Pacific Journal of Mathematics

θ-CLOSED SUBSETS OF HAUSDORFF SPACES

RAYMOND FRANK DICKMAN AND JACK RAY PORTER

Vol. 59, No. 2 June 1975

PACIFIC JOURNAL OF MATHEMATICS
Vol. 59, No. 2, 1975

θ-CLOSED SUBSETS OF HAUSDORFF SPACES

R. F. DICKMAN, JR. AND JACK R. PORTER

A topological property of subspaces of a Hausdorff space, called θ-closed, is introduced and used to prove and interrelate a number of different results. A compact subspace of a Hausdorff space is θ-closed, and a θ-closed subspace of a Hausdorff space is closed. A Hausdorff space X with property that every continuous function from X into a Hausdorff space is closed is shown to have the property that every θ-continuous function from X into a Hausdorff space is closed. Those Hausdorff spaces in which the Fomin H-closed extension operator commutes with the projective cover (absolute) operator are characterized. An H-closed space is shown not to be the countable union of θ-closed nowhere dense subspaces. Also, an equivalent form of Martin's Axiom in terms of the class of H-closed spaces with the countable chain condition is given.

1. Preliminaries. For a space X and $A \subseteq X$, the *θ-closure* of A, denoted as $\mathrm{cl}_\theta A$, is $\{x \in X$: every closed neighborhood of x meets $A\}$. The subset A is θ-closed if $\mathrm{cl}_\theta A = A$. Similarly, the *θ-interior* of A, denoted as $\mathrm{int}_\theta A$, is $\{x \in X$: some closed neighborhood of x is contained in $A\}$. Clearly, $\mathrm{cl}_\theta A$ is closed and $\mathrm{int}_\theta A$ is open. The concept of θ-closure was introduced by Velicko [15] and used by the authors in [3]. Also introduced in [15] is the concept of a H-set: a subset A of a Hausdorff space X is an *H-set* if every cover of A by sets open in X has a finite subfamily whose closures in X cover A; this concept was independently introduced in [11] and called *H-closed relative to X*. An *open filter* is a filter with a filter base consisting of open sets. A maximal open filter is called an *open ultrafilter*. A filter $\mathscr{F}$ on X is said to be *free* if $\mathrm{ad}_X \mathscr{F} \neq \varnothing$, otherwise, $\mathscr{F}$ is said to be *fixed*. A subset A of X is *far from the remainder* (f.f.r.) [1] in X if for every free open ultrafilter $\mathscr{U}$ on X, there is open $U \in \mathscr{U}$ such that $\mathrm{cl}_X U \cap A = \varnothing$; a subset A of X is *rigid in X* [3] if for every filter base $\mathscr{F}$ on X such that $A \cap \bigcap \{\mathrm{cl}_\theta F: F \in \mathscr{F}\} = \varnothing$, there is open set U containing A and $F \in \mathscr{F}$ such that $\mathrm{cl}\, U \cap F = \varnothing$. The following facts are used in the sequel:

(1.1) In $A \subseteq B \subseteq X$ and A is θ-closed in X, then A is θ-closed in B.

(1.2) A compact subset of a Hausdorff space is θ-closed.

(1.3) [15] *A* θ-closed subset of an H-closed space is an H-set.

(1.4) [3] Let A be a subset of a space X. The following are equivalent:

(a) A is rigid in X.

(b) For any filter base $\mathscr{F}$ on X, if $A \cap \cap \{\mathrm{cl}_\theta F: F \in \mathscr{F}\} = \varnothing$, then for some $F \in \mathscr{F}$, $A \cap \mathrm{cl}_\theta F = \varnothing$.

(c) For each cover $\mathscr{A}$ of A by open subsets of X, there is a finite subfamily $\mathscr{B} \subseteq \mathscr{A}$ such that $A \subseteq \mathrm{int\,cl}(\cup \mathscr{B})$.

(d) For every open filter $\mathscr{G}$ on X such that $A \cap \cap \{\mathrm{cl}\, U: U \in \mathscr{G}\} = \varnothing$, there is $U \in \mathscr{G}$ such that $A \cap \mathrm{cl}\, U = \varnothing$,

(1.5) [3] Disjoint rigid subsets in a Hausdorff space can be separated by disjoint open sets.

(1.6) [3] If A is rigid in X, then A is f.f.r. in X.

Since any closed subset of a regular Hausdorff space is θ-closed and since there are regular Hausdorff spaces with noncompact closed subsets, then the converse of 1.2 is false. In [3], it was shown that every rigid subset of a Hausdorff space is an H-set. Thus, the converse of 1.3 is false since the subset X in the space Y described in Example 1.1 in [3] is rigid in Y but is not θ-closed in Y. On the other hand, by Theorem 4 in [15] a subset of an H-closed, Urysohn space is θ-closed if and only if it is an H-set. Since an H-closed regular space is compact, then a subset of an H-closed, regular space is θ-closed if and only if it is compact. By 1.2 and 1.3, the concept of "θ-closedness" is similar to the concept of "H-closure" in the sense that both are bracketed by the concepts of "compactness" and "H-set".

Also, needed in the sequel is a few definition about semiregularity, θ-continuity, and extensions. For a space X, X_s is used to denote X plus the topology generated by the regular-open subsets (a subset is regular-open if it is the interior of the closure of itself). A space X is *semi-regular* if $X = X_s$; in particular, $(X_s)_s = X_s$.

A function $f: X \to Y$, where X and Y are spaces, is *θ-continuous* if for each $x \in X$ and open subset U of $f(x)$, there is an open subset V of x such that $f(\mathrm{cl}\, V) \subseteq \mathrm{cl}\, U$. The Katětov extension [9] (resp. Fomin extension [5]) of a Hausdorff space X is denoted as κX (resp. σX); these H-closed extensions are studied in [12, 13]. In [11], it is shown that if Y is an H-closed extension of X, then there is a continuous surjection $f: \kappa X \to Y$ such that $f(x) = x$ for $x \in X$.

2. θ-closed subsets of H-closed spaces.

For a space X and a subset $A \subseteq X$, we will let X/A denote the set X with A identified to a point and endowed with the quotient topology.

(2.1) Let X be a Hausdorff space and $A \subseteq X$. The following are equivalent:

(a) A is θ-closed in X.

(b) X/A is Hausdorff.

(c) A is the point-inverse of a continuous function from X into a Hausdorff space.

(d) A is the point-inverse of a θ-continuous function from X into a Hausdorff space.

Proof. The proof of the equivalence of (a) and (b) is straightforward to prove. Clearly, (b) implies (c) and (c) implies (d). To show (d) implies (a), let $f: X \to Y$ be a θ-continuous function into a Hausdorff space Y, $A = f^{-1}(y)$ for some $y \in Y$, and $x \notin A$. There is open set U of $f(x)$ in Y such that $y \notin \mathrm{cl}\, U$. Since there is open set V of x such that $f(\mathrm{cl}\, V) \subseteq \mathrm{cl}\, U$, then $\mathrm{cl}\, V \cap A = \varnothing$.

(2.2) Let X be a Hausdorff space and $A \subseteq X$. The following are equivalent:

(a) A is θ-closed in κX.

(b) A is rigid in X.

(c) A is f.f.r. in X and A is θ-closed in X.

Proof. (*a*) *implies* (*b*). Let $\mathscr{A}$ be a cover of A by open subsets of X. For $p \in \kappa X \backslash A$, let U_p be an open subset of κX containing p such that $\mathrm{cl}_{\kappa X} U_p \cap A = \varnothing$. There is a finite subset $\mathscr{B} \subseteq \mathscr{A}$ and finite subset $B \subseteq \kappa X \backslash A$ such that

$$\kappa X = \bigcup \{\mathrm{cl}_{\kappa X} U_p \colon p \in B\} \cup \bigcup \{\mathrm{cl}_{\kappa X} V \colon V \in \mathscr{B}\} .$$

Thus, $A \subseteq X \backslash \bigcup \{\mathrm{cl}_X (U_p \cap X) \colon p \in B\} \subseteq \bigcup \{\mathrm{cl}_X V \colon V \in \mathscr{B}\}$, and by 1.4, A is rigid in X.

(*b*) *implies* (*c*). By 1.6, A is f.f.r. in X. Suppose $p \in X \backslash A$. Then A and p are disjoint rigid subsets and, by 1.5, can be separated by disjoint open sets. Hence, A is θ-closed in X.

(*c*) *implies* (*a*). Let $p \in X \backslash A$. Since A is θ-closed in X, then there is an open set U in X such that $p \in U$ and $\mathrm{cl}_X U \cap A = \varnothing$. Since X is open in κX, then U is open in κX and $\mathrm{cl}_{\kappa X} U = \mathrm{cl}_X U \cup B$ where $B = \{q \in \kappa K \backslash K \colon U \in q\}$. Thus, $A \cap \mathrm{cl}_{\kappa X} U = \varnothing$. Suppose $p \in \kappa X \backslash X$ (thus, $p \notin A$). Then p is a free open ultrafilter on X and there is open set $U \in p$ such that $\mathrm{cl}_X U \cap A = \varnothing$. Now, $U \cup \{p\}$ is open in κX and contains p and $\mathrm{cl}_{\kappa X} (U \cup \{p\}) = \mathrm{cl}_X U \cup B$ where B is the same as above. Thus, $A \cap \mathrm{cl}_{\kappa X}(U \cup \{p\}) = \varnothing$.

By 2.2 and 1.1, it follows that a rigid subset of a Hausdorff space is θ-closed in the space.

Let X and Y be Hausdorff spaces and $f: X \to Y$ a continuous function. We say f is *absolutely closed* [17] if f cannot be continuously extended to a proper Hausdorff extension Z of X and is *regular closed* [2] if the image of the closure of an open set is closed. Dickman [2] proved that f is absolutely closed if and only if f is regular closed and point-inverses are f.f.r. in X. By 2.1 and 2.2, this statement converts into the following:

(2.3) Let $f: X \rightarrow Y$ be a continuous where X and Y are Hausdorff spaces. The following are equivalent:

(a) f is absolutely closed.

(b) f is regular closed and point-inverses are f.f.r. in X.

(c) f is regular closed and point-inverses are rigid in X.

Another consequence of 2.2, in combination with 1.5, is the following result.

(2.4) Disjoint θ-closed subsets of an H-closed space are contained in disjoint open subsets.

In [9], Katětov shows that if every closed subset of an Hausdorff space X is H-closed, then X is compact. Similarly, by 6.1.1 in [3], if every closed subset of a Hausdorff space X is rigid, then X is compact. A Hausdorff space X in which every closed subset is an H-set is called *C-compact* [16], and there are noncompact, C-compact spaces [17, Example 2]. The next result will help us prove a property possessed by C-compact spaces.

(2.5) If $f: X \rightarrow Y$ is θ-continuous where X and Y are Hausdorff and if A is H-subset of X, then $f(A)$ is an H-subset of Y.

Proof. Let $\mathscr{C}$ be cover of $f(A)$ by open subsets of Y. For each $a \in A$, there is open set $U_a \in \mathscr{C}$ such that $f(a) \in U_a$. There is an open set V_a of a such that $f(\mathrm{cl}\, V_a) \subseteq \mathrm{cl}\, U_a$. There is finite subset $B \subseteq A$ such that $A \subseteq \bigcup \{\mathrm{cl}\, V_a: a \in B\}$. It follows that $f(A) \subseteq \bigcup \{\mathrm{cl}\, U_a: a \in B\}$.

A Hausdorff space X is called *functionally compact* [4] if every continuous function from X into a Hausdorff space is closed. A C-compact space is functionally compact [4], and by 2.5, every θ-continuous function from a C-compact space into a Hausdorff space is closed. Clearly, a Hausdorff space X in which every θ-continuous function from X into a Hausdorff space is closed, is functionally compact. Surprisingly, the converse is true. We need the following definition and theorem to prove the converse.

A Hausdorff space X is called *θ-seminormal* [6] if for every θ-closed subset $A \subseteq X$ and every open set G containing A, there is regular open set R such that $A \subseteq R \subseteq G$.

(2.6) [6] A Hausdorff space is functionally compact if and only if it is H-closed and θ-seminormal.

(2.7) A Hausdorff space X is functionally compact if and only if every θ-continuous function from X into a Hausdorff space is closed.

Proof. The proof of one direction is obvious. To prove the converse, suppose X is functionally compact and $f: X \to Y$ is a θ-continuous function where Y is Hausdorff. To prove f is closed, suppose $B \subseteq X$ is a closed subset and $p \in \mathrm{cl}_Y f(B)$. By Corollary 2.1 in [4], X is H-closed. By 2.5, $f(X)$ is H-subset and, hence, closed in Y. So, $p \in f(X)$. Assume, by way of contradiction, that $p \notin f(B)$. So, $f^{-1}(p) \subseteq X\backslash B$. By 2.1, $f^{-1}(p)$, is θ-closed in X and by 2.6, there is regular open set R such that $f^{-1}(p) \subseteq R \subseteq X\backslash B$. Now, $B \subseteq X\backslash R$, but $X\backslash R$, the closure of an open set, is H-closed by 1.2 in [9]. By 2.5, $f(X/R)$ is an H-set, and hence, closed. This leads to a contradiction as $f(B) \subseteq f(X\backslash R)$ and $p \notin f(X\backslash R)$.

Problem. Characterize those Hausdorff spaces X with this property: every weakly θ-continuous function from X into a Hausdorff space is closed. A function $f: X \to Y$ is weakly θ-continuous [5, 3] if for every $x \in X$ and open set V of $f(x)$, there is open set U of x such that $f(U) \subseteq \mathrm{cl}\, V$. Every compact Hausdorff space has this property; we are unaware of any noncompact Hausdorff space with this property.

3. θ-closure in H-closed extensions. With the use of the next result, we will derive a new characterization of those subsets of a Hausdorff space X that are θ-closed in κX.

(3.1) If Y is a Hausdorff extension of X and A is a rigid subset of X, then A is rigid in Y.

Proof. By 2.2, it suffices to show that A is θ-closed in κY. By 4.4 in [11], there is a continuous surjection $f: \kappa X \to \kappa Y$ such that that $f(x) = x$ for $x \in X$. Since κX is H-closed, then f is absolutely closed. Let $z \in \kappa Y\backslash A$. Then $f^{-1}(z)$ is rigid in κX by 2.3. Using that $\kappa(\kappa X) = \kappa X$, it follows by 2.2 that A is rigid in κX. By 1.5, there is open set U in κX such that $A \subseteq U$ and $\mathrm{cl}_{\kappa X} U \cap f^{-1}(z) = \varnothing$. Let $W = \kappa Y\backslash f(\mathrm{cl}_{\kappa X} U)$. Since f is regular closed by 2.3, W is open; also, $z \in W$. Now, $f^{-1}(W)$ is open in X and $f^{-1}(W) \cap \mathrm{cl}_{\kappa X} U = \varnothing$. So $\mathrm{cl}_{\kappa X} f^{-1}(W) \cap A = \varnothing$. Since $A = f^{-1}f(A)$ by 1.8 in [13], $f(\mathrm{cl}_{\kappa X} f^{-1}(W)) \cap A = \varnothing$. Again, by 2.3, $f(\mathrm{cl}_{\kappa X} f^{-1}(W))$ is closed implying $\mathrm{cl}_{\kappa Y} W \cap A = \varnothing$. Thus, A is θ-closed in κY.

(3.2) Let X be a Hausdorff space and $A \subseteq X$. The following are equivalent:

(a) A is θ-closed in κX.

(b) A is θ-closed in every Hausdorff extension of X.

(c) A is θ-closed in σX.

(d) A is θ-closed in some H-closed extension of X.

Proof. By 3.1 and 2.2, (a) implies (b). Clearly, (b) implies (c) and (c) implies (d).

(d) implies (a). Suppose A is θ-closed in an H-closed extension Y of X. By 4.4 in [11], there is a continuous surjection $f: \kappa X \to Y$ such that $f(x) = x$ for $x \in X$. Let $z \in \kappa X \backslash A$. Since $f^{-1}f(A) = A$ by 1.8 in [13], then $f(z) \in Y \backslash A$. So, $\{f(z)\}$ and A are contained in disjoint open sets. By the continuity of f, $\{z\}$ and A are contained in disjoint open sets. So, A is θ-closed in κX.

It is not possible to replace "H-closed" in 3.4(d) by "Hausdorff" as a subset A of X can be θ-closed in some Hausdorff extension Y of X while A is not θ-closed in κX. For example, if X is Hausdorff but not H-closed, then X is θ-closed in the trival Hausdorff extension X *of* X, but X is not θ-closed in κX.

For each Hausdorff space X, we let θX denote $\{q: q$ is open ultrafilter on $X\}$. For each open set U in X, let $G(U)$ denote $\{q \in \theta X: U \in q\}$; $\{G(U): U$ open in $X\}$ forms a basis for an extremally disconnected, compact Hausdorff topology on θX [8]. By 5.2 in [13] there is a θ-continuous, perfect irreducible function $\pi: \theta X \to \sigma X$ defined by $\pi(q) = q$ for each free open ultrafilter q on X and $\pi(q) = x$ where x is the unique convergent point of the fixed open ultrafilter q.

(3.3) Let X be a Hausdorff space and U, V open subsets of X.

(a) $G(U) \cap G(V) = G(U \cap V)$ and $G(U) \cup G(V) = G(U \cup V)$.

(b) If $x \in X$ and $\pi^{-1}(x) \subseteq G(U)$, then $x \in \operatorname{int}_X \operatorname{cl}_X U$.

(3.4) If X is a Hausdorff space and $A \subseteq X$, then $\pi^{-1}(A)$ is compact if and only if A is θ-closed in κX.

Proof. Suppose $\pi^{-1}(A)$ is compact. By 3.2, it suffices to show A is θ-closed in σX. Suppose $y \in \sigma X \backslash A$. By the compactness of $\pi^{-1}(A)$ and $\pi^{-1}(y)$, the Hausdorffness of θX, and 3.3(a), there are open sets U and V in X such that $\pi^{-1}(A) \subseteq G(U)$, $\pi^{-1}(y) \subseteq G(V)$, and $G(U) \cap G(V) = \varnothing$. Now, by 3.3.(b), $A \subseteq \operatorname{int}_X \operatorname{cl}_X U$ and $y \in \operatorname{int}_X \operatorname{cl}_X V$. Since $\varnothing = G(U) \cap G(V) = G(U \cap V)$ and since every nonempty open set is contained in some open ultrafilter, then $U \cap V = \varnothing$. By 2.14 in [11], $\operatorname{int}_X \operatorname{cl}_X U \cap \operatorname{int}_X \operatorname{cl}_X V = \varnothing$. Thus, A and y are contained in disjoint open sets in X and by 4.1(c) in [11], in κX.

Conversely, suppose A is θ-closed in κX and, hence, by 3.2, θ-closed in σX. It suffices to show $\pi^{-1}(A)$ is closed in θX. Let $y \in \theta X \backslash \pi^{-1}(A)$. Then $\pi(y) \notin A$, and there is open neighborhood U of $\pi(y)$ in σX such that $\mathrm{cl}_{\sigma X} U \cap A = \varnothing$. So $\pi^{-1}(A) \cap \pi^{-1}(\mathrm{cl}_{\sigma X} U) = \varnothing$. But $y \in \pi^{-1}(\pi(y)) \subseteqq \mathrm{int}_\theta \pi^{-1}(\mathrm{cl}_{\sigma X} U)$. Hence, $\pi^{-1}(A)$ is closed in θX.

A liability of the concept "θ-continuity" is that the restriction of a θ-continuous function is not necessarily θ-continuous; this fact is emphasized by 3.4. In particular, if A is a θ-closed, but not H-closed, subspace in an H-closed space Y (e.g., the set of nonisolated points of the space Y of Example 1.1 in [3]), then by 3.4, $\pi^{-1}(A)$ is compact; however, $\pi | \pi^{-1}(A): \pi^{-1}(A) \rightarrow Y$ is not θ-continuous.

For a Hausdorff space X, let EX denote $\{q \in \theta X: q \text{ is fixed}\}$. Now, $\pi^{-1}(X) = EX$ and $\pi | EX: EX \rightarrow X$ is a θ-continuous, perfect, irreducible function (see [8, Th. 10]). Porter and Votaw [13] proved that $\sigma(EX) = E(\sigma X)$ if and only if the set of nonisolated points of EX is compact. We now characterize when σ and E commute in terms of X.

COROLLARY (3.5). *Let X be a Hausdorff space $\sigma(EX) = E(\sigma X)$ if and only if the set of nonisolated points of X is θ-closed in κX.*

Proof. Let A be the set of nonisolated points of X. By Theorem 5.8 in [13], $\pi^{-1}(A)$ is the set of nonisolated points of EX. The stated result now follows immediately by 3.4.

It is known that [10] no H-closed space is the countable union of compact nowhere dense subspaces and that [10] there exists an H-closed space that is the countable union of closed nowhere dense subspaces. An unsolved problem by Mioduszewski [10] is whether some H-closed space is the countable union of H-closed nowhere dense subspaces. We now show that no H-closed space is the countable union of θ-closed nowhere dense subspaces.

(3.6) An H-closed space is not the countable union of θ-closed nowhere dense subspaces.

Proof. Assume, by way of contradiction, that X is an H-closed space and $X = \bigcup \{A_n: n \in N\}$ where each A_n is nowhere dense and θ-closed in X. Since X is H-closed, then $X = \kappa X = \sigma X$ and $\theta X = EX$. By 3.4, $\pi^{-1}(A_n)$ is compact for each $n \in N$. If $\pi^{-1}(A_n)$ contains a nonempty open set, then by the irreducibility and closedness of π [8, Lemma 17], $\pi(\pi^{-1}(A_n)) = A_n$ contains a nonempty open set. So, each $\pi^{-1}(A_n)$ is nowhere dense. Hence, the compact Hausdorff space θX is the countable union of nowhere dense closed subsets, a contradiction.

A space has the *countable chain condition* (c.c.c.) if every family of pairwise disjoint nonempty open sets is countable. One of the equivalent forms (see [14]) of Martin's axiom is the following: Every compact Hausdorff space with ccc is not the union of less than $c(=2^{\aleph_0})$ closed nowhere dense subsets.

(3.7) Martin's axiom is equivalent to

(*) every H-closed space with c.c.c. is not the union of less than c θ-closed nowhere dense subsets.

Proof. Clearly, (*) implies the "compact Hausdorff" form of Martin's axiom. Conversely, suppose Martin's axiom is true and X is an H-closed space with c.c.c. Since X is H-closed, then $\theta X = EX$. Using the fact $\text{int}_X \pi(U) \neq \emptyset$ for every nonempty open set U of EX, it follows that EX has c.c.c. If X is the union of α, a cardinal number, θ-closed nowhere dense subsets, then, as in the proof of 3.6, the compact Hausdorff space EX with c.c.c. is also the union of α closed nowhere dense subsets. Thus, (*) is true.

REFERENCES

1. A. Blaszczyk and J. Mioduszewski, *On factorization of maps through τX*, Coll. Math., **23** (1971), 45-52.

2. R. F. Dickman, Jr., *Regular closed maps*, Proc. Amer. Math. Soc., **39** (1973), 414-416.

3. R. F. Dickman and J. R. Porter, *θ-perfect and θ-absolutely closed functions*, submitted.

4. R. F. Dickman and A. Zame, *Functionally compact spaces*, Pacific J. Math., **31** (1968), 303-311.

5. S. Fomin, *Extensions of topological spaces*, Ann. Math., **44** (1943), 471-480.

6. G. Goss and G. Viglino, *C-compact and functionally compact spaces*, Pacific J. Math., **37** (1971), 677-681.

7. H. Herrlich, *Nicht alle T_2-minimalen Räume sind von 2-Kategorie*, Math. Zeit., **91** (1966), p. 185.

8. S. Iliadis and S. Fomin, *The method of centred systems in the theory of topological spaces*, Uspekhi Mat. Nauk., **21** (1966), 47-76 = Russian Math. Surveys **21** (1966), 37-62.

9. M. Katětov, *Über H-abgeschlossene und bikompakte Räume*, Časopis Pěst. Mat. Fys., **69** (1940), 36-49.

10. J. Mioduszewski, *Remarks on Baire Theorem for H-closed spaces*, Coll. Math., **23** (1971), 39-41.

11. J. R. Porter and J. D. Thomas, *On H-closed and minimal Hausdorff spaces*, Trans. Amer. Math. Soc., **138** (1969), 159-170.

12. J. R. Porter and C. Votaw, *H-closed Extensions I*, General Topology and Appl., **3** (1973), 211-224.

13. J. R. Porter, *H*-closed Extensions *II*, Trans. Amer. Math. Soc., **202** (1975), 193-209.

14. M. E. Rudin, *Interaction of set theory and general topology*, CBMS Regional Con-

ference, August 1974; Laramie, Wyoming.
15. N. V. Velicko, *H-closed topological spaces*, Mat. Sb., **70** (112) (1966), 98-112 = Amer. Math. Soc. Transl. **78** (2) (1968), 103-118.
16. G. Viglino, *C-compact spaces*, Duke Math. J., **36** (1969), 761-764.
17. ———, *Extensions of functions and spaces*, Trans. Amer. Math. Soc., **179** (1973), 61-69.

Received September 18, 1974 and in revised form May 28, 1975. The research of the second author was partially supported by a University of Kansas research grant.

VIRGINIA POLYTECHNIC INSTITUTE AND STATE COLLEGE
AND
THE UNIVERSITY OF KANSAS—LAWRENCE

PACIFIC JOURNAL OF MATHEMATICS

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be in typed form or offset-reproduced, (not dittoed), double spaced with large margins. Underline Greek letters in red, German in green, and script in blue. The first paragraph or two must be capable of being used separately as a synopsis of the entire paper. Items of the bibliography should not be cited there unless absolutely necessary, in which case they must be identified by author and Journal, rather than by item number. Manuscripts, in triplicate, may be sent to any one of the editors. Please classify according to the scheme of Math. Reviews, Index to Vol. **39**. All other communications should be addressed to the managing editor, or Elaine Barth, University of California, Los Angeles, California, 90024.

The Pacific Journal of Mathematics expects the author's institution to pay page charges, and reserves the right to delay publication for nonpayment of charges in case of financial emergency.

100 reprints are provided free for each article, only if page charges have been substantially paid. Additional copies may be obtained at cost in multiples of 50.

The *Pacific Journal of Mathematics* is issued monthly as of January 1966. Regular subscription rate: $72.00 a year (6 Vols., 12 issues). Special rate: $36.00 a year to individual members of supporting institutions.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley, California, 94708.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), 8-8, 3-chome, Takadanobaba, Shinjuku-ku, Tokyo 160, Japan.

Pacific Journal of Mathematics
Vol. 59, No. 2 June, 1975