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**A SHEAF THEORETIC REPRESENTATION OF RINGS WITH  
BOOLEAN ORTHOGONALITIES**

PATRICK NOBLE STEWART

## A SHEAF THEORETIC REPRESENTATION OF RINGS WITH BOOLEAN ORTHOGONALITIES

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**It is shown that certain associative rings with Boolean  
 orthogonalities are isomorphic to rings of global sections.**

Let  $A$  be a ring and  $\perp$  a relation on  $A$ . For each subset  $S$  of  $A$  define

$$S^\perp = \{x \in A \mid x \perp s \text{ for all } s \in S\} \text{ and } S^{\perp\perp} = (S^\perp)^\perp.$$

When  $S = \{s\}$  we write  $s^\perp$  and  $s^{\perp\perp}$  instead of  $\{s\}^\perp$  and  $\{s\}^{\perp\perp}$ . Subsets of  $A$  of the form  $S^\perp$  are *polars*. The relation  $\perp$  is a *Boolean orthogonality* if all polars are two-sided ideals and if, for all  $x, y \in A$ ,

1.  $x \perp y \rightarrow y \perp x$ ,    2.  $x \perp x \rightarrow x = 0$ , and
3.  $x^{\perp\perp} \cap y^{\perp\perp} = (0) \rightarrow x \perp y$ .

The set of polars is a Boolean algebra (see [3]) with meet and join defined by

$$B \wedge C = B \cap C \quad \text{and} \quad B \vee C = (B^\perp \wedge C^\perp)^\perp.$$

Boolean orthogonalities have been studied by Davis [3], Cornish [1] and by Cornish and Stewart [2].

Throughout this paper we shall assume that  $A$  is an associative ring with an identity and with a Boolean orthogonality  $\perp$ . We shall also assume that the following *finiteness condition* is satisfied:

*for any two elements  $x, y \in A$  there is a finite set  $F \subseteq A$   
 such that  $x^{\perp\perp} \wedge y^{\perp\perp} = F^{\perp\perp}$ .*

Notice that if  $F = \{f_1, \dots, f_n\}$ , then  $F^\perp = f_1^\perp \wedge \dots \wedge f_n^\perp$  and  $F^{\perp\perp} = f_1^{\perp\perp} \vee \dots \vee f_n^{\perp\perp}$ .

An ideal  $I$  of  $A$  is a  $\perp$ -ideal if  $F^{\perp\perp} \subseteq I$  for every finite set  $F \subseteq I$ , and  $I$  is  $\perp$ -prime if  $I \neq A$  and whenever the intersection of two polars  $B$  and  $C$  is contained in  $I$ , either  $B \subseteq I$  or  $C \subseteq I$ .

**LEMMA.** *Assume that  $P$  is either a  $\perp$ -prime ideal or  $P = A$ , that  $I$  is a  $\perp$ -ideal and that  $x \in A \setminus I$  is such that  $x^{\perp\perp} \wedge x^{\perp\perp} \subseteq I$  implies that  $a \in P$ . Then there is a  $\perp$ -prime  $\perp$ -ideal  $Q$  such that  $I \subseteq Q \subseteq P$  and  $x \notin Q$ .*

*Proof.* Using Zorn’s Lemma select a  $\perp$ -ideal  $Q \supseteq I$  maximal with respect to the property “ $x \notin Q$  and  $x^{\perp\perp} \wedge x^{\perp\perp} \subseteq Q$  implies that  $a \in P$ ”. Clearly  $I \subseteq Q \subseteq P$ .

Suppose that  $B$  and  $C$  are polars neither of which is contained in  $Q$ . Choose  $b \in B \setminus Q$  and  $c \in C \setminus Q$ . Then

$$B' = \cup \{F^{\perp\perp} \mid F \text{ is a finite subset of } \{b\} \cup Q\}$$

and

$$C' = \cup \{G^{\perp\perp} \mid G \text{ is a finite subset of } \{c\} \cup Q\}$$

are  $\perp$ -ideals which properly contain  $Q$ . By the maximality of  $Q$  either  $x^{\perp\perp} \subseteq B'$  or  $x^{\perp\perp} \wedge b_1^{\perp\perp} \subseteq B'$  for some  $b_1 \in A \setminus P$ , and  $x^{\perp\perp} \subseteq C'$  or  $x^{\perp\perp} \wedge c_1^{\perp\perp} \subseteq C'$  for some  $c_1 \in A \setminus P$ . Thus we obtain finite sets  $\{b, f_1, \dots, f_n\} \subseteq \{b\} \cup Q$  and  $\{c, g_1, \dots, g_m\} \subseteq \{c\} \cup Q$  such that one of  $x^{\perp\perp}$ ,  $x^{\perp\perp} \wedge b_1^{\perp\perp}$ ,  $x^{\perp\perp} \wedge c_1^{\perp\perp}$  or  $x^{\perp\perp} \wedge b_1^{\perp\perp} \wedge c_1^{\perp\perp}$  is contained in

$$\begin{aligned} & \{b, f_1, \dots, f_n\}^{\perp\perp} \wedge \{c, g_1, \dots, g_m\}^{\perp\perp} \\ &= (b^{\perp\perp} \vee f_1^{\perp\perp} \vee \dots \vee f_n^{\perp\perp}) \wedge (c^{\perp\perp} \vee g_1^{\perp\perp} \vee \dots \vee g_m^{\perp\perp}) \\ &= (b^{\perp\perp} \wedge c^{\perp\perp}) \vee H^{\perp\perp} \end{aligned}$$

where  $H$  is a finite subset of  $Q$  (we have used the distributivity of the Boolean algebra of polars and also the finiteness condition). If  $b^{\perp\perp} \wedge c^{\perp\perp} \subseteq Q$ , then  $x^{\perp\perp} \subseteq Q$  or  $x^{\perp\perp} \wedge l^{\perp\perp} \subseteq Q$  for some  $d \in A \setminus P$  both of which contradict the choice of  $Q$ . Thus  $B \cap C \not\subseteq Q$  and we conclude that  $Q$  is  $\perp$ -prime.

For the remainder of this paper  $\bar{X}$  will be fixed set of  $\perp$ -prime ideals which contains all  $\perp$ -prime  $\perp$ -ideals and which is full (that is, if  $I$  is a sum of polars and  $I \neq A$ , then  $I \subseteq P$  for some  $P \in \bar{X}$ ).

PROPOSITION 2. (Cornish [1]). For each  $P \in \bar{X}$ ,

$$\{x \in A \mid x^{\perp} \not\subseteq P\} = \cap \{R \in \bar{X} \mid R \subseteq P\} = \cap \{Q \in \bar{X} \mid Q \subseteq P$$

and  $Q$  is a  $\perp$ -prime  $\perp$ -ideal}.

*Proof.* Suppose that  $x^{\perp} \not\subseteq P$  and  $R$  is a  $\perp$ -prime ideal contained in  $P$ . Then  $x^{\perp\perp} \wedge x^{\perp} = (0) \subseteq R$  and so  $x \in R$ .

If  $x^{\perp} \subseteq P$ , then by Lemma 1 (take  $I = x^{\perp}$ ) there is a  $\perp$ -prime  $\perp$ -ideal  $Q \subseteq P$  such that  $x \notin Q$ . This establishes the result.

The set described in the proposition will be denoted by  $O_p$ . We note that  $O_p = P$  if and only if  $P$  is minimal in  $\bar{X}$ .

Let  $P \in \bar{X}$ . The set  $O_p$ , being an intersection of  $\perp$ -ideals, is itself a  $\perp$ -ideal. Define a relation (also denoted by  $\perp$ ) on  $A/O_p$  by

$$(x + O_p) \perp (y + O_p) \leftrightarrow x^{\perp\perp} \wedge y^{\perp\perp} \subseteq O_p.$$

This relation is well-defined because if  $x_1 = x + a$  and  $y_1 = y + b$  where  $a, b \in O_p$ , then

$$x_1^{\perp\perp} \wedge y_1^{\perp\perp} = (x + a)^{\perp\perp} \wedge (y + b)^{\perp\perp} \subseteq (x^{\perp\perp} \vee a^{\perp\perp}) \wedge (y^{\perp\perp} \vee b^{\perp\perp})$$

and so  $x_1^{\perp\perp} \wedge y_1^{\perp\perp} \subseteq (x^{\perp\perp} \wedge y^{\perp\perp}) \vee F^{\perp\perp}$  where  $F$  is a finite subset of  $O_p$ . It is routine to check that

$$x^{\perp} + O_p \subseteq (x + O_p)^{\perp} \quad \text{and} \quad x^{\perp\perp} + O_p \subseteq (x + O_p)^{\perp\perp}$$

for each  $x \in A$ , and that the relation  $\perp$  is a Boolean orthogonality on  $A/O_p$ .

**PROPOSITION 3.** *For each  $P \in \bar{X}$ ,  $\bar{P} = P/O_p$  is a  $\perp$ -prime ideal of  $A/O_p$  which contains all proper polars of  $A/O_p$ .*

*Proof.* Let  $\bar{B}$  and  $\bar{C}$  be polars in  $A/O_p$  such that  $\bar{B} \cap \bar{C} \subseteq \bar{P}$ . Suppose that  $\bar{B} \not\subseteq \bar{P}$ . Then there is an element  $b \in A$  such that  $b + O_p \in \bar{B} \setminus \bar{P}$ . Let  $c + O_p \in \bar{C}$ . Then

$$(b^{\perp\perp} + O_p) \cap (c^{\perp\perp} + O_p) \subseteq (b + O_p)^{\perp\perp} \cap (c + O_p)^{\perp\perp} \subseteq \bar{B} \cap \bar{C} \subseteq \bar{P}$$

and so  $b^{\perp\perp} \cap c^{\perp\perp} \subseteq P$ . Since  $b \notin P$  we conclude that  $c \in P$  and so  $\bar{C} \subseteq \bar{P}$ . Thus  $\bar{P}$  is  $\perp$ -prime.

Suppose that  $a^{\perp\perp} \wedge b^{\perp\perp} \subseteq O_p$ . Then there is a finite set  $\{f_1, \dots, f_n\} \subseteq O_p$  such that

$$a^{\perp\perp} \wedge b^{\perp\perp} = \{f_1, \dots, f_n\}^{\perp\perp} = f_1^{\perp\perp} \vee \dots \vee f_n^{\perp\perp}.$$

For each  $i = 1, \dots, n$ ,  $f_i \in O_p$  and so  $f_i^{\perp} \not\subseteq P$ . Thus  $f_1^{\perp} \wedge \dots \wedge f_n^{\perp} \not\subseteq P$ . Also,  $b^{\perp\perp} \wedge f_1^{\perp} \wedge \dots \wedge f_n^{\perp} \subseteq a^{\perp}$  because  $a^{\perp\perp} \wedge b^{\perp\perp} \wedge f_1^{\perp} \wedge \dots \wedge f_n^{\perp} = (0)$ . If  $a \notin O_p$ , then  $a^{\perp} \subseteq P$  and so, since  $f_1^{\perp} \wedge \dots \wedge f_n^{\perp} \not\subseteq P$ ,  $b^{\perp\perp} \subseteq P$ . Thus  $\bar{P}$  contains  $(a + O_p)^{\perp}$  for all  $a \notin O_p$ . It follows that  $\bar{P}$  contains all proper polars of  $A/O_p$ .

Let  $S$  be the disjoint union of the factor rings  $A/O_p$ . The relation (also denoted by  $\perp$ ) on the product

$$\begin{aligned} &\Pi\{A/O_p \mid P \in \bar{X}\} \\ &= \{f: \bar{X} \rightarrow S \mid f(P) \in A/O_p \text{ for all } P \in \bar{X}\} \end{aligned}$$

defined by

$$f \perp g \leftrightarrow f(P) \perp g(P) \text{ in } A/O_p \text{ for all } P \in \bar{X}$$

is a Boolean orthogonality. Each  $a \in A$  determines a function  $\hat{a} \in \Pi\{A/O_p \mid P \in \bar{X}\}$  defined by  $\hat{a}(P) = a + O_p$ . It follows from Lemma 1 that  $\cap\{P \mid P \text{ is a } \perp\text{-prime } \perp\text{-ideal}\} = (0)$  and so  $\cap\{O_p \mid P \in \bar{X}\} = (0)$ . Thus we obtain the usual embedding

$$A \xrightarrow{\hat{\cdot}} \hat{A} \subseteq \Pi\{A/O_p \mid P \in \bar{X}\}.$$

This embedding respects orthogonalities; that is,  $a \perp b$  in  $A$  if and only if  $\hat{a} \perp \hat{b}$  in the product.

We define a topology on  $\bar{X}$  by declaring the basic open sets to be the subsets of the form

$$\bar{X}(a) = \{P \in \bar{X} \mid a^{\perp\perp} \not\subseteq P\}.$$

Notice that  $\bar{X}(a) \cap \bar{X}(b) \supseteq \bar{X}(c)$  for all  $c \in a^{\perp\perp} \wedge b^{\perp\perp}$  and so these sets do qualify as a topological base.

Suppose that  $\{\bar{X}(a) \mid a \in C\}$  is a cover of  $\bar{X}$  consisting of basic open sets. Then  $\Sigma\{a^{\perp\perp} \mid a \in C\} = A$  because  $\bar{X}$  is full. Since  $A$  has an identity there is a finite set  $F \subseteq C$  such that  $\Sigma\{a^{\perp\perp} \mid a \in F\} = A$ . Thus  $\{\bar{X}(a) \mid a \in F\}$  covers  $\bar{X}$  and so  $\bar{X}$  is quasi-compact.

Give  $S$  the topology generated by sets of the form  $\hat{a}[U] = \{a + O_p \mid P \in U\}$  where  $U$  is open in  $\bar{X}$  and  $a \in A$ . We obtain a sheaf of rings  $(S, \pi, \bar{X})$  where  $\pi: S \rightarrow \bar{X}$  is the projection onto  $\bar{X}$ .

Let  $\Gamma = \{f \mid f \in \Pi\{A/O_p \mid P \in \bar{X}\} \text{ is continuous}\}$  be the ring of global sections. The following observation shows that  $\hat{A} \subseteq \Gamma$ : for all  $x, y \in A$ ,  $\{P \in \bar{X} \mid x - y \in O_p\}$  is open in  $\bar{X}$ . To see this notice that if  $x - y \in O_Q$ , then  $Q \in \bar{X}(u) \subseteq \{P \in \bar{X} \mid x - y \in O_p\}$  where  $u$  is any element in  $(x - y)^{\perp} \setminus Q$ .

**THEOREM 4.**  $\hat{A} = \Gamma$ .

*Proof.* Let  $f \in \Gamma$ . Since  $\bar{X}$  is quasi-compact there are finite sets  $\{a_1, \dots, a_n\}$  and  $\{v_1, \dots, v_n\}$  such that  $\bar{X} = \bar{X}(a_1) \cup \dots \cup \bar{X}(a_n)$  and  $f(P) = v_i + O_p$  for all  $P \in \bar{X}(a_i)$ .

Notice that  $v_i - v_j \in \cap\{O_p \mid P \in \bar{X}(a_i) \cap \bar{X}(a_j)\}$ , so  $(v_i - v_j)^{\perp} \subseteq$

$Q \in \bar{X}$  implies that  $a_i^{\perp\perp} \wedge a_j^{\perp\perp} \subseteq Q$ . It follows from Lemma 1 (take  $P = A$  and  $I = (v_i - v_j)^\perp$  for each  $x \notin (v_i - v_j)^\perp$ ) that

$$(v_i - v_j)^\perp = \cap \{Q \mid (v_i - v_j)^\perp \subseteq Q \in \bar{X}\}$$

and so  $a_i^{\perp\perp} \wedge a_j^{\perp\perp} \subseteq (v_i - v_j)^\perp$ . Thus  $(v_i - v_j)^{\perp\perp} \wedge a_j^{\perp\perp} \subseteq a_i^{\perp\perp}$ .

Since  $\bar{X} = \bar{X}(a_1) \cup \dots \cup \bar{X}(a_n)$  and  $\bar{X}$  is full,  $a_1^{\perp\perp} + \dots + a_n^{\perp\perp} = A$ . Choose  $u_i \in a_i^{\perp\perp}$  such that  $1 = u_1 + \dots + u_n$  and let  $v = u_1 v_1 + \dots + u_n v_n$ . Then

$$v - v_j = u_1(v_1 - v_j) + \dots + u_n(v_n - v_j) \in a_j^\perp \subseteq O_P$$

for all  $P \in \bar{X}(a_j)$ . Thus  $f(P) = v_j + O_P = v + O_P$  for all  $P \in \bar{X}(a_j)$  and so  $f = \hat{v} \in \hat{A}$ .

*f-rings* (Keimal [4]). Let  $A$  be an  $f$ -ring with identity. The relation defined by  $x \perp y \leftrightarrow |x| \wedge |y| = 0$  is a Boolean orthogonality and  $x^{\perp\perp} \wedge y^{\perp\perp} = (|x| \wedge |y|)^{\perp\perp}$ . Let  $\bar{X}$  be the set of irreducible  $\ell$ -ideals. Then  $\bar{X}$  is full because polars are  $\ell$ -ideals and sums of  $\ell$ -ideals are again  $\ell$ -ideals. Also, all  $\perp$ -prime  $\perp$ -ideals are irreducible  $\ell$ -ideals and so  $A$  is isomorphic to the  $f$ -ring of all global sections of the sheaf  $(S, \pi, \bar{X})$ .

*Reduced rings* (Koh [5]). Let  $A$  be a ring with identity and no nonzero nilpotent elements. The relation defined by  $x \perp y \leftrightarrow xy = 0$  is a Boolean orthogonality and  $x^{\perp\perp} \wedge y^{\perp\perp} = (xy)^{\perp\perp}$ . Let  $\bar{X}$  be the set of all prime ideals of  $A$ . Clearly  $\bar{X}$  is full. Also, all  $\perp$ -prime  $\perp$ -ideals are completely prime and so  $A$  is isomorphic to the ring of global sections of the sheaf  $(S, \pi, \bar{X})$ . Each stalk  $A/O_P$  is reduced (Proposition 2) and the prime ideal  $P/O_P$  contains all zero divisors (Proposition 3).

*Semiprime rings*. Let  $A$  be a semiprime ring with identity. The relation defined by  $x \perp y \leftrightarrow (x)(y) = (0)$  is a Boolean orthogonality. However, the finiteness condition may not be satisfied as the following example shows.

Let  $R$  be a semiprime ring with identity,  $R'$  the ring of  $3 \times 3$  matrices with entries from  $R$ ,

$$x = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Define  $\bar{x}$  and  $\bar{y}$  in  $P = \Pi\{R_n \mid R_n = R' \text{ for } n = 1, 2, \dots\}$  by

$$\begin{aligned} \bar{x}(n) &= \begin{cases} x & \text{if } n \equiv 1 \pmod{2} \\ 0 & \text{if } n \not\equiv 1 \pmod{2} \end{cases} \\ \bar{y}(n) &= \begin{cases} y & \text{if } n \equiv 1 \pmod{3} \\ 0 & \text{if } n \not\equiv 1 \pmod{3} \end{cases} \end{aligned}$$

Notice that  $\bar{x}\bar{y} = \bar{y}\bar{x} = \bar{x}^2 = \bar{y}^2 = 0$ . Let  $E$  be the subring of  $P$  which is generated by the identity of  $P$ ,  $\bar{x}$ ,  $\bar{y}$  and

$$\Sigma\{R_n \mid R_n = R' \text{ for } n = 1, 2, \dots\}. \text{ Then}$$

$$\bar{x}^{\perp\perp} = \{f \in E \mid f(n) = 0 \text{ for } n \not\equiv 1 \pmod{2}\},$$

$$\bar{y}^{\perp\perp} = \{f \in E \mid f(n) = 0 \text{ for } n \not\equiv 1 \pmod{3}\},$$

and so

$$\bar{x}^{\perp\perp} \wedge \bar{y}^{\perp\perp} = \{f \in E \mid f(n) = 0 \text{ for } n \not\equiv 1 \pmod{6}\}.$$

If  $\bar{x}^{\perp\perp} \wedge \bar{y}^{\perp\perp} = \{f_1, \dots, f_n\}^{\perp\perp}$ , then at least one of the  $f_i$  must satisfy  $f_i(n) \neq 0$  for infinitely many positive integers  $n$ . But then there are integers  $\alpha, \beta$  and  $\gamma$  such that  $f_i(n) = (\alpha + \beta\bar{x} + \gamma\bar{y})(n)$  for all but a finite number of positive integers  $n$ . This is incompatible with the requirement that  $f_i(n) = 0$  for  $n \not\equiv 1 \pmod{6}$ .

When the finiteness condition is satisfied (for instance, when  $A$  satisfies the maximum condition on annihilators),  $A$  is isomorphic to the ring of all global sections of the sheaf  $(S, \pi, \bar{X})$  where  $\bar{X}$  is the set of prime ideals of  $A$ . Each stalk  $A/O_p$  is semiprime (Proposition 1) and the prime ideal  $P/O_p$  contains all two-sided annihilator ideals of  $A/O_p$  (Proposition 2).

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