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**FINDING A MAXIMAL SUBALGEBRA ON WHICH THE TWO  
ARENS PRODUCTS AGREE**

JULIEN O. HENNEFELD

## FINDING A MAXIMAL SUBALGEBRA ON WHICH THE TWO ARENS PRODUCTS AGREE

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**Arens has given two ways of defining a Banach algebra product on the second dual of a Banach algebra  $\mathcal{A}$ . In this paper we give a construction for finding a maximal subalgebra on which the two Arens products agree. Moreover, we give an example which shows that there is not necessarily a unique maximal subalgebra on which the two Arens products agree. This example is a Banach algebra whose second dual has a *nonunique* element  $I$  which is simultaneously a right identity under the first Arens product and a left identity under the second Arens product.**

**1. Preliminaries.** The two Arens products are defined according to the following rules. Let  $\mathcal{A}$  be a Banach algebra. Let  $A, B \in \mathcal{A}$ ,  $f \in \mathcal{A}^*$ ,  $F, G \in \mathcal{A}^{**}$ .

DEFINITION 1.1.

$(f*_1A)B = f(AB)$  This defines  $f*_1A$  as an element of  $\mathcal{A}^*$ .

$(G*_1f)A = G(f*_1A)$  This defines  $G*_1f$  as an element of  $\mathcal{A}^*$ .

$(F*_1G)f = F(G*_1f)$  This defines  $F*_1G$  as an element of  $\mathcal{A}^{**}$ .

We will call  $F*_1G$  the first or the  $m_1$  product.

DEFINITION 1.2.  $(f*_2A)B = f(BA)$ ;  $(F*_2f)A = F(f*_2A)$ ;  $(F*_2G)f = G(F*_2f)$ .

We will call  $F*_2G$  the second or the  $m_2$  product.

PROPOSITION 1.3. *If  $\mathcal{A}$  has an approximate identity, then  $\mathcal{A}^{**}$  has an element  $I$  which is simultaneously a right  $m_1$  identity and a left  $m_2$  identity. Call such an element  $I$  a simultaneous identity.*

*Proof.*  $\mathcal{A}^{**}$  has a right  $m_1$  identity by [2, p. 146] the proof that it also has a left  $m_2$  identity is similar.

EXAMPLE 1.4. A simultaneous right  $m_1$  and left  $m_2$  identity, unlike a two-sided identity, is not necessarily unique.

Let  $X = c_0 \oplus_{sup} \ell^1$ . Let  $\{x_1, x_2, x_3, x_4, \dots\}$  be the basis  $\{d_1, e_1, d_2, e_2, \dots\}$  where  $\{d_i\}$  and  $\{e_i\}$  are the canonical bases for  $c_0$  and  $\ell^1$  respectively. Let  $\mathcal{D}$  be the norm closure of operators in  $\mathcal{B}(X)$  which have a finite matrix with respect to  $\{x_i\}$ . For each  $f \in \mathcal{D}^*$  we can associate a matrix  $(f_{ij})$  by defining  $f_{ij} = f(E_{ij})$  when

$E_{i,j}$  is the matrix in  $\mathcal{D}$  with a 1 in the  $ij^{\text{th}}$  place and 0's elsewhere.  $\mathcal{D}$  has an approximate identity, namely the operators  $E_n$  with 1's down the first  $n$  entries on the diagonal and 0's elsewhere.

Let  $T_n$  be the matrix with 1's in the  $j+1, j^{\text{th}}$  slots for  $j = 1, 3, 5, \dots, 2n-1$  and 0's elsewhere. Clearly  $\|T\| = n$  and so by the Hahn Banach theorem there exists an  $f_n \in \mathcal{D}^*$  of norm one with  $f_n(T_n) = n$ . Since  $f_n$  has norm one, each of its entries must have modulus  $\leq 1$ . This can be seen directly or from [7, Prop. 2.6]. Hence the matrix for  $f_n$  must have  $j+1, j^{\text{th}}$  entries = 1 for  $j = 1, 3, \dots, 2n-1$ .

By the weak star compactness of the unit ball of  $\mathcal{D}^*$  there exists an  $f$  which is a weak star cluster point of the  $f_n$ . Note that the  $j+1, j^{\text{th}}$  entries of  $f$  must all be 1, because if  $f_{m+1,m} \neq 1$  for some  $m$ , then the weak star neighborhood of  $f$  given by  $\mathcal{N}(f; E_{m+1,m}; \varepsilon)$  would not contain infinitely many  $f_n$  for  $\varepsilon$  small. It is clear that  $f$  is not in the subspace of  $\mathcal{D}^*$  spanned by those functionals whose matrices have either a finite number of rows or columns. Hence, there exists and  $H \in \mathcal{D}^{**}$  such that  $H(f) = 1$  and  $H(g) = 0$  if the matrix for  $g$  has a finite number of rows or a finite number of columns.

Note that  $I_*H = 0$  because for arbitrary  $g \in \mathcal{D}^*$ ,  $(I_*H)g = \lim_n (E_n *_1 H)g$  by the left weak star continuity of  $m_1$ . See [1]. This equals  $\lim_n E_n(H *_1 g) = \lim_n (H *_1 g)E_n = \lim_n H(g *_1 E_n)$ . But it is easily seen that for each  $n$ , the matrix for the functional  $g *_1 E_n$  has the same first  $n$  rows as that of  $g$  and zeroes elsewhere. This can be computed directly. Hence  $H(g *_1 E_n) = 0$  and so  $I_*H = 0$ . Similarly it can be seen that  $(H *_2 I)g = \lim (H *_2 E_n)g = \lim E_n(H *_2 g) = \lim (H *_2 g)E_n = \lim H(g *_2 E_n)$  and that the functional  $g *_2 E_n$  has as its matrix the first  $n$  columns of  $g$  and zeroes elsewhere. Thus  $H *_2 I = 0$ .

From the fact that  $I_*H = 0$  it follows that  $G *_1 H = 0$  for all  $G \in \mathcal{D}^{**}$  since  $G *_1 H = (G *_1 I) *_1 H = G *_1 (I_*H)$ . Similarly,  $H *_2 I = 0$  implies  $H *_2 G = 0$  for all  $G \in \mathcal{D}^{**}$ . It is easy to see that  $H + I$  is a simultaneous right  $m_1$  and left  $m_2$  identity.

2. The main result. Let  $\mathcal{A}$  be a Banach algebra and suppose the two Arens products agree on  $\mathcal{B}$  where  $\mathcal{A} \subset \mathcal{B} \subset \mathcal{A}^{**}$ . Then by Zorn's lemma, it follows that there exists an algebra  $\mathcal{M}$  with  $\mathcal{B} \subset \mathcal{M} \subset \mathcal{A}^{**}$  such that the two Arens products agree on  $\mathcal{M}$  and  $\mathcal{M}$  is maximal with respect to this property.

EXAMPLE 2.1. Let  $\mathcal{D}$  be the same Banach algebra as in Example 1.4. Then there is not a unique maximal subalgebra of  $\mathcal{D}^{**}$  on which the Arens products agree. Note that the Arens products agree on the algebra generated by  $[\mathcal{D}, I]$ . Also they agree on the

algebra generated by  $[\mathcal{D}, H]$ , since they agree if one factor is in  $\mathcal{D}$ , and also  $H*_1H = H*_2H = 0$ . However the Arens products cannot agree on any algebra containing  $I$  and  $H$ , since  $I*_1(I + H) = I$  but  $I*_2(I + H) = I + H$ .

**DEFINITION 2.2.** Let  $\mathcal{A}$  be a Banach algebra and  $E_\alpha$  an approximate identity with weak star limit  $I$  in  $\mathcal{A}^{**}$ . Then  $E_\alpha$  is called *projecting* if for each  $F \in \mathcal{A}^{**}$ ,  $E_\alpha*_1F*_1E_\beta$  is in  $\mathcal{A}$  for  $E_\alpha$  and  $E_\beta$  sufficiently far out.

**THEOREM 2.3.** Let  $E_\alpha$  be a projecting weak identity for  $\mathcal{A}$  and let  $I*_1(F*_2I) = F*_2I$  for all  $F \in \mathcal{A}^{**}$ . Then

(1)  $m_1 = m_2$  on  $\mathcal{N}$  where  $\mathcal{N} = \{F*_2I: F \in \mathcal{A}^{**}\}$

(2)  $\mathcal{N}$  is an algebra which is maximal with respect to the property that  $m_1 = m_2$ .

*Proof.* One of the difficulties is the fact that mixed Arens products like  $(F*_1G)*_2H$  are not necessarily associative. In this proof all limits will be in the weak star topology on  $\mathcal{A}^{**}$ . We will make frequent use of the fact that the two Arens products agree if one of the factors is in  $\mathcal{A}$ . Also we will make very careful use of the left weak star continuity of  $m_1$  and the right weak star continuity of  $m_2$ . Furthermore note that by the hypothesis on  $I$ , it follows that  $I*_1V = V$  for any  $V \in \mathcal{N}$ .

Given  $S = F*_2I$  and  $T = G*_2I$  we must show that  $S*_2T = S*_1T$ . Note that  $S*_2T = I*_1(S*_2T)$  since  $S*_2T$  is in  $\mathcal{N}$  and equals

$$(\lim_{\alpha} E_\alpha)*_1(S*_2T) = \lim_{\alpha} [E_\alpha*_1(S*_2T)] .$$

Note also that

$$S*_1T = (I*_1S)*_1T = \lim (E_\alpha*_1S)*_1T = \lim [E_\alpha*_1(S*_1T)] .$$

Hence it is sufficient to show that  $E_\beta*_1(S*_2T) = E_\beta*_1(S*_1T)$  for all  $E_\beta$  far enough out.

$$\begin{aligned} &\text{But since } E_\beta \in \mathcal{A}, E_\beta*_1(S*_2T) = E_\beta*_2(S*_2T) \\ &= (E_\beta*_2S)*_2T = (E_\beta*_2S)*_2(I*_1T) \\ &= (E_\beta*_2S)*_2 \lim_{\alpha} (E_\alpha*_2T) \text{ by the left weak star continuity of } m_1 \\ &= \lim_{\alpha} [(E_\beta*_2S)*_2(E_\alpha*_2T)] \text{ by the right weak star continuity of } m_2 \\ &= \lim_{\alpha} [((E_\beta*_2S)*_2E_\alpha)*_2T] \\ &= \lim_{\alpha} [((E_\beta*_2S)*_2E_\alpha)*_1T] \text{ since } E_\beta*_2S*_2E_\alpha \text{ is in } \mathcal{A} \text{ for } E_\beta \text{ and } E_\alpha \\ &\text{far enough out} \\ &= \lim_{\alpha} [(E_\beta*_2S)*_2E_\alpha]*_1T = [(E_\beta*_2S)*_2I]*_1T \text{ by weak star continuity} \\ &= (E_\beta*_2(S*_2I))*_1T = (E_\beta*_2S)*_1T \text{ since } S \in \mathcal{N} \\ &= (E_\beta*_1S)*_1T = E_\beta*_1(S*_1T) \end{aligned}$$

and this concludes the proof of part (1).

For part (2)  $\mathcal{N}$  is an algebra because  $(F*_2I)*_2(G*_2I) = (F*_2G)*_2I$  by the associativity of  $m_2$ , and is thus in  $\mathcal{N}$ . Next suppose that  $F \notin \mathcal{N}$ . Then  $F*_2I \neq F$  and yet  $F*_1I = F$  and so  $\mathcal{N}$  is maximal.

3. Applications. For an infinite, Abelian group it is well known [3] that the Arens products never agree on all of  $L(G)**$ .

COROLLARY 3.1. *If  $G$  is a compact Abelian group, then  $L(G)$  satisfies the hypotheses of the above theorem.*

*Proof.* Let  $E_\alpha$  be an approximate identity for  $L(G)$  with weak star limit  $I$ . By [3, Thm. 2.4]  $L(G)$  is a two-sided ideal in  $L(G)**$ . So in particular  $E_\alpha$  will be projecting. It is easily observed that if a Banach algebra  $\mathcal{A}$  is commutative, then  $F*_2A = A*_2F$  for all  $A \in \mathcal{A}$  and  $F \in \mathcal{A}**$ . Then

$$\begin{aligned} I*_1(F*_2I) &= \lim_{\alpha} \lim_{\beta} [E_{\alpha}*_2(F*_2E_{\beta})] \\ &= \lim_{\alpha} \lim_{\beta} [(E_{\alpha}*_2F)*_2E_{\beta}] = \lim_{\alpha} \lim_{\beta} [(F*_2E_{\alpha})*_2E_{\beta}] \\ &= \lim_{\alpha} \lim_{\beta} [F*_2(E_{\alpha}*_2E_{\beta})] = \lim_{\alpha} [F*_2(E_{\alpha}*_2I)] \\ &= \lim_{\alpha} [F*_2E_{\alpha}] = F*_2I. \end{aligned}$$

DEFINITION. A shrinking basis  $\{e_j\}$  for a Banach space is called boundedly growing if there exists an  $\varepsilon > 0$  and a positive integer  $n$  such that  $\|x_1 + \dots + x_n\| < n - \varepsilon$  whenever the  $x_i$ 's have norm 1 and are distinct block basic vectors.

COROLLARY 3.2. *If  $X$  has an unconditionally monotone, boundedly growing bases  $\{e_j\}$  then  $\mathcal{C}$  the algebra of compact linear operators satisfies the hypotheses of the theorem, and  $\mathcal{N}$  will consist of those  $F \in \mathcal{C}**$  for which each of the "rows" of  $F$  are elements of  $X^*$  (as opposed to  $X**$ ).*

*Proof.* The operators  $E_n$ , with ones down the first  $n$  slots of the diagonal and zeroes elsewhere, form an approximate identity for  $\mathcal{C}$ . For any  $F \in \mathcal{C}**$  and integers  $n, m$  we claim that  $E_n*_1F*_1E_m$  is in  $\mathcal{C}$ . To see this first note that for  $f \in \mathcal{C}^*$   $(E_n*_1F*_1E_m)f = E_n[(F*_1E_m)*_1f] = [(F*_1E_m)*_1f]E_n = (F*_1E_m)(f*_1E_n) = F[E_m*_1(f*_1E_n)]$ . But  $E_m*_1(f*_1E_n)$  which is an element of  $\mathcal{C}^*$  has as its matrix, the matrix obtained from  $f$  by replacing by zeroes all rows after the  $n^{\text{th}}$  row and all columns after the  $m^{\text{th}}$  column. This can be observed directly. Thus  $(E_n*_1F*_1E_m)f = \check{C}(f)$  where  $C$  is the compact operator with matrix  $(C_{ij})$  where  $C_{ij} = F(g_{ij})$  and  $g_{ij}$  has matrix with a one in the  $ij^{\text{th}}$  place and zeroes elsewhere. Hence  $E_n*_1F*_1E_m = C$ .

From the proof of [7, Prop. 3.3 and Cor. 4.2] it follows that if  $X$  has an unconditionally monotone, boundedly growing basis then the matrices with a finite number of rows are dense in  $\mathcal{E}^*$ . See the correction at the end of this paper for details. Thus  $I_{*1}F = F$  for any  $F \in \mathcal{E}^{**}$  since  $(I_{*1}F)f = \lim (E_n{}_{*1}F)f = \lim F(f_{*1}E_n)$  and the matrix for  $f_{*1}E_n$  can be obtained from that of  $f$  by replacing with zeroes all rows after then  $n^{\text{th}}$ .

To identify  $\mathcal{N}$ , first note that each  $F \in \mathcal{E}^{**}$  can be regarded as having “rows” which are elements of  $X^{***}$  and “columns” which are elements of  $X^{**}$ . The  $n^{\text{th}}$  “row” of  $F$  is the restriction of  $F$  to the elements of  $\mathcal{E}^*$  whose matrices have zeros outside the  $n^{\text{th}}$  row; “columns” are similarly defined. (Of course, a “row” of  $F$  in this sense does not have a sequence of numbers associated with it.)

Then note that  $(F{}_{*2}I)f = \lim F(f{}_{*2}E_n)$  and recall that  $f{}_{*2}E_n$  has as its matrix the first  $n$  columns of  $f$ . Recall also that the hypotheses imply that the matrices with a finite number of rows are norm dense in  $\mathcal{E}^*$ . Thus  $\lim F(f{}_{*2}E_n) = F(f)$  for all  $f$  in  $\mathcal{E}^*$  if and only if each row of  $F$  is in  $X^*$ , since by hypothesis the basis for  $X$  is shrinking.

**EXAMPLE 3.3.** For  $X = c_0$  or  $X = c_0 \oplus \ell^p$  with  $1 < p < \infty$  the natural basis is boundedly growing. Moreover,  $\mathcal{N}$  is strictly contained between  $\mathcal{B}(X)$  and  $\mathcal{E}^{**}$ , because it will have some elements (with “columns” in  $X^{**}$ ) which won’t be in  $\mathcal{B}(X)$ .

*Correction.* In [7, Props. 3.2 and 3.3] the assumption that  $X$  is reflexive was mistakenly omitted. Of course, the main Theorem 3.2 is not affected, since there  $X$  was uniformly convex. Also, in the proof of [7, Cor. 4.2] it was stated that: If  $X$  has a boundedly growing, unconditionally monotone basis then the matrices with a finite number of rows are dense in  $\mathcal{E}^*$ . Here is a proof of that fact: Suppose the matrices with a finite number of rows are not dense in  $\mathcal{E}^*$ . We will show that this implies that the basis is not boundedly growing.

First note that there exists and  $f \in \mathcal{E}^*$  such that  $f^N$  does not approach 0, where  $f^N$  is the matrix formed from  $f$  by deleting the first  $N$  rows and columns. To see this observe that for  $g \in \mathcal{E}^*$ , if  $g^N \rightarrow 0$  then  $g - g^N$  approaches  $g$ . Thus for  $\lambda > 0$ ,  $\exists K: \|g - (g - g^K)\| < \lambda/2$ . Then since each column of  $g$  can be regarded as an element of  $X^*$  and the basis for  $X$  is shrinking, there exists an  $M$  such that the matrix consisting of the first  $M$  rows of  $g - g^K$  will be within  $\lambda/2$  of  $g - g^K$ . Therefore, since the matrices with a finite number of rows are assumed to be non dense in  $\mathcal{E}^*$ , there must exist an  $f$  for which  $f^N$  does not approach 0. Without loss of generality [7,

Prop. 2.6] we can assume that  $\|f^{N'}\| \downarrow 1$ .

Given  $\varepsilon$  and  $n$ , let  $\delta > 0$ . Pick  $N_1: \|f^{N_1}\| < 1 + \delta$ . Since the basis is shrinking, the finite operators are dense in  $\mathcal{E}$ . Thus there exists an integer  $N'_1 > N_1$  and a finite operator  $T_1$  of norm 1 such that  $T_1$  is concentrated on the manifold  $X_1 = [e_{N_1}, \dots, e_{N'_1}]$  and  $f^{N'_1}(T_1) > 1$ . Let  $N_2 = N'_1 + 1$ . There exists an operator  $T_2$  of norm 1, concentrated on the manifold  $X_2 = [e_{N_2}, \dots, e_{N'_2}]$  such that  $f^{N'_2}(T_2) > 1$ . Repeating this process  $n$  times, we can find  $T_1, \dots, T_n$  such that  $f^{N'_k}(T_k) > 1$ , and the  $T_k$  are concentrated on disjoint basic blocks. Hence

$$\begin{aligned} n &< f^{N'_1}(T_1) + \dots + f^{N'_n}(T_n) = f^{N'_1}(T_1 + \dots + T_n) \\ &\leq \|f^{N'_1}\| \|T_1 + \dots + T_n\| \end{aligned}$$

thus  $n/(1 + \delta) < \|T_1 + \dots + T_n\|$  and there exists an  $x$  of norm 1, where  $x = x_1 + \dots + x_n$  and each  $x_i$  in  $X_i$  such that  $n/(1 + \delta) < \|(T_1 + \dots + T_n)x\| = \|T_1x_1 + \dots + T_nx_n\|$ . However,  $\delta < 0$  was arbitrary. By picking  $\delta$  small enough we can assure that  $\|T_1x_1 + \dots + T_nx_n\|$  is as close to  $n$  as we wish. By unconditional monotonicity, each  $\|x_i\| \leq 1$ . Thus each  $\|T_ix_i\| \leq 1$  and since the  $T_ix_i$  are from disjoint blocks the basis won't be boundedly growing.

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