $(ab^{p-1})^p = a^{p^2} = e$ ,  $\langle ab^{p-1} \rangle \cap Z(H) = E$  and trivially  $\mu(H) = p^2$ . By [3],  $\mu(G) = \mu(H) + \mu(K) = p^2 + p$ .  $G_2 \cong \langle c \rangle$  so  $Z(H) \cap G_2 = E$  and  $\{H, G_2\}$  is faithful. Therefore,  $|G:H| + |G:G_2| \geq \mu(G) = p^2 + p$  and  $|G:G_2| \geq p^2$ . Hence  $\text{deg}\{G_1, G_2\} = |G:G_1| + |G:G_2| \geq 2p^2 > \mu(G)$  proving  $\{G_1, G_2\}$  is not minimal.

Case II.  $p = 2$ ,  $\mu(H) = 8$  and  $\mu(G) = \mu(H) + \mu(K) = 10$ , by [3]. (\*) becomes

$$(a^i b^j c^k)^2 = a^{(i+j)2 + ij^2} = \begin{cases} e, & i, j \text{ both even} \\ a^2, & \text{otherwise.} \end{cases}$$

One easily checks that  $G_1 = \langle a^2 c \rangle$ ,  $G_2 = \langle c \rangle$  and  $\text{deg}\{G_1, G_2\} = 16 > \mu(G)$  which proves the lemma.

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