

# Pacific Journal of Mathematics

## **CERTAIN REPRESENTATIONS OF INFINITE GROUP ALGEBRAS**

INDRANAND SINHA

# CERTAIN REPRESENTATIONS OF INFINITE GROUP ALGEBRAS

I. SINHA

For any group  $G$ , let  $\rho$  be an irreducible representation of the group algebra  $\mathfrak{F}G$  over a field  $\mathfrak{F}$ . Then by Schur's lemma, the center  $\mathcal{A}$  of its commuting ring, is a field containing  $\mathfrak{F}$ . If  $\rho$  is finite-dimensional over  $\mathcal{A}$ , then it is called finite and if it is finite-dimensional over  $\mathfrak{F}$  itself, then it is called strongly finite. In this paper, certain conditions are given for finiteness of  $\rho$ . Also it is shown that for some types of groups, finiteness of  $\rho$  is related to the existence of abelian subgroups of finite index in certain quotient of the group. Conditions under which finiteness and strongly finiteness are equivalent, are given. Finally, consequences of  $\rho$  being faithful on  $G$ , or being faithful on  $\mathfrak{F}G$ , are studied.

Study of finiteness of irreducible representations was initiated by Kaplansky in [3], and later carried to a great extent by Passman, Issacs, and others: {see [5] and relevant references therein}. Finiteness and strong finiteness were studied in [6]. Using a slight modification of the technique of [4] to suit our nonsemisimple case, we get Theorem 1 which includes the results of [4] and gives us Theorem 2 whose corollaries contains the result of [3].

We further recall the well-known result that for a finite group  $G$ , if the kernel of an irreducible representation  $\rho$  contains the commutator subgroup  $G'$ , then the representation is 1-dimensional over  $\mathcal{A}$ . As corollary to our Theorem 3, we prove that in general, if  $G'$  is contained in the kernel of  $\rho$ , then  $\rho$  is finite whether  $G$  is finite or not.

2. **Finiteness of representation.** In this section we study conditions under which a given irreducible representation is finite, and also the conditions under which all irreducible representations are finite. We need the following:

DEFINITIONS. 1. Let  $\rho$  be a representation of  $\mathfrak{F}G$ . Then  $G_\rho = \{g \in G \mid \rho(g) = 1\}$ , and  $\text{Kern } \rho = \text{kernel } \rho = \{x \in \mathfrak{F}G \mid \rho(x) = 0\}$ .

Thus  $\rho$  is  $G$ -faithful if  $G_\rho = 1$ , while  $\rho$  is  $\mathfrak{F}G$ -faithful if  $\text{Kern } \rho = 0$ .

2. Let  $\mathfrak{B} \leq \text{Aut } G$ . For  $S \leq G$ , we shall write  $\mathfrak{A}_s(S)$  for the left-ideal  $\{\sum x_i(\mathcal{S}_i^{\beta_i} - 1) \mid x_i \in \mathfrak{F}G, \mathcal{S}_i \in S, \beta_i \in \mathfrak{B}\}$ . {For a general study of such ideals we may refer to [6] and [8].} We write  $\mathfrak{A}(S)$ , if  $\mathfrak{B} = \{\text{identity}\}$ .

3. We also define the  $\mathfrak{B}$ -kernel of  $\rho$  in  $H \leq G$  to be

$$\{h \in H \mid \rho(h^\beta) = 1, \forall \beta \in \mathfrak{B}\},$$

and set  $K_n^{\mathfrak{B}}(H) = \bigcap \{\mathfrak{B}\text{-kernels of } \rho \text{ in } H\}$ , where the intersection runs through all irreducible representations  $\rho$  of  $G$  for which  $\dim_{\Delta} \rho > n^2$ , where  $\Delta$  is the center of the commuting ring of  $\rho$ . If no such  $\rho$  exists, then we put  $K_n^{\mathfrak{B}}(H) = G$ .

4. Let  $S_{2n}$  be the symmetric group of degree  $2n$ . Then for an algebra  $A$ , the sums

$$\sigma_n = \sum_{\sigma \in S_{2n}} (\text{sgn } \sigma) x_{\sigma(1)} \cdot x_{\sigma(2)} \cdots x_{\sigma(2n)}, \quad x_\rho \in A,$$

are called the Standard Monomial Sums (of parity  $n$ ).

5. Define  $\sum_n(G)$  to be the  $\mathfrak{F}$ -space spanned by all the  $\sigma_n$  in  $\mathfrak{F}G$ . {We shall frequently write  $\sum_n$  wherever the group in question is clear from context.}

This  $\sum_n$  plays a significant role in determining the degrees of irreducible representations.

Specifically we have:

**PROPOSITION 1.** *Let  $\rho$  be an irreducible representation of  $\mathfrak{F}G$ . Then  $\dim_{\Delta} \rho \leq n^2$  if and only if  $\sum_n \subseteq \text{Kern } \rho$ .*

*Proof.* If  $\dim_{\Delta} \rho \leq n^2$  then  $\mathfrak{F}G/\text{Kern } \rho$  is a primitive algebra of matrices of  $\dim n$  over  $\Delta$ . By Theorem 1 of [1], for any  $\sigma_n \in \mathfrak{F}G$ ,  $\rho(\sigma_n) = 0$ , whence  $\sigma_n \in \text{Kern } \rho$  so that  $\sum_n \subseteq \text{Kern } \rho$ .

Conversely, suppose  $\sum_n \subseteq \text{Kern } \rho$ . Then  $\mathfrak{F}G/\text{Kern } \rho$  is a primitive algebra satisfying  $\sigma_n = 0$  for every  $\sigma_n$  in  $\mathfrak{F}G/\text{Kern } \rho$ . Then by Theorem 1 of [2], p. 226,  $\mathfrak{F}G/\text{Kern } \rho$  is a central simple algebra of  $\dim \leq n^2$ . Hence  $\dim_{\Delta} \rho \leq n^2$ .

Using this result we obtain:

**THEOREM 1.** *Let  $S \leq H \leq G$ . Then  $S \subseteq K_n^{\mathfrak{B}}(H)$  if and only if  $\mathfrak{A}_{\mathfrak{B}}(S) \cdot \sum_n \subseteq \text{Rad } \mathfrak{F}G$ .*

*Proof.* We observe that  $\mathfrak{A}_{\mathfrak{B}}(S) = \{\sum x_i (s_i^{\beta} - 1) \mid x_i \in \mathfrak{F}G, s_i \in S, \beta_i \in \mathfrak{B}\}$ . Thus, to show that  $\mathfrak{A}_{\mathfrak{B}}(S) \cdot \sum_n \subseteq \text{Rad } \mathfrak{F}G$ , it suffices to show that  $(s^\beta - 1) \cdot \sum_n \subseteq \text{Rad } \mathfrak{F}G$ , for  $\forall \beta \in \mathfrak{B}, \forall s \in S$ . Now let  $h \in K_n^{\mathfrak{B}}(H)$  and  $\rho$  be an irreducible representation of  $G$ . If  $\dim_{\Delta} \rho > n^2$ , then  $\rho(h^\beta) = 1$  or  $\rho(h^\beta - 1) = 0, \forall \beta \in \mathfrak{B}$ , by the very definition of  $K_n^{\mathfrak{B}}(H)$ . On the other hand, if  $\dim_{\Delta} \rho \leq n^2$ , then by Proposition 1,  $\rho(\sum_n) = 0$ .

Thus, in both cases,  $\rho[(h^\beta - 1) \cdot \sum_n] = 0$ . Since  $\rho$  is arbitrary, so  $(h^\beta - 1) \cdot \sum_n \subseteq \text{Rad } \mathfrak{F}G$ . Hence  $S \subseteq K_n^{\mathfrak{B}}(H)$  implies  $\mathfrak{A}_{\mathfrak{B}}(S) \cdot \sum_n \subseteq \text{Rad } \mathfrak{F}G$ .

Conversely, suppose  $\mathfrak{A}_{\mathfrak{B}}(S) \cdot \sum_n \subseteq \text{Rad } \mathfrak{F}G$ . Then, in particular,

$$(s^\beta - 1) \cdot \sum_n \subseteq \text{Rad } \mathfrak{F}G, \forall \beta \in \mathfrak{B}, s \in S.$$

We define the left-idealizer [7], of  $\sum_n$  into  $\text{Rad } \mathfrak{F}G$ , by  $L_{\text{Rad}(\sum_n)\mathfrak{F}G} = L(\sum_n) = \{x \in \mathfrak{F}G \mid x \cdot \sum_n \subseteq \text{Rad } \mathfrak{F}G\}$ . This is clearly a left-ideal. Also  $[L(\sum_n) \cdot g] \cdot \sum_n = L(\sum_n)[g \cdot \sum_n \cdot g^{-1}] \cdot g = [L(\sum_n) \cdot \sum_n] \cdot g \subset \text{Rad } \mathfrak{F}G \cdot g = \text{Rad } \mathfrak{F}G$ , for  $\forall g \in G$ . Hence  $L(\sum_n)$  is a two-sided ideal of  $\mathfrak{F}G$ .

Now let  $\rho$  be an irreducible representation of  $G$ , afforded by the  $\mathfrak{F}G$ -module  $\mathfrak{M}$ . Since  $L(\sum_n)$  is a two-sided ideal, so  $\text{Ann } L(\sum_n) = \{m \in \mathfrak{M} \mid L(\sum_n)m = 0\}$  is an  $\mathfrak{F}G$ -submodule of  $\mathfrak{M}$ . Thus either  $\text{Ann } L(\sum_n) = 0$  or  $\mathfrak{M}$ . Now assume that  $\dim_{\Delta} \rho > n^2$ . Again, by Proposition 1,  $\rho(\sum_n) \neq 0$  so that  $\sum_n \cdot \mathfrak{M} \neq 0$ , whence  $\sum_n \not\subseteq \text{Rad } \mathfrak{F}G$ . But  $L(\sum_n) \cdot [\sum_n \cdot \mathfrak{M}] = [L(\sum_n) \cdot \sum_n] \cdot \mathfrak{M} = 0$ , since  $L(\sum_n) \cdot \sum_n \subseteq \text{Rad } \mathfrak{F}G$ . Thus  $\text{Ann } L(\sum_n) = \mathfrak{M}$ . Then, as by hypothesis  $s^\beta - 1 \in L(\sum_n)$ , so  $(s^\beta - 1) \cdot \mathfrak{M} = 0$ ; or  $\rho(s^\beta - 1) = 0$ . As  $\rho$  was arbitrary with  $\dim_{\Delta} \rho > n^2$ ; so  $s \in K_n^{\mathfrak{B}}(H)$ .

Letting  $\mathfrak{F} = \text{complex-field}$ , we have  $\text{Rad } \mathfrak{F}G = 0$ . Taking  $\mathfrak{B} = \{1\}$  in this case, we obtain the result of Passman [4]:

**COROLLARY.**  $g \in K_n(G)$  if and only if  $(g - 1) \cdot \sum_n = 0$ .  
We also deduce:

**THEOREM 2.** Let  $S \subseteq G$  and  $\mathfrak{B} \subseteq \text{Aug } G$  such that  $S^{\mathfrak{B}} = G$ . Then  $\mathfrak{A}_{\mathfrak{B}}(S) \cdot \sum_n \subseteq \text{Rad } \mathfrak{F}G$  if and only if  $\dim_{\Delta} \rho \leq n^2$  for every irreducible representation  $\rho$  of  $G$ . {Of course,  $\Delta$  depends on  $\rho$ .}

*Proof.* By Theorem 1,  $\mathfrak{A}_{\mathfrak{B}}(S) \cdot \sum_n \subseteq \text{Rad } \mathfrak{F}G$  if and only if  $S \subseteq K_n^{\mathfrak{B}}(G)$ :  $\{G = H\}$ .

The latter condition is equivalent to the statement that for every irreducible representation  $\rho$  with  $\dim_{\Delta} \rho > n^2$ , we have  $\rho(s^\beta) = 1$ ,  $\forall \beta \in \mathfrak{B}, s \in S$ . Since  $S^{\mathfrak{B}} = G$ , so we deduce that  $\rho = 1$ .

**COROLLARY 1.** If  $\sum_n \subseteq \text{Rad } \mathfrak{F}G$  for some  $n$ , then every irreducible representation of  $\mathfrak{F}G$  is finite.

**COROLLARY 2.** [3]. If in  $\mathfrak{F}G$ ,  $\sum_n = 0$  for some  $n$ , then every irreducible representation of  $G$  is finite.

Next recall that if  $|G| < \infty$  then  $G' \subseteq G_\rho$  for any irreducible

representation  $\rho$ , if and only if  $\rho$  is of dim. 1. A generalization of sort, is obtained in the corollary to:

**THEOREM 3.** *Let  $\rho$  be an irreducible representation of  $\mathfrak{F}G$ .*

(a) *If either (i)  $\sum_n(G/G_\rho) = 0$  for some  $n$ , or (ii)  $\exists A \leq G \ni \cdot$ ,  $G_\rho \leq A$ ,  $|G:A| < \infty$  and  $A/G_\rho$  is abelian, then  $\rho$  is finite.*

(b) (Conversely) *If  $\rho$  is finite and  $\mathfrak{F}G$  satisfies either of the following conditions:*

(i)  *$G/G_\rho$  is periodic and  $\mathfrak{F}(G/G_\rho)$  is nonmodular;*

(ii)  *$G/G_\rho$  is periodic with a finite  $p$ -Sylow subgroup for Char.  $\mathfrak{F} = p \neq 0$ ;*

(iii)  *$G/G_\rho$  satisfies minimum-condition on subgroups; then  $\exists A \leq G \ni \cdot$ ,  $G_\rho \leq A$ ,  $|G:A| < \infty$  and  $A/G_\rho$  is abelian.*

*Proof.* (a) Suppose (i) holds. In the notation of [6],  $G_\rho = \mathfrak{X}^{-1}(\text{Kern } \rho)$  where for any ideal  $I$  of  $\mathfrak{F}G$ ,  $\mathfrak{X}^{-1}(I) = \{g \in G \mid g - 1 \in I\}$ , and hence  $\mathfrak{X}(G_\rho)$  is a sub-ideal in  $\text{Kern } \rho$ . Since  $\mathfrak{X}(G_\rho)$  is the kernel of the linear extension of the canonical map  $G \rightarrow G/G_\rho$ , so  $\mathfrak{F}G/\mathfrak{X}(G_\rho) \cong \mathfrak{F}(G/G_\rho)$ . Therefore,  $\sum_n(G/G_\rho) = 0$  implies that the standard monomialsum, in  $\mathfrak{F}G/\mathfrak{X}(G_\rho)$ , all vanish. Now  $\mathfrak{F}G/\text{Kern } \rho \cong \mathfrak{F}G/\mathfrak{X}(G_\rho)/\text{Kern } \rho/\mathfrak{X}(G_\rho)$ ; therefore, the same holds for  $\mathfrak{F}G/\text{Kern } \rho$ . In particular,  $\sum_n(G) \subseteq \text{Kern } \rho$ . Then, by Proposition 1,  $\rho$  is finite. Next let (ii) hold. Then  $|G/G_\rho : A/G_\rho| = n < \infty$ , and  $A/G_\rho$  is abelian. Therefore, by the result of Kaplansky mentioned before, or by Theorems. 5.1, 8.1 of [5], all the irreducible representations of  $G/G_\rho$  are finite.

Now if  $\rho$  is afforded by the  $\mathfrak{F}G$ -module  $\mathfrak{M}$ , then putting  $\bar{\rho}(\bar{g}) \cdot m = \rho(g) \cdot m$ , for  $\bar{g} \in G/G_\rho$ , and observing that  $G_\rho = \{g \in G \mid \rho(g) = 1\}$ , we get a representation  $\bar{\rho}$  of  $G/G_\rho$ , such that  $\bar{\rho}$  is irreducible and the commuting ring of  $\bar{\rho}$  in  $\text{Hon}_{\mathfrak{F}}(\mathfrak{M}, \mathfrak{M})$ , is the same as that of  $\rho$ .

Thus the finiteness of  $\bar{\rho}$  implies the finiteness of  $\rho$ .

(b)  $G/G_\rho \cong S \leq GL(n, A)$  and any such  $S$  satisfying either of the conditions (i), (ii) or (iii), is abelian by finite: {see [9], Corollaries 9.4, 9.7, 9.8, and 9.23}. We then get our  $A$ , by taking the complete inverse-image of the abelian part of  $G/G_\rho$ .

Since the group-algebra of an abelian group always satisfies  $\sum_n = 0$ , so we obtain:

**COROLLARY.** *If  $G' \subseteq G_\rho$ , then  $\rho$  is finite.*

**3. Strong finiteness of representations.** In this section we give a result which shows the equivalence of finiteness and strong-finiteness in certain conditions.

**THEOREM 4.** *Under either of the following conditions, an irreducible representation  $\rho$  of  $G$  is finite if and only if it is strongly finite:*

- (i)  $G$  is finitely generated;
- (ii)  $\rho$  is absolutely irreducible;
- (iii)  $\exists H \trianglelefteq G \ni |G:H| < \infty$  and  $\rho_H$  has a strongly finite constituent.

*Proof.* (i) This is the content of Lemma 7 of [6].

(ii) Let the absolutely irreducible finite representation  $\rho$ , be afforded by the  $\mathfrak{F}G$ -module  $\mathfrak{M}$ . Since  $\Delta \cong \text{Hom}_{\mathfrak{F}}(\mathfrak{M}, \mathfrak{M})$ ,  $\mathfrak{F} \cong \Delta$ , so we can make  $\Delta \otimes_{\mathfrak{F}} \mathfrak{M}$  into a  $\Delta G$ -module by letting  $g \cdot (d \otimes m) = d \otimes \rho(g)m$ .

Define  $\psi: \Delta \otimes_{\mathfrak{F}} \mathfrak{M} \rightarrow \mathfrak{M}$  by  $\psi(d \otimes m) = dm$ . Since  $\rho$  and  $d$  commute, so

$$\psi(g \cdot (d \otimes m)) = \psi(d \otimes \rho(g)m) = d(\rho(g)m) = \rho(g)(dm).$$

Thus  $\psi$  is a  $\Delta G$ -homomorphism. Since  $\mathfrak{M}$  is absolutely irreducible, so  $\Delta \otimes_{\mathfrak{F}} \mathfrak{M}$  is irreducible. So  $\psi$  is an isomorphism. Then  $\dim_{\Delta}(\Delta \otimes_{\mathfrak{F}} \mathfrak{M}) = \dim_{\Delta} \mathfrak{M} < \infty$ . Thus  $\rho$  is also finite-dimensional over  $\mathfrak{F}$ .

(iii) By Clifford's theorem,  $\rho_H = \bigoplus \sum_{i=1}^{[G:H]} \rho_i$ , where  $\rho_i$  are all conjugate irreducible-representations of  $H$ . Hence, if one of them is finite-dimensional over  $\mathfrak{F}$ , then so are all; and hence  $\rho$ .

**4. Faithful representation.** Finally, let  $\rho$  be a representation (not necessarily irreducible) of the group algebra  $\mathfrak{F}G$ . For any left-ideal  $I$  we shall write  $\rho^I$  for the representation afforded by the module  $I \cdot \mathfrak{M}$ , where  $\rho$  is afforded by  $\mathfrak{M}$ . We shall let  $\mathfrak{A} = \mathfrak{A}(G)$  denote the augmentation-ideal of  $\mathfrak{F}G$  and  $J = [\mathfrak{F}G, \mathfrak{F}G]$ . Let  $\text{Char } \mathfrak{F} = p \neq 0$ .

We then investigate the consequences of  $\rho$  being faithful as a representation of  $G$  and as a representation of  $\mathfrak{F}G$  respectively. Recalling that if  $H \trianglelefteq G$ , then  $\mathfrak{A}(H)$  is the left-ideal in  $\mathfrak{F}G$  generated by  $\{h - 1 \mid h \in H\}$ : [6], we have the following:

**THEOREM 5.** (a) *If  $\rho$  is faithful on  $G$ , then  $A = \mathfrak{A}^{-1}(\text{Kern } \rho^{\mathfrak{A}})$  is an elementary abelian normal  $p$ -subgroup which is central if  $\mathfrak{A} \cong \text{Kern } \rho^{\mathfrak{A}}$ .*

(b) *If  $\rho$  is faithful on  $\mathfrak{F}G$ , then*

(i)  $A = 1$  unless  $|G| = 2$ ,  $p = 2$  in which case  $A = G$ ;

(ii)  $B = \mathfrak{A}^{-1}(\text{Kern } \rho^J) = 1$  unless  $p = 2$  and  $B = G$  is abelian, or  $p = 2$ ,  $G$  is nonabelian and  $B$  is central.

*Proof.* (a) Since  $\text{Kern } \rho^{\mathfrak{A}} \trianglelefteq \mathfrak{F}G$  so  $A \trianglelefteq G$ .

Let  $\mathfrak{M}$  afford  $\rho$  so that  $\mathfrak{X}\cdot\mathfrak{M}$  affords  $\rho^{\mathfrak{X}}$ . Hence  $g \in A$  if and only if  $(g - 1) \cdot \sum_{\substack{x \in G \\ \lambda_x \in \mathfrak{F}}} \lambda_x(x - 1)m = 0$  for each  $m \in \mathfrak{M}$ . Since  $p \neq 0$ , so  $(g^p - 1)m = (g - 1)^p m = (g - 1)[(g - 1)^{p-1}m] = 0$  as  $(g - 1)^{p-1}m \in \mathfrak{X}\cdot\mathfrak{M}$ . Thus  $g^p m = m$  and faithfulness implies that  $g^p = 1$ . Further, if  $h \in A$  then  $(h - 1)m$  and  $(g - 1)m$  are both in  $\mathfrak{X}\cdot\mathfrak{M}$ . Then,

$$(g - 1)(h - 1)m = 0 = (h - 1)(g - 1)m$$

or

$$g^{-1}h^{-1}gh m = m .$$

Again faithfulness gives that  $gh = hg$ ; i.e.,  $A$  is abelian.

If  $\mathfrak{X} \subseteq \text{Kern } \rho^{\mathfrak{X}}$ , then  $g \in G, h \in A$  implies

$$\begin{aligned} (gh - 1)m &= [(g - 1)(h - 1) + (g - 1) + (h - 1)]m \\ &= (g - 1)m + (h - 1)m = (hg - 1)m . \end{aligned}$$

Thus  $gh m = hg m$  and faithfulness gives that  $A \subseteq Z(G)$ .

(b) (i) Now let  $\rho$  be  $\mathfrak{F}G$ -faithful. Then the Kern  $\rho^{\mathfrak{X}} = \text{Ann } \mathfrak{X}$  in  $\mathfrak{F}G$ . It is well-known that this annihilator is 0 unless  $|G| < \infty$  and  $\text{Ann } \mathfrak{X} = \mathfrak{F} \cdot (\sum_{g \in G} g)$ . Now if  $g_i \in A$ , then  $g_i - 1 = a \in \text{Kern } \rho^{\mathfrak{X}}$  so that  $g_i - 1 = k \cdot \sum g, k \in \mathfrak{F}$ . Linear independence of the group elements, gives us that  $g_i = 1$ , and  $k = 0$ , or  $|G| = 2, g_i = 1, i = 2, k = 1$ , and  $+1 = -1$  in  $\mathfrak{F}$ .

(ii) Again by faithfulness  $\text{Kern } \rho' = \text{Ann } J$ . So  $g \in B$  implies that  $(g - 1)(hk - kh) = 0, \forall h, k \in G$ , i.e.,  $ghk - gkh - hk + kh = 0$ . If  $\text{Char } \mathfrak{F} \neq 2$ , then we must either have  $ghk = gkh$  in which case  $hk = kh$ , or  $ghk = hk$  in which case  $g = 1$ .

In case  $\text{Char } \mathfrak{F} = 2$  and  $g$  is noncentral then choose  $k \in G$  such that  $gk \neq kg$ . Put  $h = g^{-1}$ . Then the above identity gives,

$$k - gkg^{-1} - g^{-1}k + kg^{-1} = 0 .$$

Since  $gkg^{-1} \neq k$ , so either  $k = g^{-1}k$  or  $k = kg^{-1}$ , both leading to  $g = 1$ , a contradiction. Thus in this case  $g \in Z(G)$ .

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