

# Pacific Journal of Mathematics

**NOTES ON STABLE CURRENTS**

HIROSHI MORI

## NOTES ON STABLE CURRENTS

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**With additional assumptions we answer a conjecture proposed by Lawson and Simons.**

In a work [5], H. B. Lawson, Jr. and J. Simons proved that there exist no stable rectifiable currents on an  $n$ -dimensional unit sphere  $S^n$  in the  $(n + 1)$ -dimensional Euclidean space  $R^{n+1}$ . And concerning to this fact, they conjectured as follows.

*Conjecture.* Let  $M$  be a compact, simply-connected Riemannian manifold with the sectional curvature satisfying  $1/4 < K_s \leq 1$  for all tangent two planes  $\sigma$ . Then there exist no stable rectifiable currents on  $M$ .

We obtain the following results with respect to this conjecture.

Let  $M$  be a compact, connected  $n$ -dimensional Riemannian manifold isometrically immersed in  $(n + 1)$ -dimensional Euclidean space  $R^{n+1}$ . Let  $\delta$  be a constant with  $0 < \delta \leq 1$ , and suppose that at each point  $x$  of  $M$ , with respect to a suitable unit normal, every principal curvature  $\lambda_j$  of  $M$  satisfies

$$\sqrt{\delta} \leq \lambda_j \leq 1$$

$$j = 1, \dots, n.$$

**THEOREM.** *Let  $M$  be a compact, connected Riemannian manifold satisfying the conditions expressed above. Associate to each  $\mathcal{S} \rightarrow \mathcal{R}_p(M)$  a quadratic form  $Q_{\mathcal{S}}$  on  $\mathcal{V}$  as follows. For  $V \in \mathcal{V}$ , let  $\phi_t$  be the flow generated by  $V$  and set*

$$Q_{\mathcal{S}}(V) = \frac{d^2}{dt^2} M(\phi_{t*} \mathcal{S})|_{t=0}.$$

Then for all  $\mathcal{S} \in \mathcal{R}_p(M)$

$$\text{tr } Q_{\mathcal{S}} \leq p(p + 1 - n\delta - \delta)M(\mathcal{S}).$$

(For definitions of  $\mathcal{V}$  and  $\mathcal{R}_p(M)$ , see below.)

**COROLLARY 1.** *Under the assumptions of the Theorem, for all  $p$  with  $1 \leq p < n\delta + \delta - 1$ , any rectifiable  $p$ -current  $\mathcal{S} \leftarrow \mathcal{R}_p(M)$  is not stable. If  $\delta$  satisfies  $n/(n + 1) < \delta \leq 1$ , then any rectifiable  $p$ -current  $\mathcal{S} \in \mathcal{R}_p(M)$  is not stable for each  $p$  with  $1 \leq p \leq n - 1$ .*

**COROLLARY 2.** *Under the assumptions of the Theorem, if  $\delta$  satisfies  $n/(n + 1) < \delta \leq 1$ , then*

$$H_p(M; Z) = H_p(S^n; Z)$$

for each  $p$  with  $0 \leq p \leq n$ . Therefore, in particular, if  $n = 2$  or  $n \geq 5$ , then  $M$  is homeomorphic to  $S^n$ . (This conclusion follows from weaker conditions.)

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1. In the following, we use the same notation as in [5]. Also see [5] for detailed definitions. Let  $M$  be a compact  $n$ -dimensional Riemannian manifold with Riemannian metric  $g$  and canonical connection  $\nabla$ . For a point  $x \in M$ ,  $T_x(M)$  denotes the tangent space of  $M$  at  $x$ . Let  $\mathcal{R}_p(M)$  be the set of all rectifiable  $p$ -currents on  $M$ , where  $0 \leq p \leq n$ . For a current  $\mathcal{S} \in \mathcal{R}_p(M)$ ,  $\vec{\mathcal{S}}_x$  denotes an orientation of  $\mathcal{S}$ , that is, for  $\mathcal{H}^p$ -almost all  $x \in \mathcal{S}$ ,  $\vec{\mathcal{S}}_x$  is a simple  $p$ -vector of unit length which represents  $T_x(\mathcal{S})$ , where  $\mathcal{H}^p$  is the Hausdorff  $p$ -measure on  $M$ . Let  $V$  be a smooth vector field on  $M$ . We define a linear mapping  $\mathcal{S}^V: T_x(M) \rightarrow T_x(M)$  by  $\mathcal{S}^V(X) := \nabla_X V$  for  $X \in T_x(M)$ . This mapping can be extended uniquely as a derivation to  $\Lambda^p T_x(M)$ , that is, as a linear map  $\mathcal{S}^V: \Lambda^p T_x(M) \rightarrow \Lambda^p T_x(M)$  which for simple vectors is given by

$$\mathcal{S}^V(X_1 \wedge \cdots \wedge X_p) = \sum_{i=1}^p X_1 \wedge \cdots \wedge X_{i-1} \wedge \mathcal{S}^V X_i \wedge X_{i+1} \wedge \cdots \wedge X_p.$$

At  $x \in M$ , we define also the linear map  $\nabla_v, \cdot V: T_x(M) \rightarrow T_x(M)$  by  $\Delta_{v, X} V := \nabla_v \nabla_{\tilde{X}} V - \nabla_{\Delta_v \tilde{X}} V$  for  $X \in T_x(M)$ , where  $\tilde{X}$  is any extension of  $X$  to a local vector field. The definition is independent of any extension  $\tilde{X}$ , and the map carries over uniquely as a derivation to  $\Lambda^p T_x(M)$ . Consider a current  $\mathcal{S} \in \mathcal{R}_p(M)$  and a vector field  $V$  on  $M$ . Let  $\phi_t: M \rightarrow M$ ,  $t \in \mathbb{R}$  be the 1-parameter group of diffeomorphisms generated by  $V$ . Then for each  $t \in \mathbb{R}$  we have the rectifiable current  $\phi_{t\#}(\mathcal{S})$  which, as a linear functional on  $\Lambda^p(M)$ , is given by

$$(\phi_{t\#} \mathcal{S})(\omega) = \mathcal{S}(\phi_t^* \omega)$$

for  $\omega \in \Lambda^p(M)$ , where  $\Lambda^p(M)$  is the space of all smooth  $p$ -forms on  $M$ . Let  $\|\cdot\|$  denote the usual norm of a linear functional on  $\Lambda^p(M)$  which has the usual Fréchet topology. Then,

$$\|\phi_{t\#} \mathcal{S}\| = \int_{\mathcal{S}} \sqrt{(\phi_t^* g)(\vec{\mathcal{S}}, \vec{\mathcal{S}})} d\|\mathcal{S}\|$$

where  $\|\mathcal{S}\|$  is a measure on  $M$  defined, by using the  $p$ -dimensional Hausdorff measure  $\mathcal{H}^p$  on  $M$ , as follows: for a Borel set  $X \subset M$ ,  $\|\mathcal{S}\|(X) = \mathcal{H}^p(X \cap \mathcal{S})$ .

DEFINITION. A rectifiable  $p$ -current  $\mathcal{S} \in \mathcal{R}_p(M)$  is said to be stable if, for each vector field  $V$  the following two conditions hold:

$$(s_1) \quad \frac{d}{dt} M(\phi_{t\#} \mathcal{S})|_{t=0} = 0,$$

$$(s_2) \quad \frac{d^2}{dt^2} M(\phi_{t\#} \mathcal{S})|_{t=0} \geq 0.$$

The following is obtained by Lawson and Simons in [5].

PROPOSITION 1. Let  $M$  be a compact Riemannian manifold and  $V$  a vector field on  $M$  with associated flow  $\phi_t$ . Then for any rectifiable  $p$ -current  $\mathcal{S} \in \mathcal{R}_p(M)$ ,

$$(1) \quad \frac{d}{dt} M(\phi_{t\#} \mathcal{S})|_{t=0} = \int_M \langle \mathcal{A} \vec{\mathcal{S}}, \vec{\mathcal{S}} \rangle d\|\mathcal{S}\|,$$

$$(2) \quad \frac{d^2}{dt^2} M(\phi_{t\#} \mathcal{S})|_{t=0} = \int_M \{ -\langle \mathcal{A}^V \vec{\mathcal{S}}, \vec{\mathcal{S}} \rangle^2 + \langle \mathcal{A}^V \mathcal{A}^V(\vec{\mathcal{S}}), \vec{\mathcal{S}} \rangle + |\mathcal{A}^V(\vec{\mathcal{S}})|^2 + \langle \nabla_{V, \vec{\mathcal{S}}} V, \vec{\mathcal{S}} \rangle \} d\|\mathcal{S}\|.$$

REMARK. In the special case that  $V = \nabla f$  (= the gradient of  $f$ ) for some  $f \in C^3(M)$ , the transformation  $\mathcal{A}^V$  is symmetric and (2) simplifies to

$$(2)' \quad \frac{d^2}{dt^2} M(\phi_{t\#} \mathcal{S})|_{t=0} = \int_M \{ -\langle \mathcal{A}^V \vec{\mathcal{S}}, \vec{\mathcal{S}} \rangle^2 + 2|\mathcal{A}^V(\mathcal{S})|^2 + \langle \nabla_{V, \vec{\mathcal{S}}} V, \mathcal{S} \rangle \} d\|\mathcal{S}\|.$$

For future reference we shall write the integrand of (2)' at  $x \in M$  in terms of tangent vectors at  $x$ . Let  $\{\bar{e}_1, \dots, \bar{e}_p, \bar{e}_{p+1}, \dots, \bar{e}_n\}$  be an orthonormal basis of  $T_x(M)$  and set  $\xi = \bar{e}_1 \wedge \dots \wedge \bar{e}_p$ . Then

$$(3) \quad \begin{aligned} & -\langle \mathcal{A}^V \xi, \xi \rangle^2 + 2|\mathcal{A}^V(\xi)|^2 + \langle \nabla_{V, \xi} V, \xi \rangle \\ & = \left\{ \sum_{j=1}^p \langle \mathcal{A}^V(\bar{e}_j), \bar{e}_j \rangle \right\}^2 + 2 \sum_{j=1}^p \sum_{\alpha=p+1}^n \langle \mathcal{A}^V(\bar{e}_j), \bar{e}_\alpha \rangle^2 \\ & \quad + \sum_{i=1}^p \langle \nabla_{V, \bar{e}_i} V, \bar{e}_i \rangle, \end{aligned}$$

where  $|\mathcal{A}^V(\xi)|$  denotes the length of  $p$ -vector  $\mathcal{A}^V(\xi)$ .

2. Now we assume that  $M$  is isometrically immersed in  $(n + 1)$ -

dimensional Euclidean space  $R^{n+1}$  with canonical Riemannian metric  $\langle, \rangle$  and canonical Riemannian connection  $\bar{\nabla}$ . For all local formulas we may consider the isometric immersion  $f$  of  $M$  into  $R^{n+1}$  as an imbedding and thus identify  $x \in M$  with  $f(x) \in R^{n+1}$ . The tangent space  $T_x(M)$  is identified with a subspace of the tangent space  $T_x(R^{n+1})$ . The normal space  $T_x^\perp$  is the subspace of  $T_x(R^{n+1})$  consisting of all  $\zeta \in T_x(R^{n+1})$  which are orthogonal to  $T_x(M)$  with respect to the Riemannian metric  $\langle, \rangle$ . For each point  $x$  of  $M$ , choose a field  $\zeta$  of unit normal vectors defined on a neighborhood  $U$  of  $x$ . Then we have the basic formulas

$$\begin{aligned}\bar{\nabla}_x Y &= \nabla_x Y + \langle A_\zeta X, Y \rangle \zeta \\ \bar{\nabla}_x \zeta &= -A_\zeta X\end{aligned}$$

where  $X$  and  $Y$  are smooth vector fields tangent to  $M$ , and  $A_\zeta$  is a tensor field of type  $(1, 1)$ , called the second fundamental form associated with  $\zeta$ . The Gauss equation expresses the curvature tensor  $R$  of  $M$  as follows.

$$R(X, Y)Z = \langle A_\zeta Y, Z \rangle A_\zeta X - \langle A_\zeta X, Z \rangle A_\zeta Y$$

where  $X, Y$  and  $Z$  are smooth vector fields tangent to  $M$ .

Let  $\delta$  be a constant with  $0 < \delta \leq 1$ , and suppose that at each point  $x$  of  $M$ , with respect to a suitable field  $\zeta$  of unit normals, every principal curvature  $\lambda_j$  of  $M$  satisfies  $\sqrt{\delta} \leq \lambda_j \leq 1, j = 1, \dots, n$ .

REMARK. The above assumption implies that  $M$  has the sectional curvature satisfying  $\delta \leq K_\sigma \leq 1$  for all tangent two planes  $\sigma$ . And from the continuity of the eigen-values of the linear map  $A_\zeta: T_x(M) \rightarrow T_x(M)$ , called the principal curvatures of  $M$ , the above assumption also implies that  $M$  is orientable. Therefore we can choose a global field  $\zeta$  of unit normals on  $M$  which satisfies the above condition, and then we can write  $A_\zeta = A$ .

3. To estimate the left hand side of  $(s_2)$  we begin with the space of functions  $\mathcal{F} = \{\psi | M; \psi: R^{n+1} \rightarrow R \text{ is linear}\}$ , and define

$$\mathcal{V} = \{\nabla \psi; \psi \in \mathcal{F}\}.$$

Then there is a natural isomorphism

$$(4) \quad \mathcal{V} \cong R^{n+1}$$

which associates to  $v \in R^{n+1}$  the gradient of the function  $\psi_v(x) = \langle v, x \rangle$  on  $M$ . This identification introduces a natural inner product on  $\mathcal{V}$ .

To any simple unit  $p$ -vector  $\xi \in \Lambda^p T_x(M)$ , at any  $x \in M$ , we can associate a quadratic form  $Q_\xi$  on  $\mathcal{V}$  as follows. For  $V \in \mathcal{V}$ , let  $\phi_t$

be the flow generated by  $V$ , and define

$$Q_\varepsilon(V) = \frac{d^2}{dt^2} \Big|_{t=0} \phi_{t\#} \xi \Big|_{t=0}.$$

Then we have the following.

PROPOSITION 2. *Under the assumptions as expresses above, we have*

$$\text{tr } Q_\varepsilon \leq p(p + 1 - n\delta - \delta).$$

*Proof.* Suppose  $V \in \mathcal{V}$  corresponds to  $v \in R^{n+1}$  under the isomorphism (4). Then at any  $y \in M$

$$V(y) = v - \langle v, \zeta_y \rangle \zeta_y,$$

and then for  $X \in T_x(M)$ ,  $\nabla_x V = (\bar{\nabla}_x V)^T = \langle v, \zeta_x \rangle AX$ , where  $( )^T$  denotes orthogonal projection  $T_x(R^{n+1}) \rightarrow T_x(M)$ . Thus,

$$(5) \quad \mathcal{A}^V(X) = \nabla_x V = \langle v, \zeta_x \rangle AX.$$

And it follows easily that

$$(6) \quad \nabla_{v,x} V = -\langle V, AV \rangle AX + \langle v, \zeta_x \rangle \nabla_v(A\tilde{X}) - \langle v, \zeta_x \rangle A(\nabla_v \tilde{X})$$

where  $\tilde{X}$  is any extension of  $X$  to a local vector field.

We now choose an orthonormal basis  $\{x_0 = \zeta_x, x_1 = e_1, \dots, x_n = e_n\}$  for  $R^{n+1}$ , where  $e_j$  is an eigenvector corresponding to the eigenvalue  $\lambda_j$  of  $A$ ,  $j = 1, \dots, n$ . Via (4) this fixes an orthonormal basis  $\{V_0, V_1, \dots, V_n\}$  of  $\mathcal{V}$ . It then follows from (5) and (6) that  $\nabla_{v_0} \cdot V_0 = \mathcal{A}^V 1 = \dots = \mathcal{A}^V n = 0$  and  $\mathcal{A}^V o = A$ ,  $\nabla_{v_j} \cdot V_j = -\lambda_j A$ ,  $j = 1, \dots, n$ , as transformations of  $T_x(M)$ . For given simple unit  $p$ -vector  $\xi \in \Lambda^p T_x(M)$ , we can choose an orthonormal basis  $\{\bar{e}_1, \dots, \bar{e}_p, \bar{e}_{p+1}, \dots, \bar{e}_n\}$  of  $T_x(M)$  with  $\xi = \bar{e}_1 \wedge \dots \wedge \bar{e}_p$ . It then follows from (2), (2)', (3) and above formulas that

$$\begin{aligned} \text{tr } (Q_\xi) &= \sum_{i=0}^n Q_\xi(V_i) \\ &= \sum_{i=0}^n \left\{ \left( \sum_{j=1}^p \langle \mathcal{A}^V l \bar{e}_j, \bar{e}_j \rangle \right)^2 + 2 \sum_{j=1}^p \sum_{\alpha=p+1}^n \langle \mathcal{A}^V l \bar{e}_j, \bar{e}_\alpha \rangle^2 \right. \\ &\quad \left. + \sum_{j=1}^p \langle \nabla_{v_l, \bar{e}_j} V_i, \bar{e}_j \rangle \right\} \\ &= \left( \sum_{j=1}^p \langle A \bar{e}_j, \bar{e}_j \rangle \right)^2 + 2 \sum_{j=1}^p \sum_{\alpha=p+1}^n \langle A \bar{e}_j, \bar{e}_\alpha \rangle^2 - \sum_{i=1}^n \sum_{j=1}^p \langle \lambda_i A \bar{e}_j, \bar{e}_j \rangle \\ &= \left( \sum_{j=1}^p \langle A \bar{e}_j, \bar{e}_j \rangle \right)^2 + 2 \sum_{j=1}^p \left( |A \bar{e}_j|^2 - \sum_{i=1}^p \langle A \bar{e}_j, \bar{e}_i \rangle^2 \right) \\ &\quad - \sum_{i=1}^n \sum_{j=1}^p \lambda_i \langle A \bar{e}_j, \bar{e}_j \rangle \end{aligned}$$

$$\begin{aligned}
&= 2 \sum_{j=1}^p |A\bar{e}_j|^2 + \sum_{\substack{i,j=1 \\ i \neq j}}^p (\langle A\bar{e}_i, \bar{e}_i \rangle \langle A\bar{e}_j, \bar{e}_j \rangle - 2\langle A\bar{e}_j, \bar{e}_i \rangle^2) \\
&\quad - \sum_{j=1}^p \langle A\bar{e}_j, \bar{e}_j \rangle^2 - \sum_{l=1}^n \sum_{j=1}^p \lambda_l \langle A\bar{e}_j, \bar{e}_j \rangle.
\end{aligned}$$

By the assumption,  $\sqrt{\delta} \leq \lambda_j \leq 1$ ,  $j = 1, \dots, n$ , we get  $|A\bar{e}_j|^2 \leq 1$ , and  $\sqrt{\delta} \leq \langle A\bar{e}_j, \bar{e}_j \rangle \leq 1$  for  $1, \dots, n$ . Thus we have

$$\begin{aligned}
\text{tr}(Q_\varepsilon) &\leq 2p + p(p-1) - p\delta - np\delta \\
&= p(p+1 - n\delta - \delta).
\end{aligned}$$

Combining Proposition 1 and Proposition 2 we get the theorem and the Corollary 1. And by virtue of the basic theorems on integral currents, we have the Corollary 2, see [2] or [5].

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